Mental exercise: ordering

Say you have 3 students to feed, and 3 box lunches: beef, turkey, and veggie.

How many ways can you feed the students?

Say you have 7 students to feed, and 7 distinct box lunches.

How many ways can you feed the students?

Put the students in an order - any order, just hold the order fixed. Now feed mouths:

7 options for student 1

6 options for student 2

5 options for student 3

4 options for student 4

3 options for student 5

2 options for student 6

1 option left for student 7

Total possibilities: 7 * 6 * 5 * 4 * 3 * 2 * 1

= 5040

Say you have 7 students to feed, and 7 distinct box lunches.

How many ways can you feed the students?

This sequential multiplication is called factorial

7! = 7*6*5*4*3*2*1

Say you have 12 nodes in a brain, 12 nodes in another brain, and you want to see the best match.

How many ways can you look for a match?

Hold one order fixed, in any order

12 options for node 1

11 options for node 2

10 options for node 3

…

12! = 479,001,600

Intimidating?

264 node network:
264!= “Inf”

64 node network:
64!= 1.2e89

Current estimate of atoms in the universe:
1e78 to 1e82

5-card poker hands: ~300M

Question:

Practically, how can we compare orders of things?

How could I match 200 and 200 people into couples?

How can I match brain network nodes?

Up til now, we’ve been counting how many ways things can be ordered.

But sometimes we only care about part of an order.

5-card poker hands: ~300M

You get 5 cards. How many ways might they have appeared?

5*4*3*2*1 = 5! = 120

There are ~300M ways to receive the first 5 cards of a 52-card deck.

But for each 5-card hand, there are 120 ways it could have arrived to you

So there are 300M/120 = ~2.5M distinct hands in poker

Let’s look again at this example

possible dealings of 5 cards: 52*51*50*49*48 = ~300M

Note that this is 52!/47! - the 47! on the bottom would cancel out everything on top but 48-52

So 300M = 52!/47!

But for each 5-card hand, there are 5! = 120 ways it could have arrived to you

So there are 52!/(47!*5!) distinct hands in poker

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Note that there is a flip-side: this is also “52 choose 47”

Which makes sense: if there are 2.5M distinct hands, there are 2.5M distinct non-hands

A symmetry…

Take a different approach for a moment:

Suppose you had 5 things, how many ways can you sample from them?

	Write an example	How many of this kind?
None	NNNNN	1
Singletons	YNNNN	5
Pairs	YYNNN	?
Triplets	YYYNN	?
Quartets	YYYYN	5
Quintets (all)	YYYYY	1

Take a different approach for a moment:

Suppose you had 5 things, how many ways can you sample from them?

Could you immediately know how many ways to sample?

Consider a set of N items

How many subsets are possible (none, singletons, pairs, … all)?

Each item is in or out - a binary decision

For N items, 2^N possibilities

For 5 items, 2^5 = 32 possibilities

1+5+10+10+5+1=32

Ways to sample 5 items? 32

Ways to order 5 items? 5! = 120

5c0 = 1	**5c1 = 5**	**5c2 = 10**	**5c3 = 10**	**5c4 = 5**	**5c5 = 1**
00000	10000	11000	00111	01111	11111
	01000	10100	01011	10111	
	00100	10010	01101	11011	
	00010	10001	01110	11101	
	00001	01100	10011	11110	
		01010	10101		
		01001	10110		
		00110	11001		
		00101	11010		
		00011	11100		

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		00100	10010	01101	11011	
		00010	10001	01110	11101	
		00001	01100	10011	11110	
			01010	10101		
			01001	10110		
			00110	11001		
			00101	11010		
			00011	11100		
Ways to order each sample	**0!=1**	**1!=1**	**2!=2**	**3!=6**	**4!=24**	**5!=120**

Instead of retrospective counting, consider prospective view

first choice	another	another	
0	00	000 001	
	01	010 011	
			...
1	10	100 101	
	11	110 111	

We have just discussed how to map a group of 5 things into a binary code

We have gone from a world of 5 things to a world of 2 things

There were 2^5 = 32 codes to map one to the other (a.k.a, ‘characteristic fn’)

This binary mapping is very common: N items can form 2^N possible groups

A set of N items has 2^N possible subsets (none… all N items) (a.k.a, ‘power set’)

Consider N neurons - how many distinct firing codes may they form? 2^N

Consider N neurons - in how many orders may they each fire once? N!

It matters greatly whether you want order in a grouping

The difference between 2^N possibilities (groupings) vs N! possibilities (orderings):

N	2^N	N!
0	1	1
1	2	1
2	4	2
3	8	6
4	16	24
5	32	120
6	64	720
7	128	5040

When does order matter in our scientific world?

Matching nodes of a graph
Matching people in cohorts

The possible orders for any reasonable size set are virtually endless, uncomputable.
There must be workarounds.

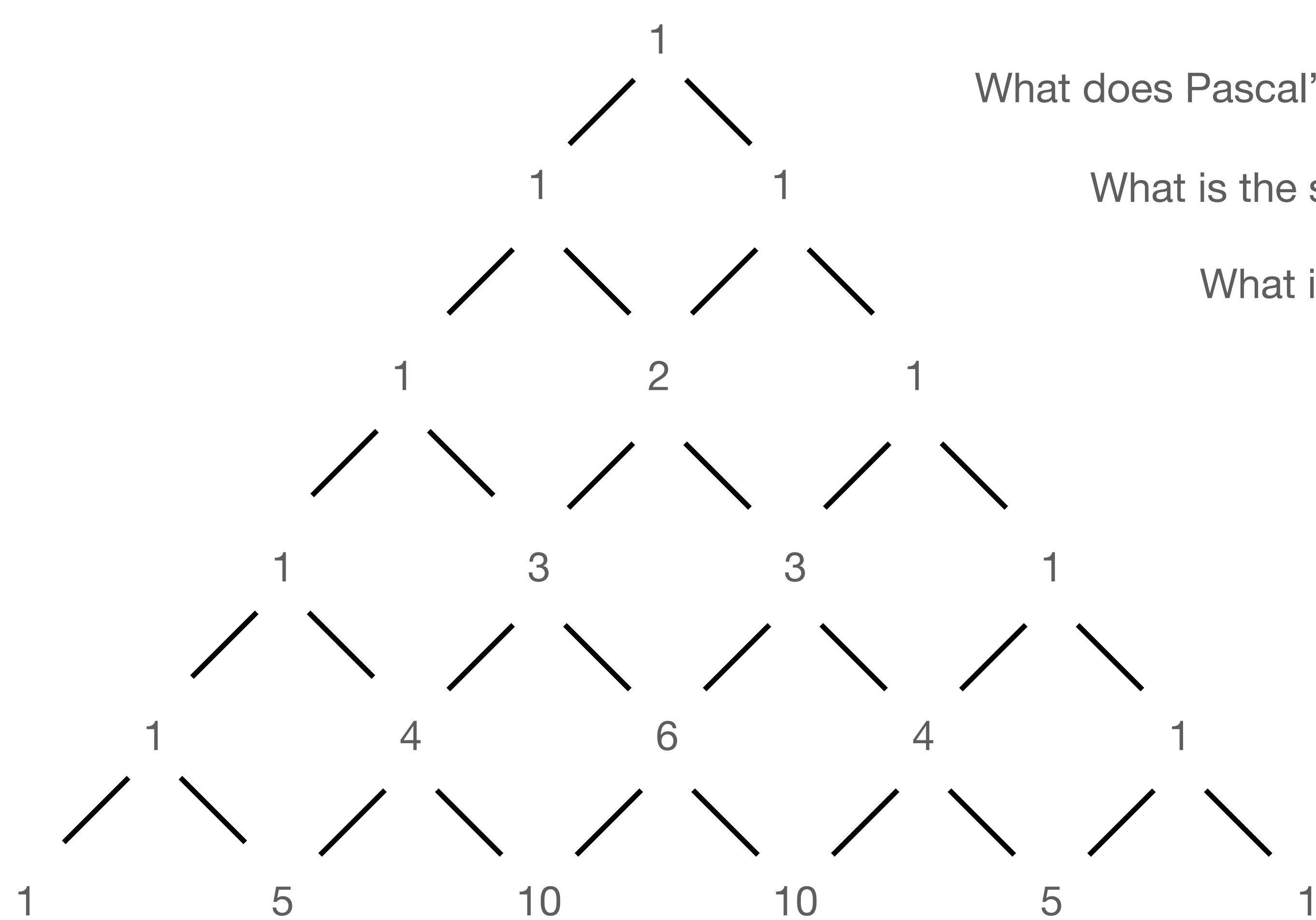
Challenge:

You are a researcher. You have a synthetic brain. It has N neurons.

Cancer will be cured when the N neurons fire in the right order, once each. You can run the brain 3M times.

What is the likelihood you cure cancer as a function of N?

Question



What does Pascal’s triangle have to do with today?

What is the second diagonal (1,2,3…)?

What is the third diagonal?

The fourth?