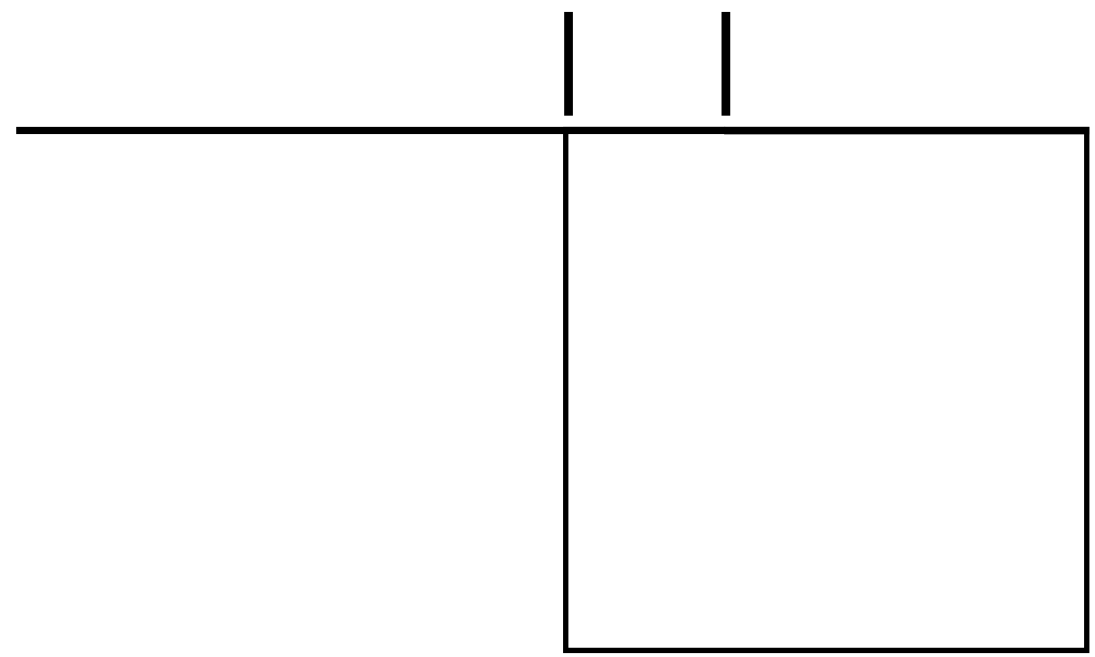
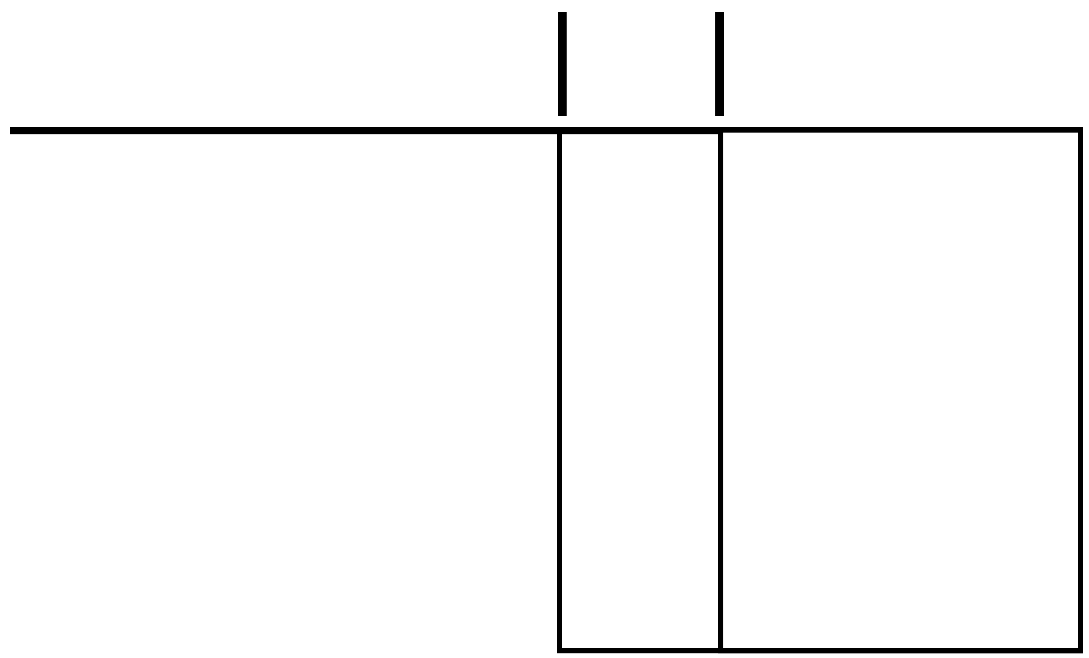
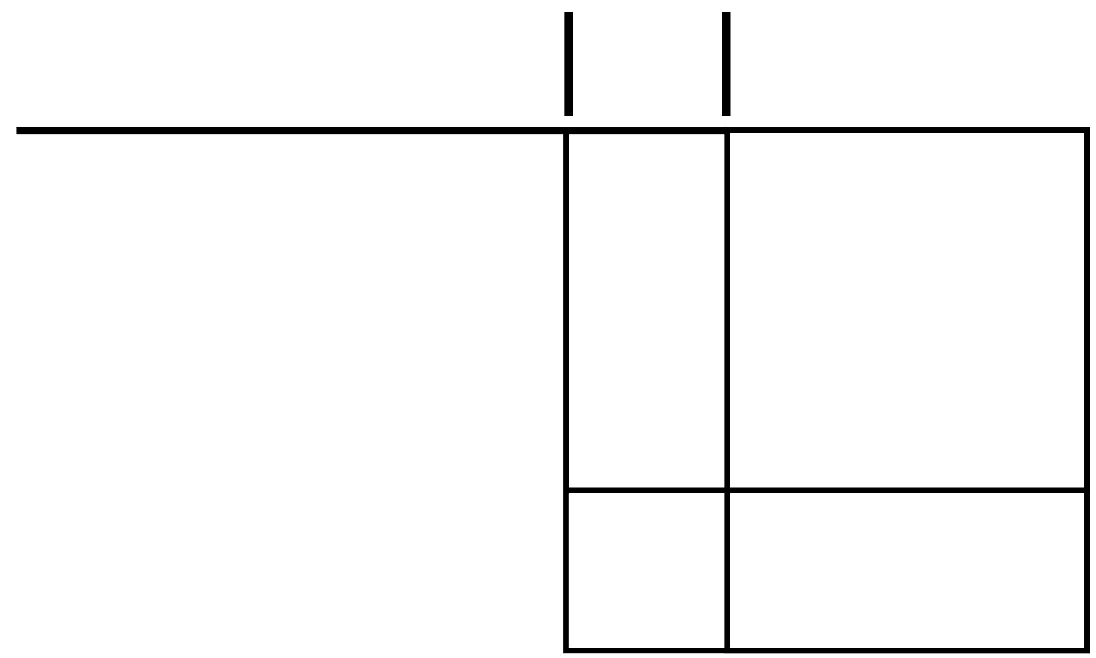
Mental exercise: finding square roots after the apocalypse

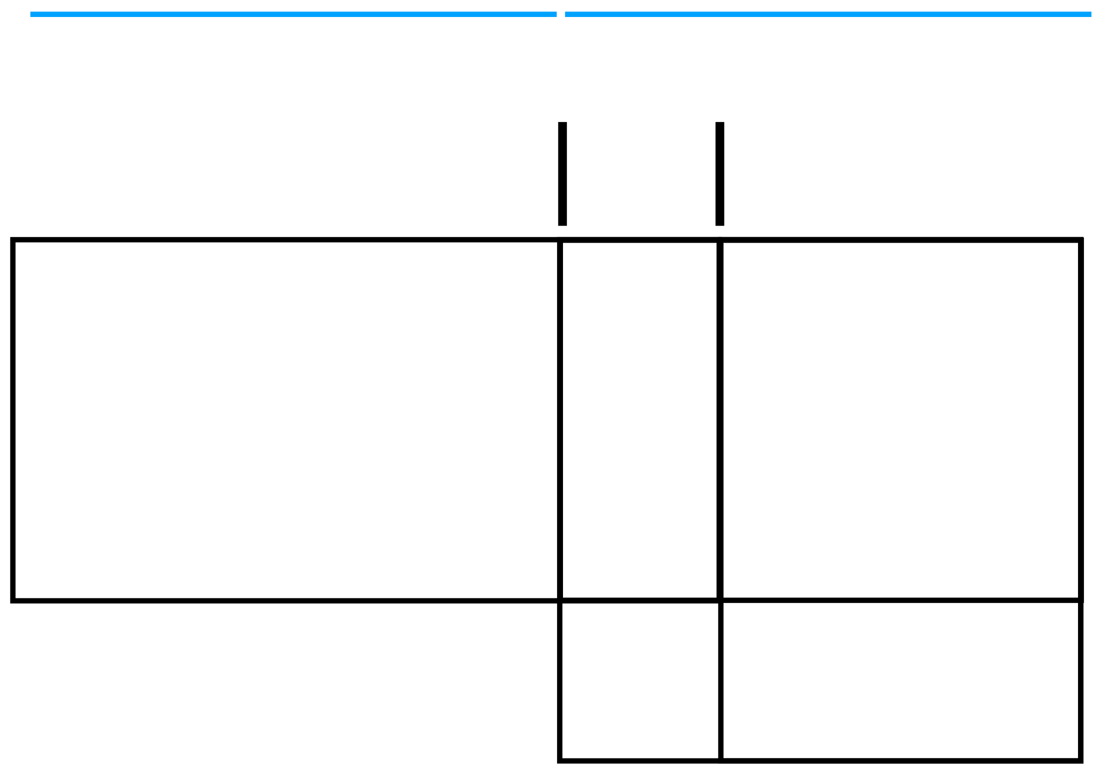
Find:

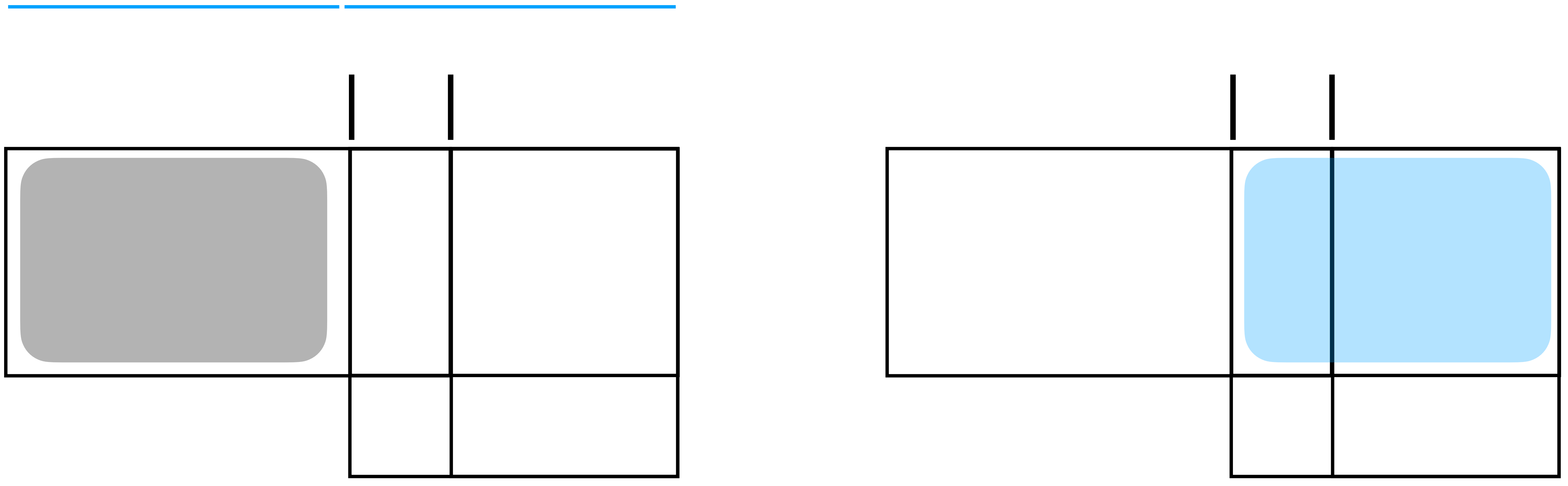
$\sqrt{4}$ $\sqrt{5}$ $\sqrt{6}$

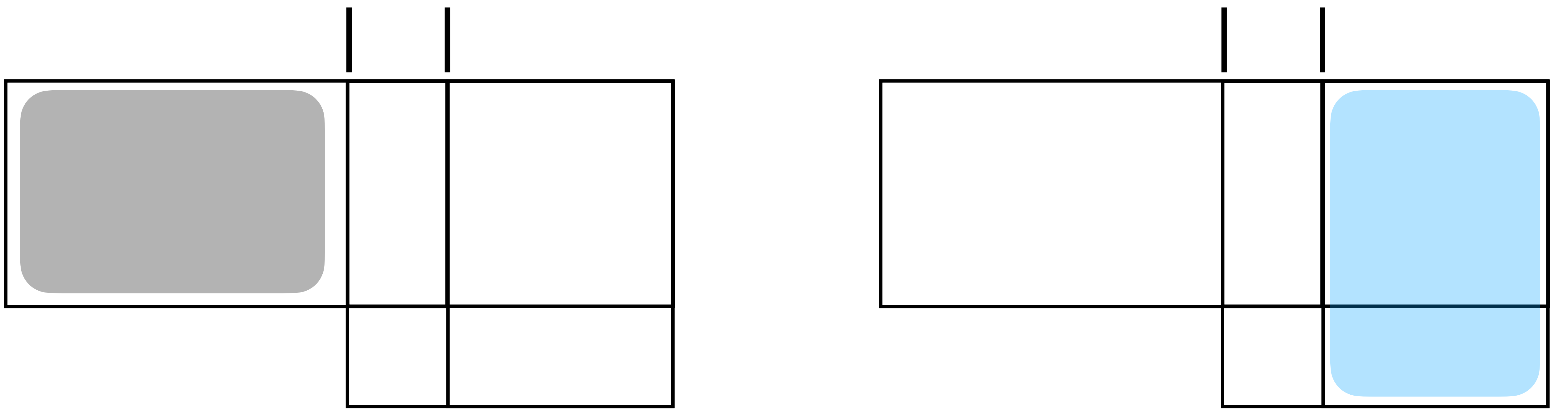


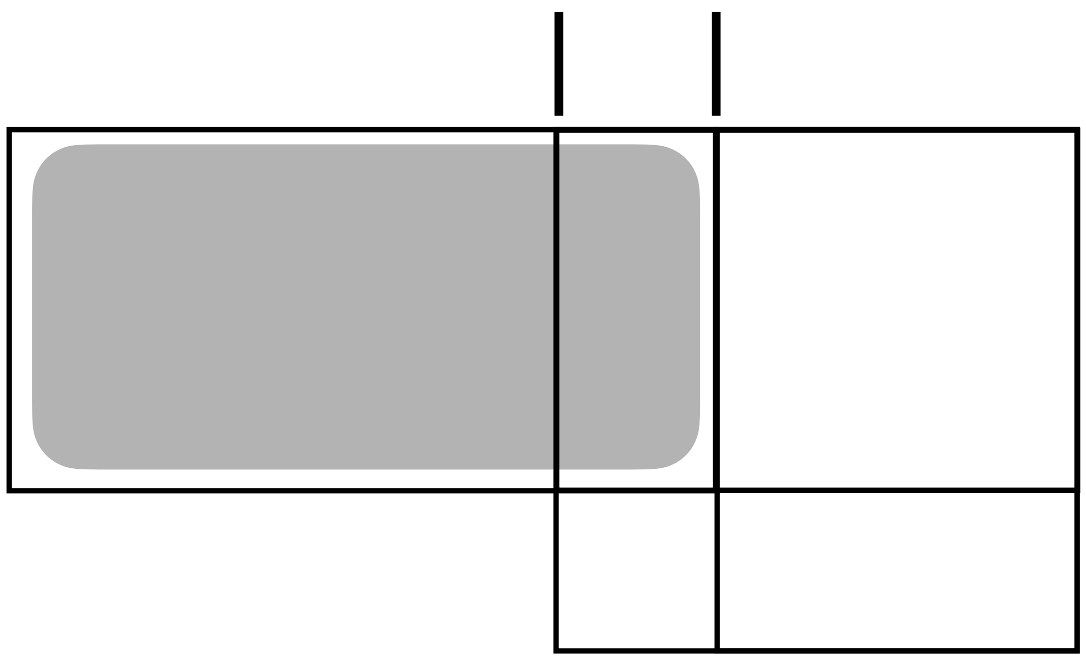


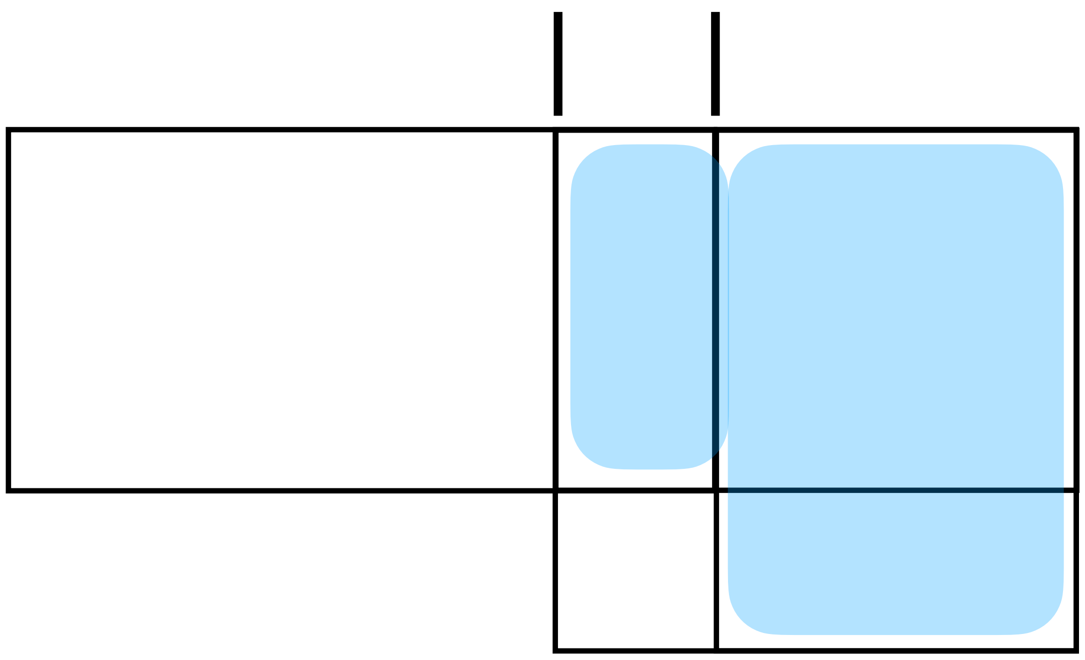


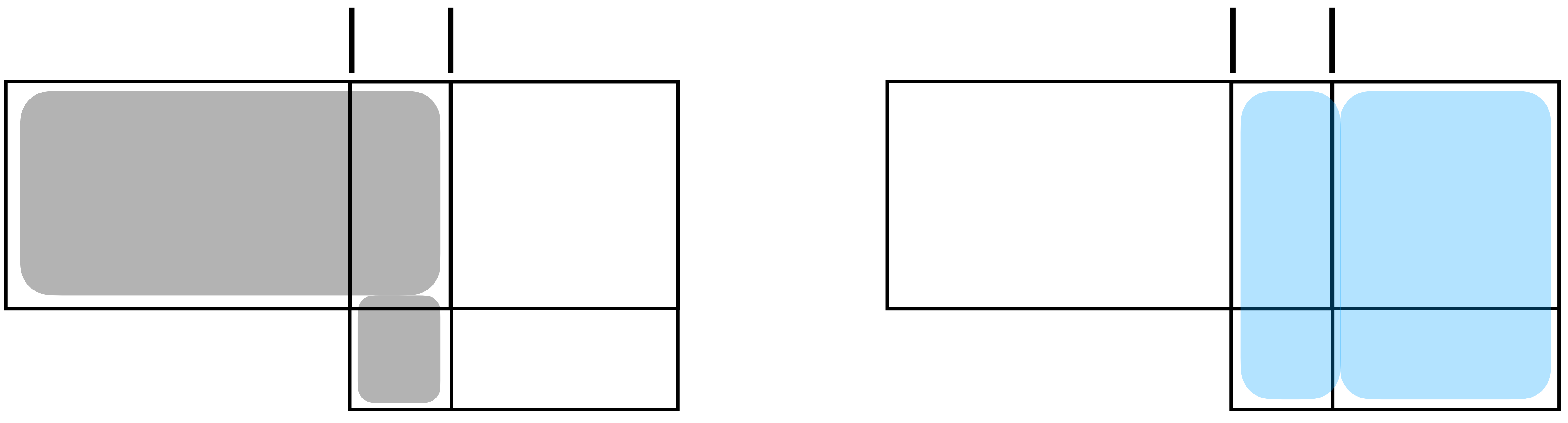


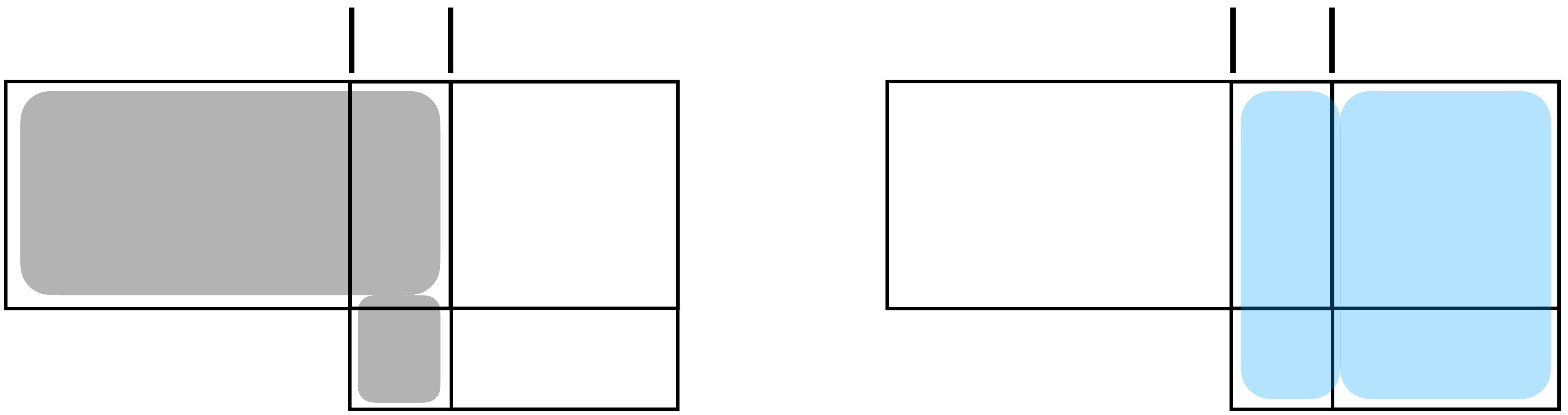




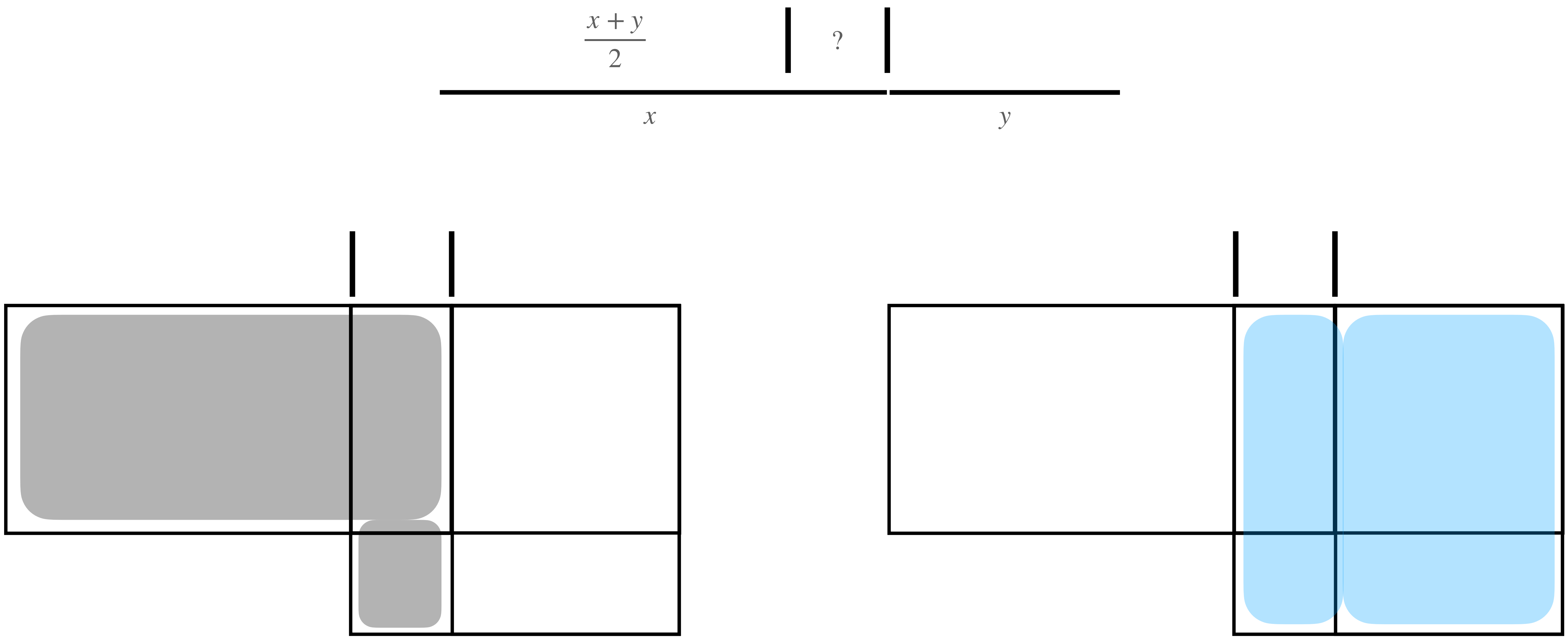








If a line be cut in equal and unequal segments,
the square on the half is equal to
the rectangle of the unequal segments and the square on the segment between cuts



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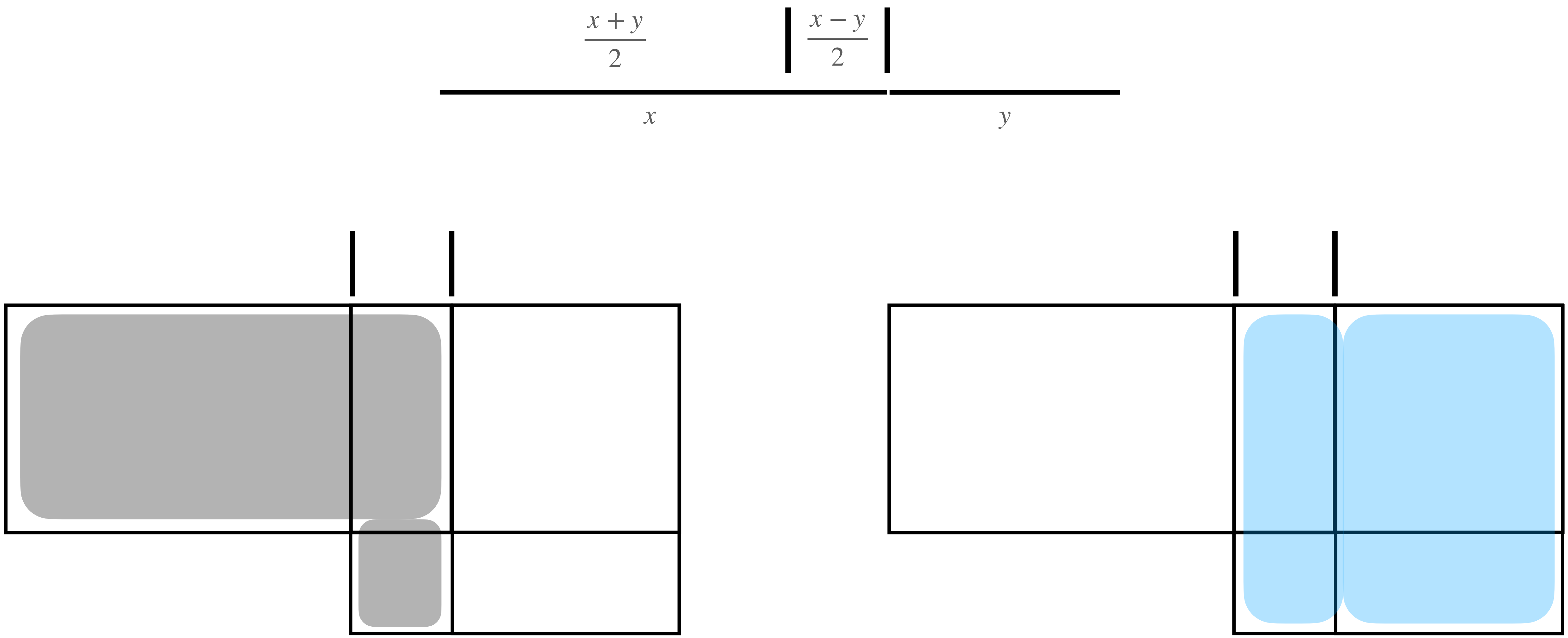
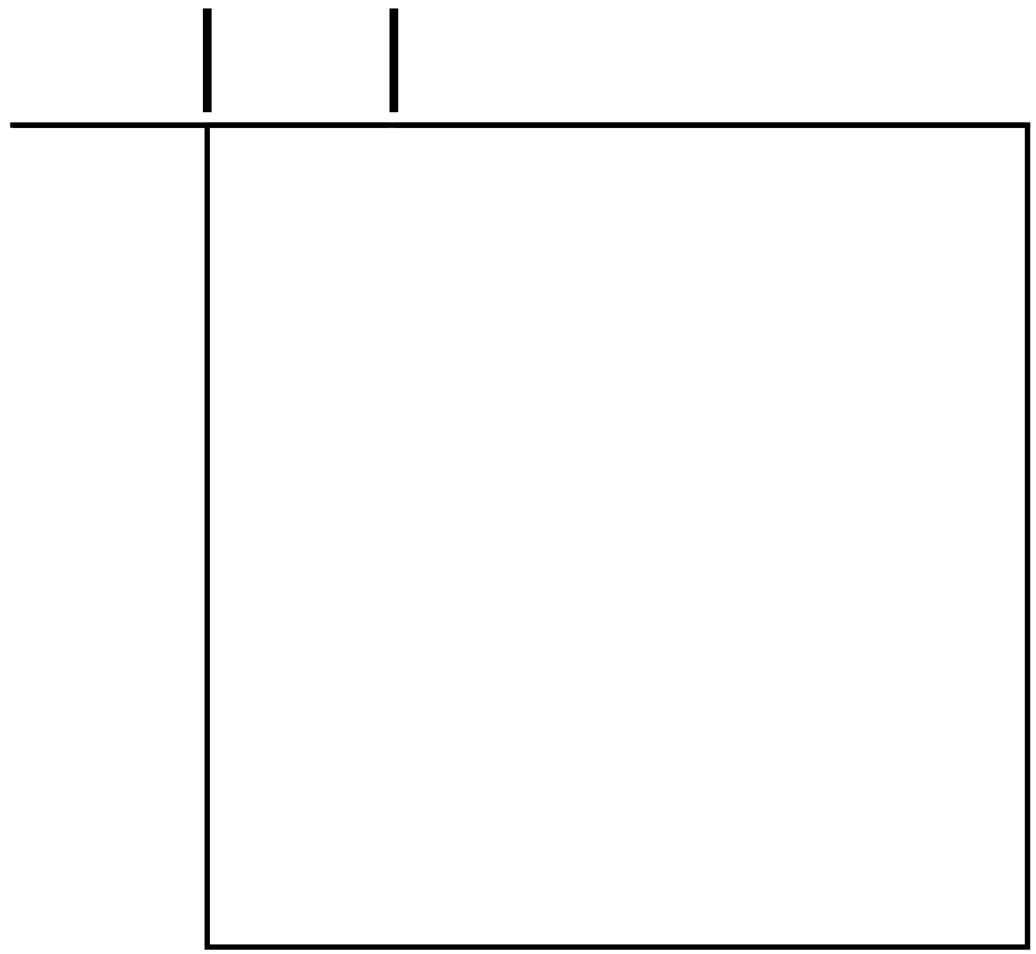
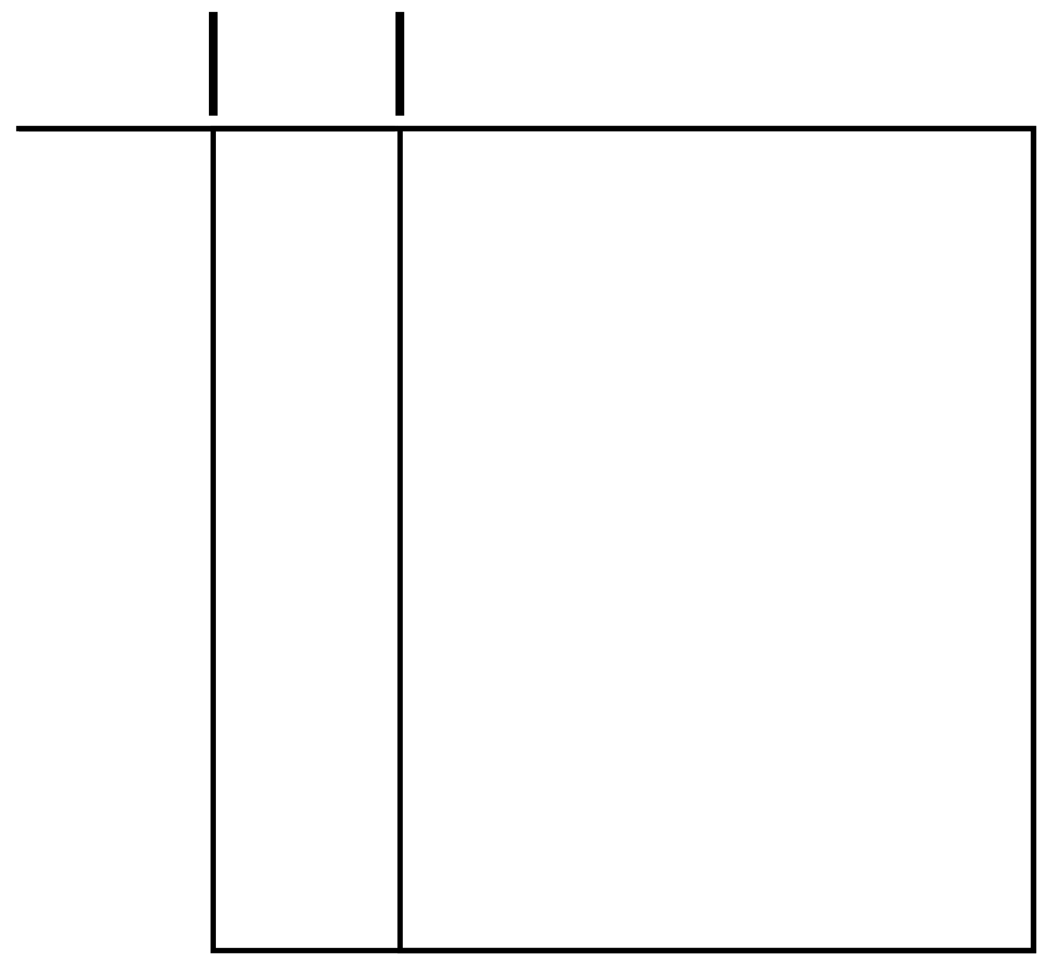
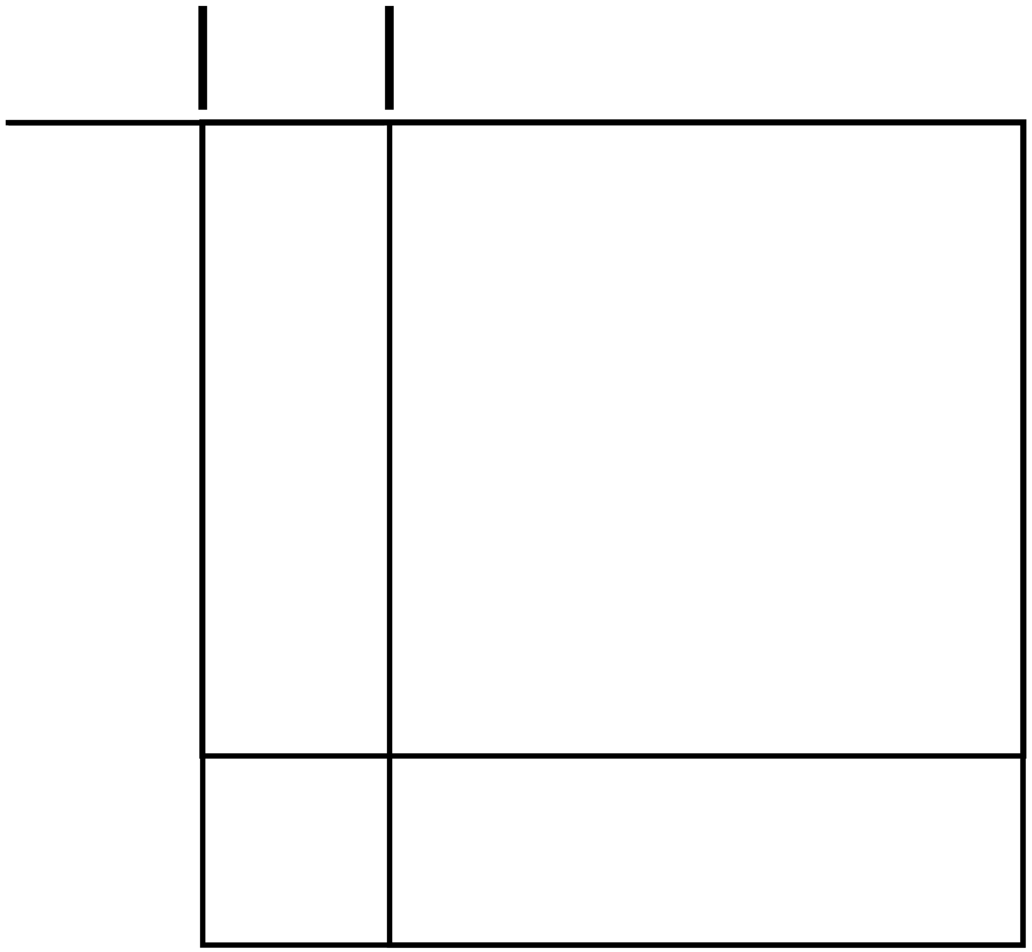


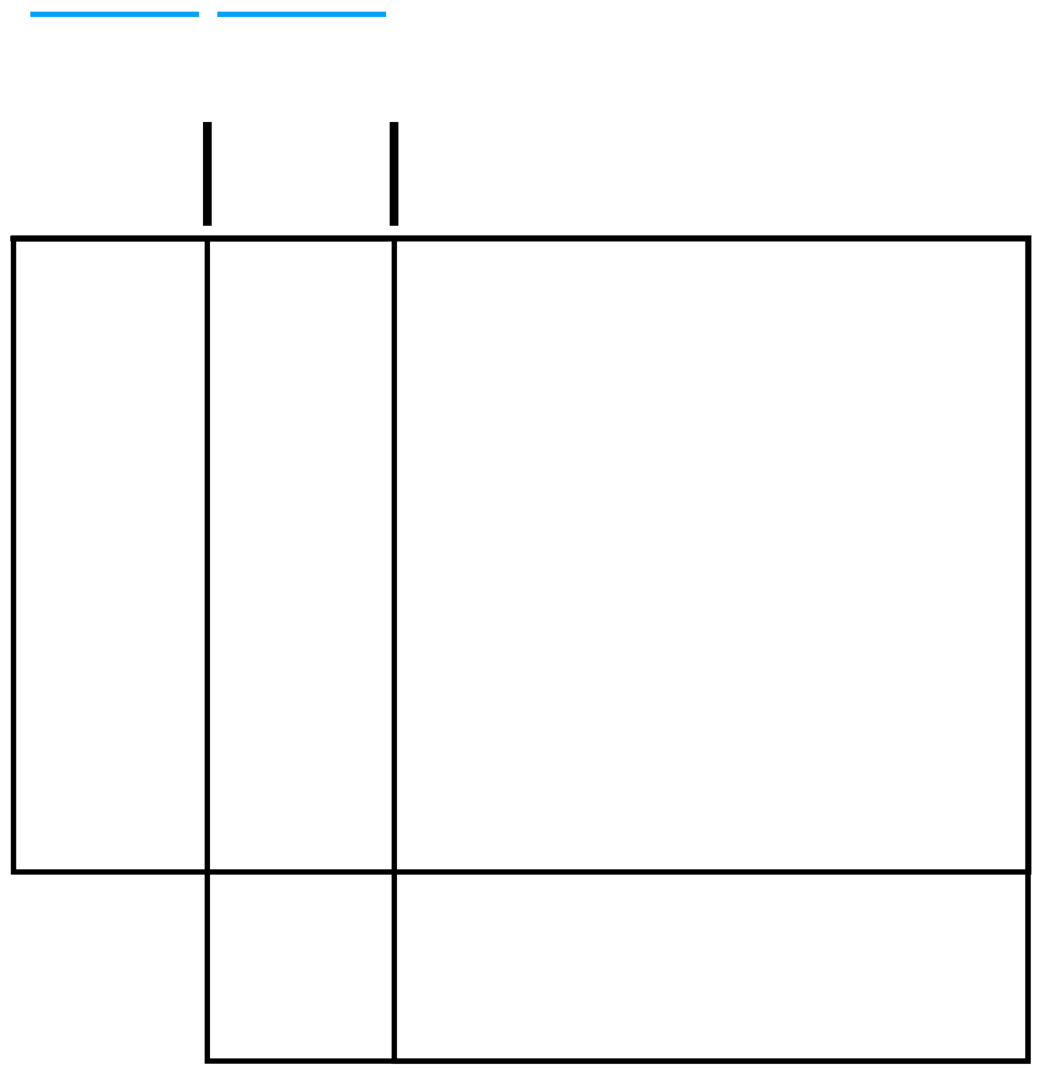
If a line be cut in equal and unequal segments,
the square on the half is equal to
the rectangle of the unequal segments and the square on the segment between cuts

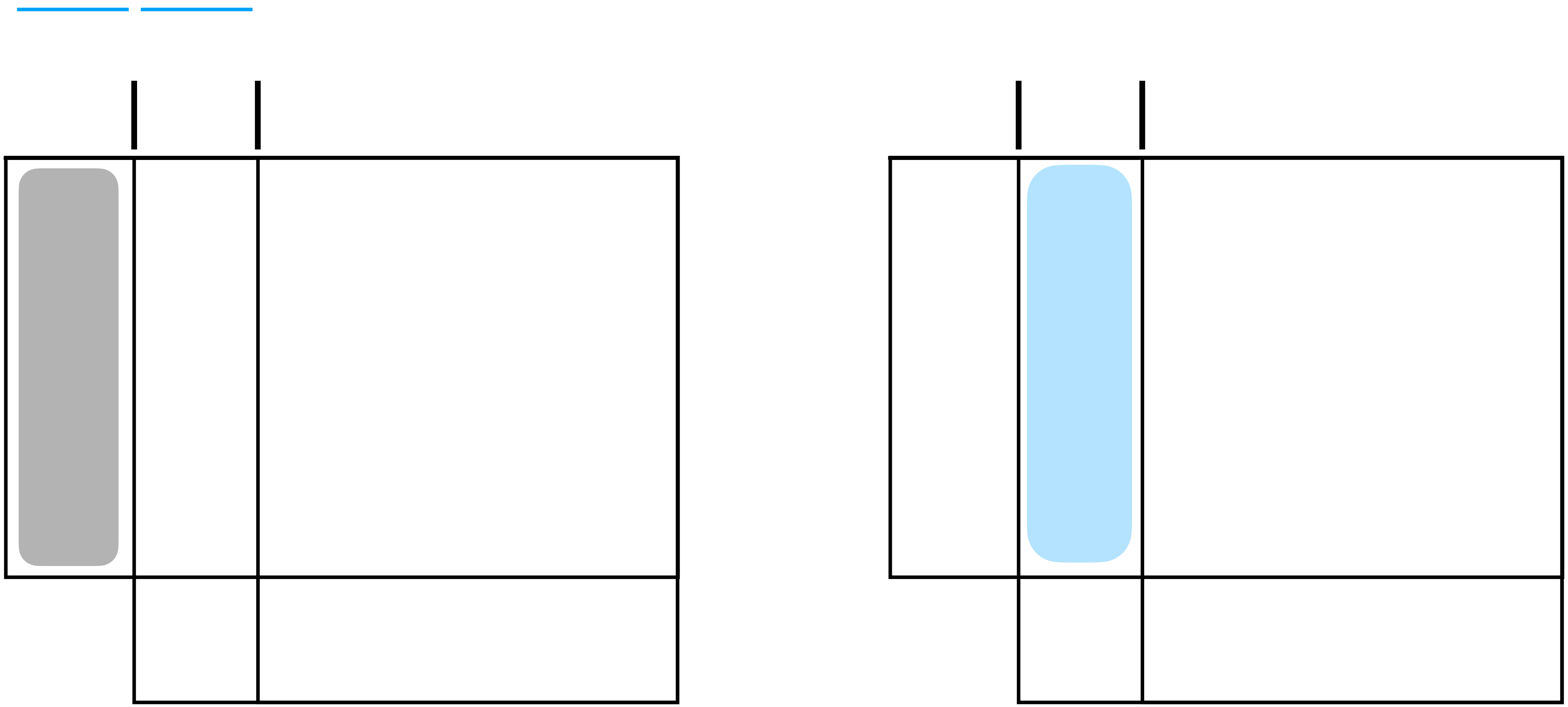
$$\left(\frac{x+y}{2}\right)^2 = xy + \left(\frac{x-y}{2}\right)^2$$

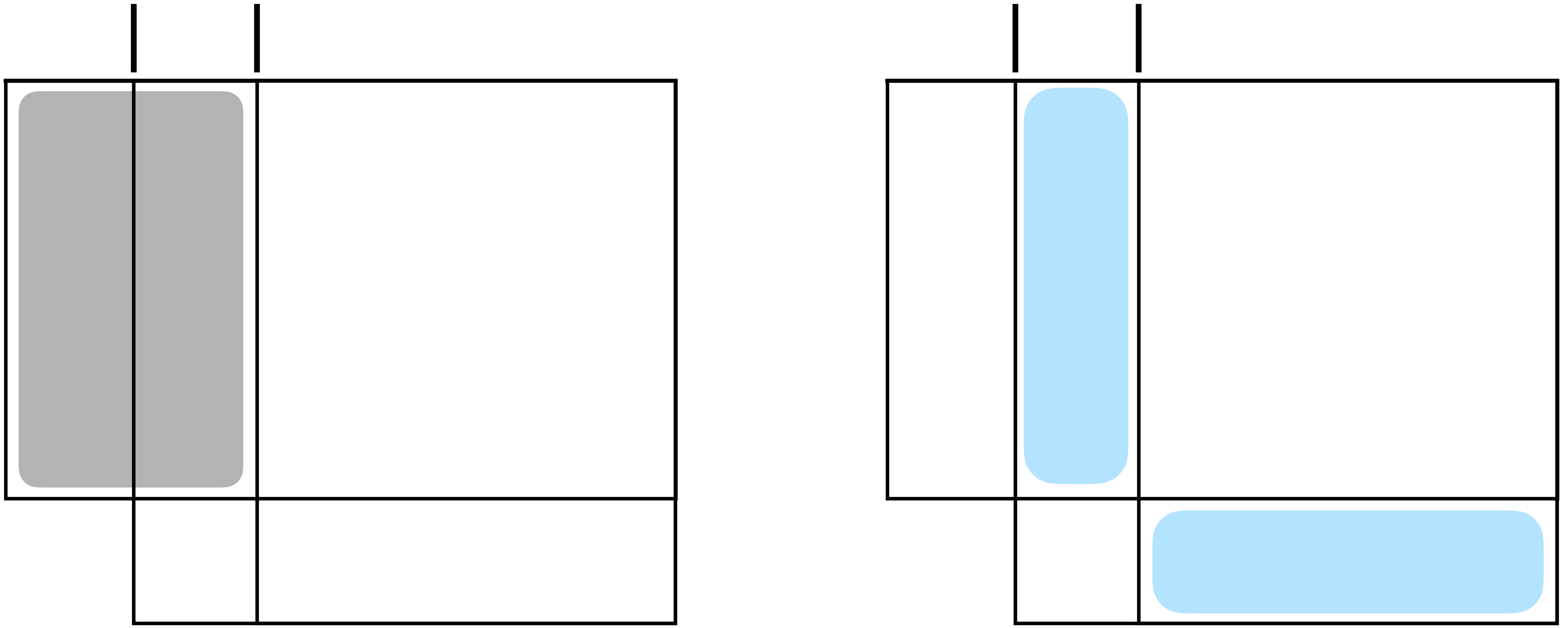


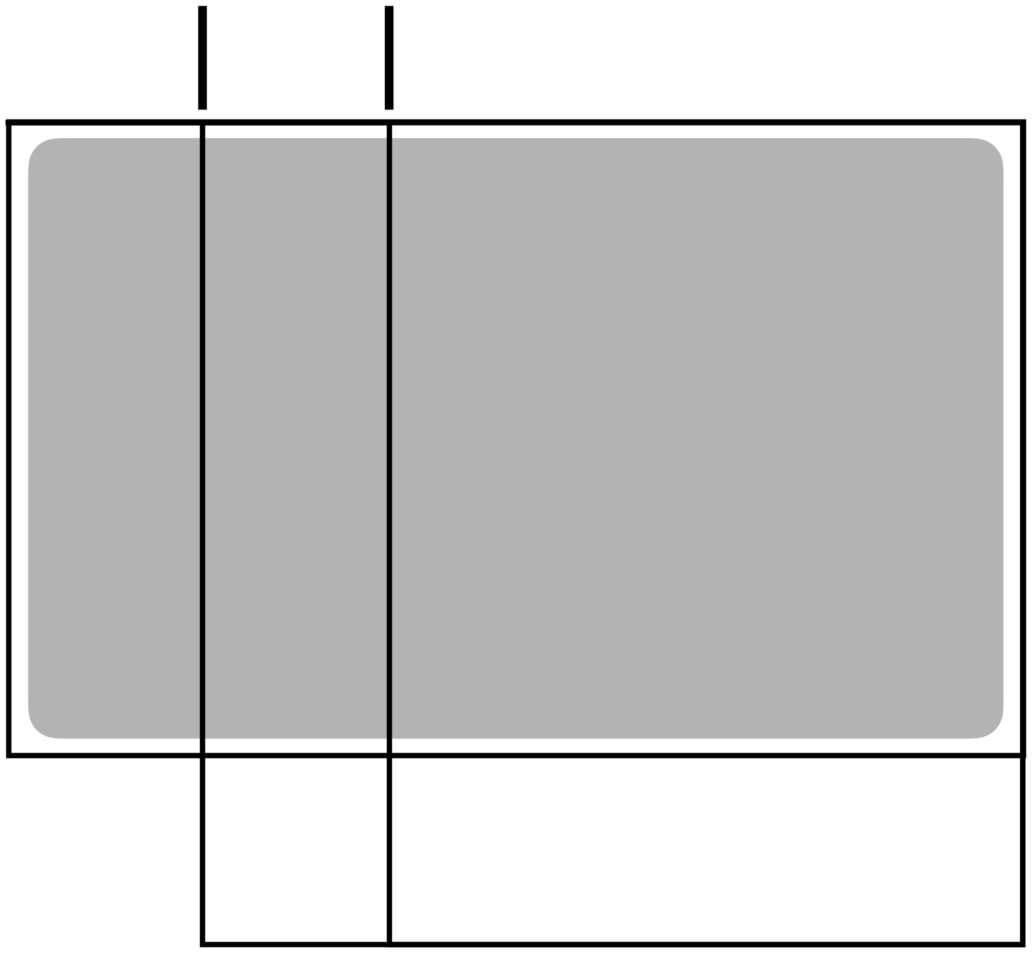


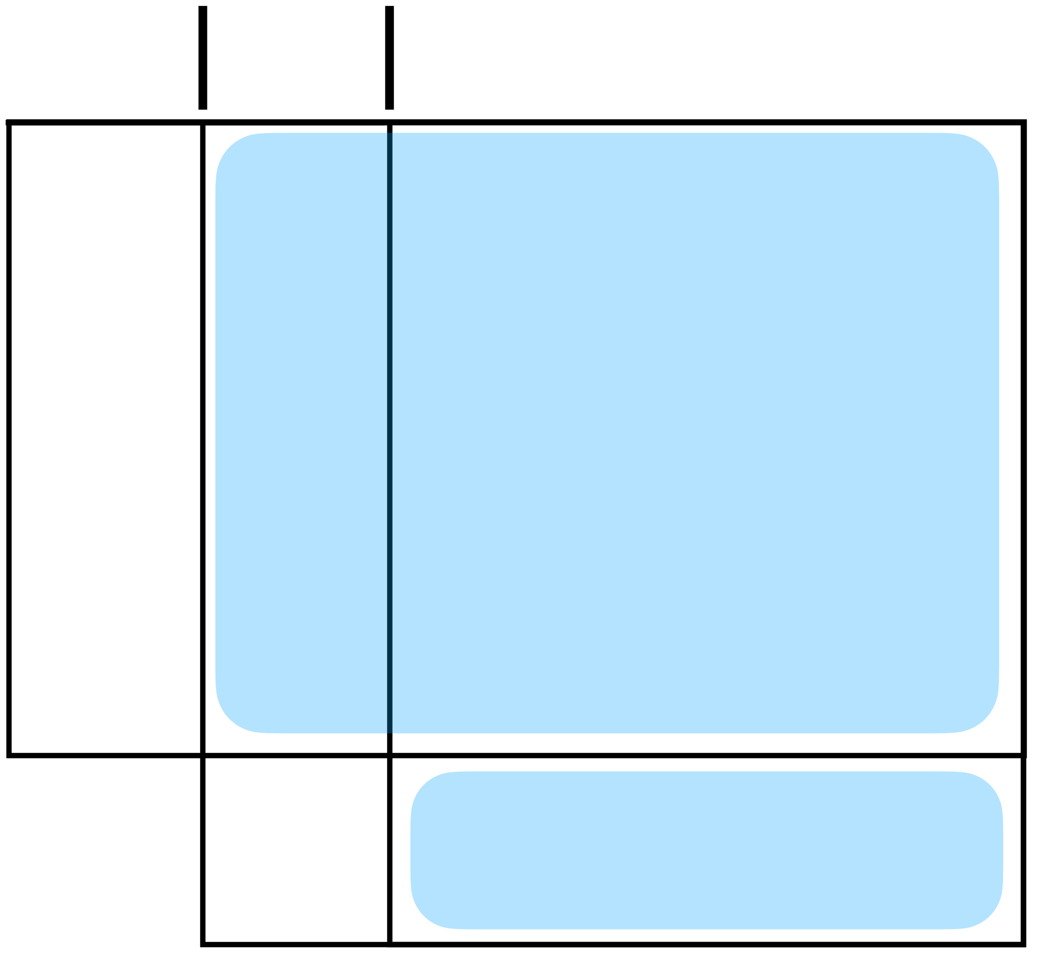


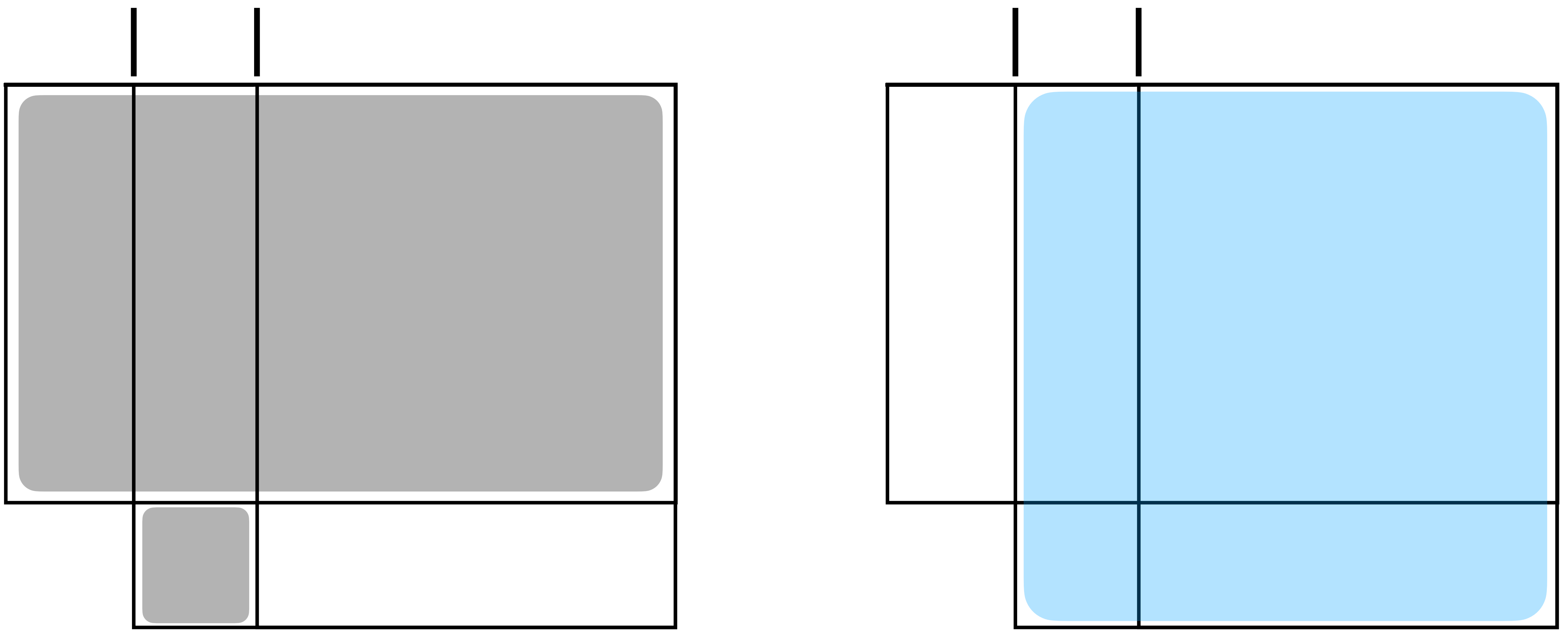


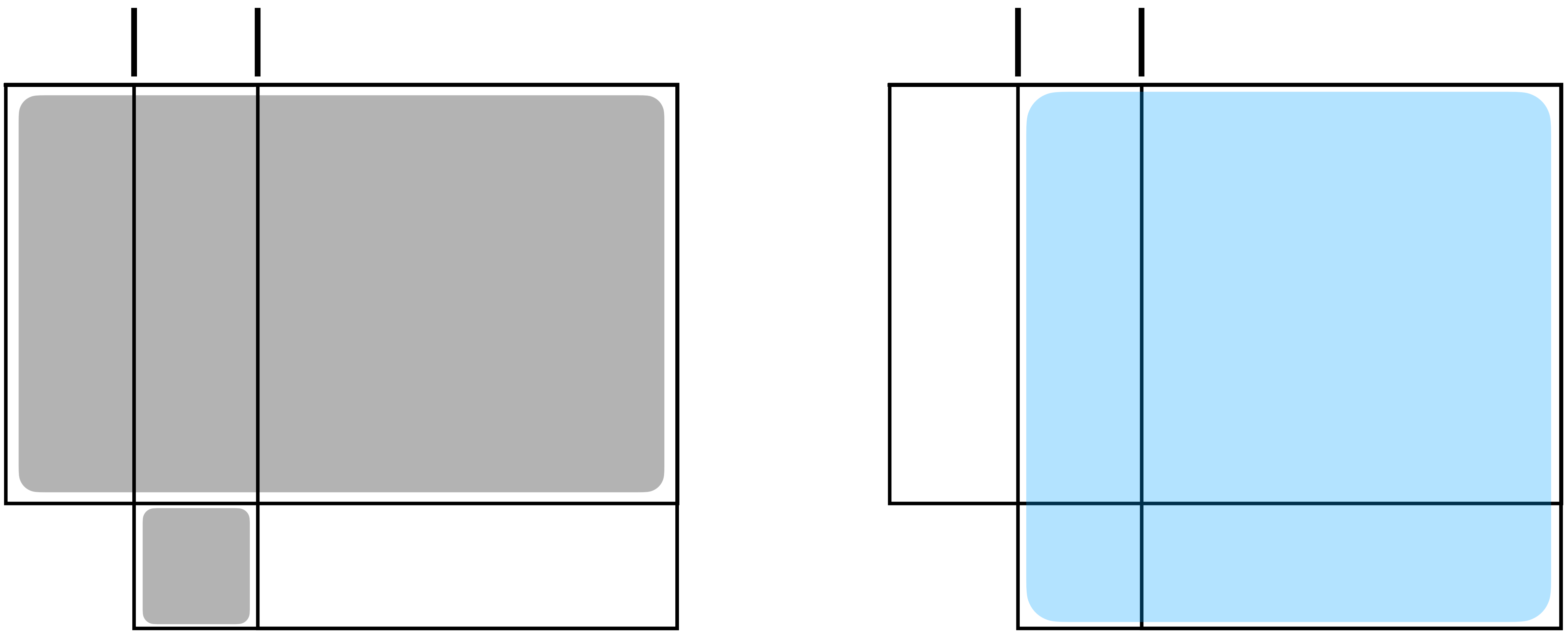




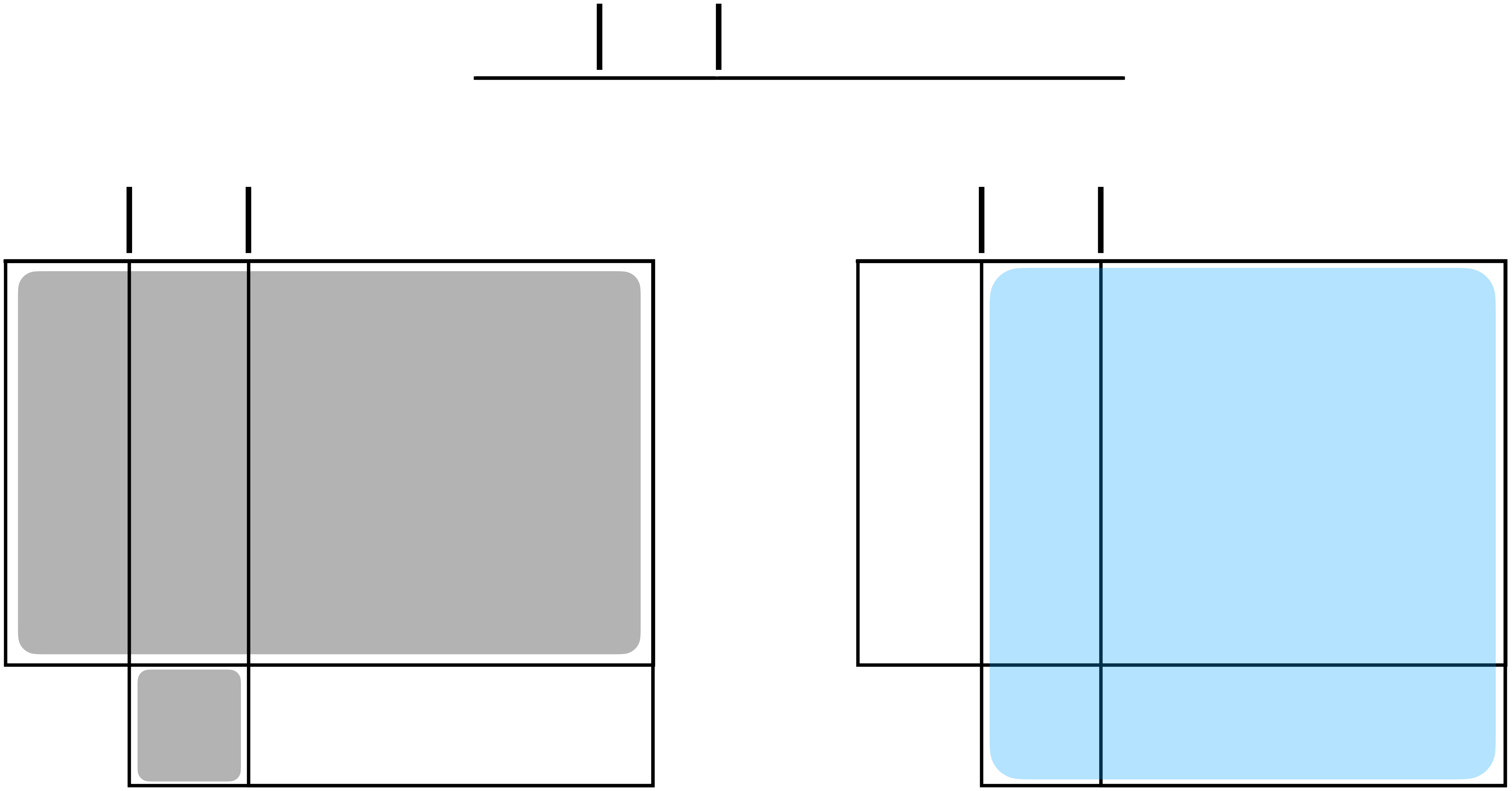








If a line be cut in equal parts, and to that another line be added,
the square on the half and added part is equal to
the rectangle of the original and added segments with the added sement and the square on half the original segment



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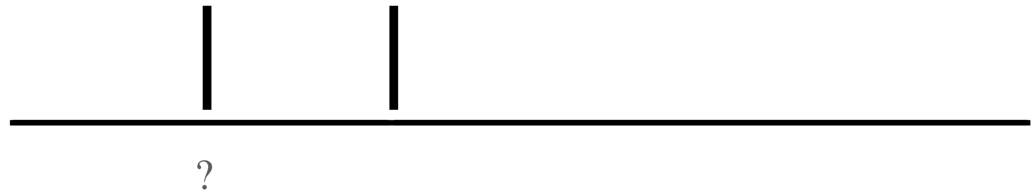

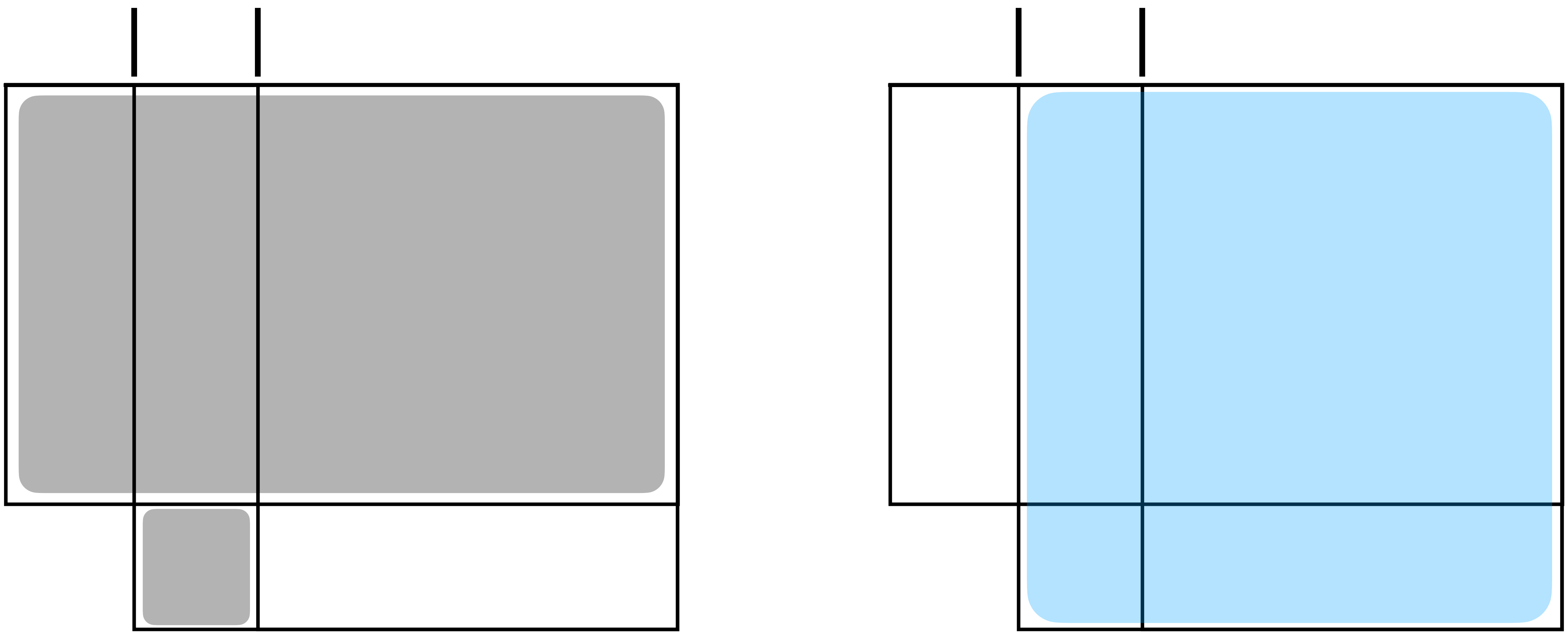

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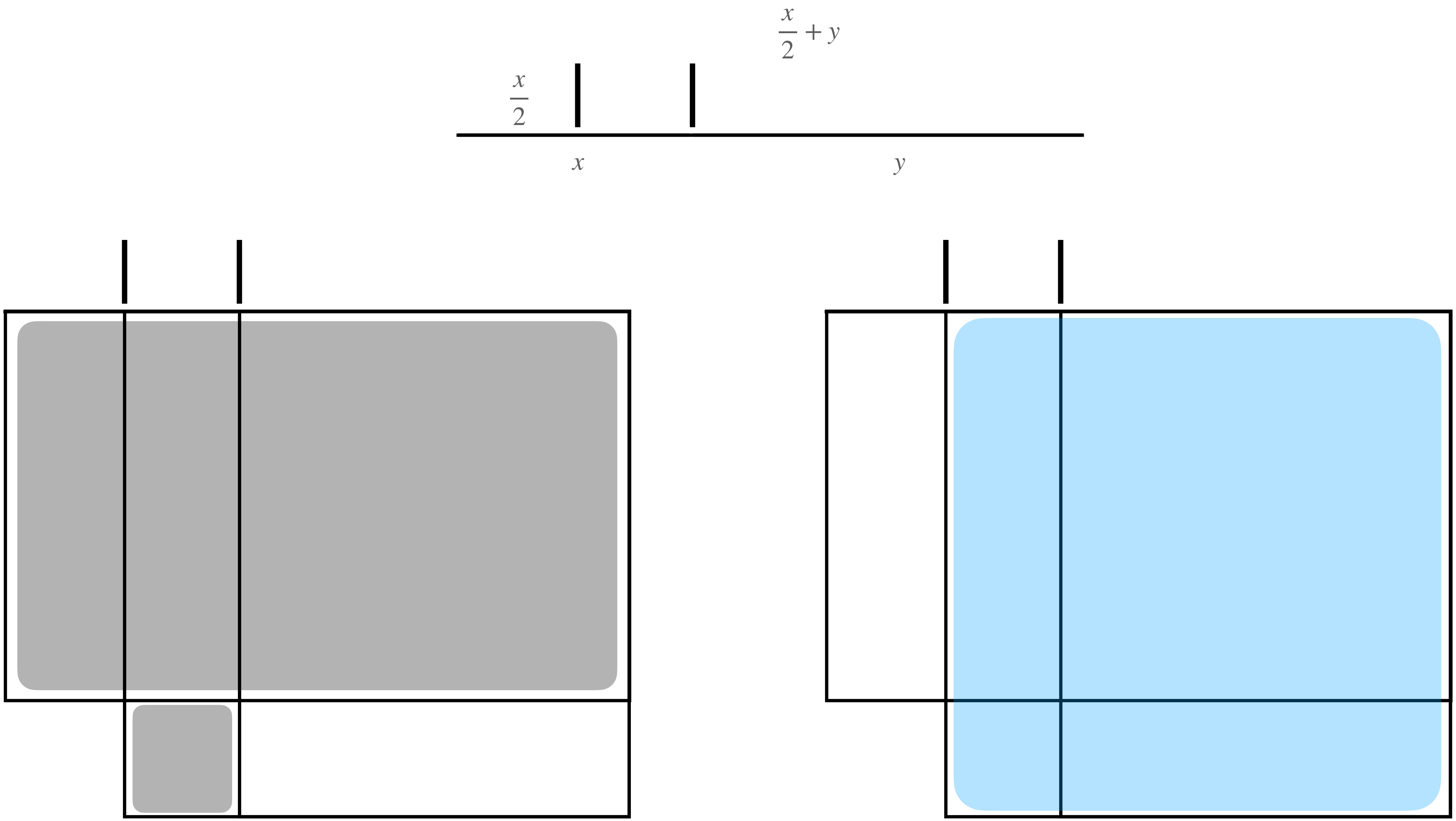

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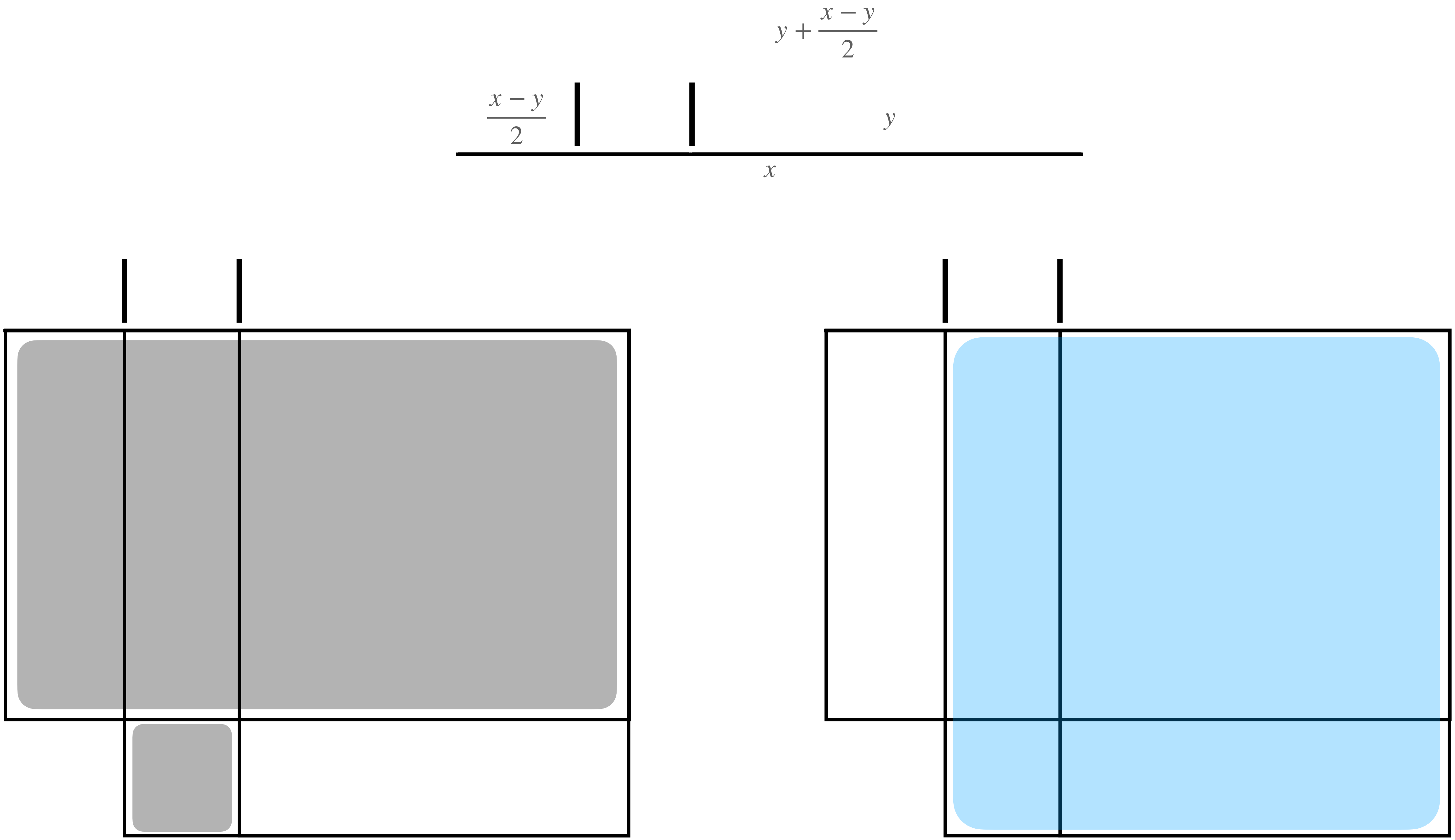
$$\left(\frac{x}{2} + y\right)^2 = (x + y)y + \left(\frac{x}{2}\right)^2$$

True. But it doesn’t seem nice. Let’s try again.



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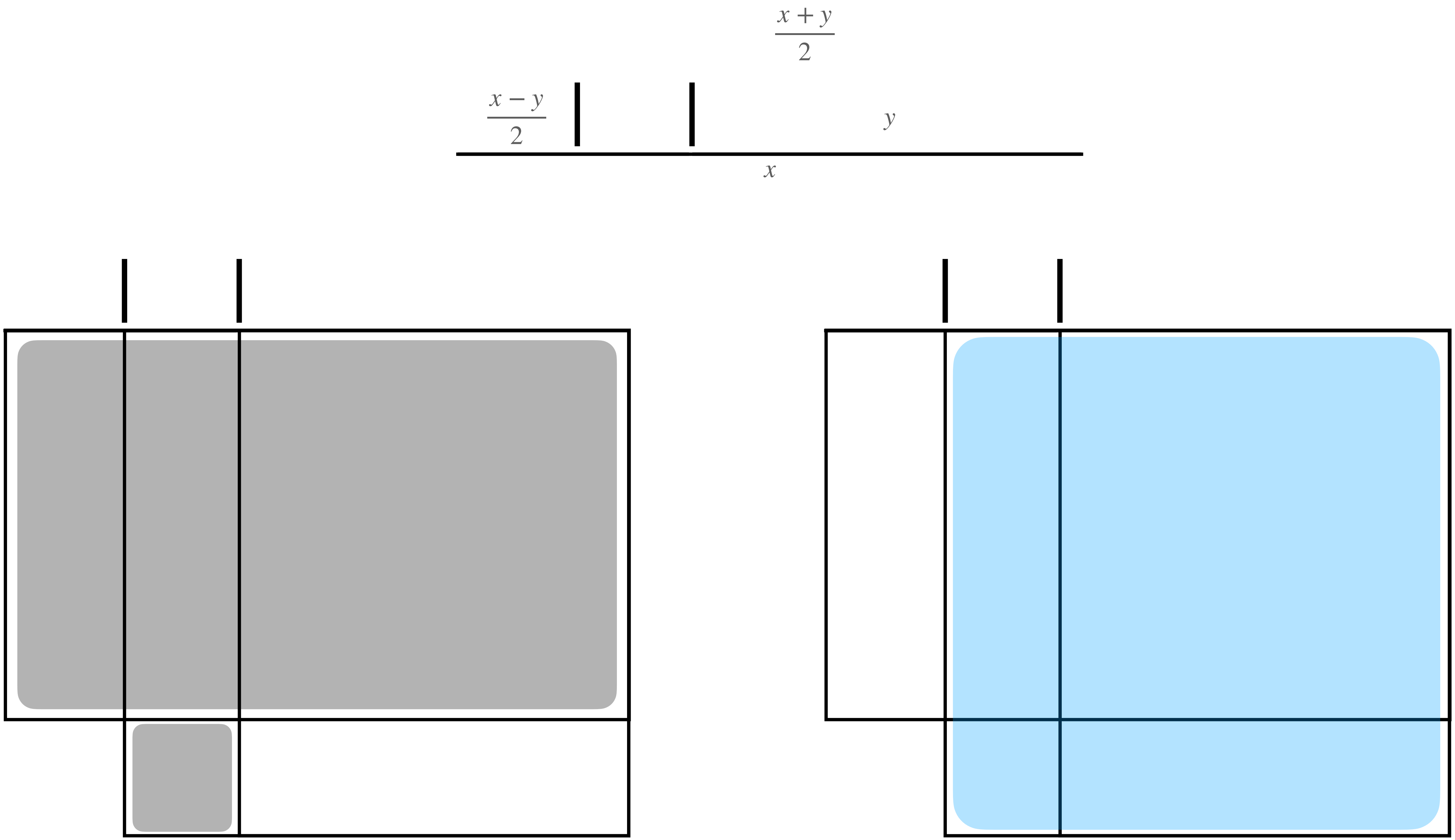

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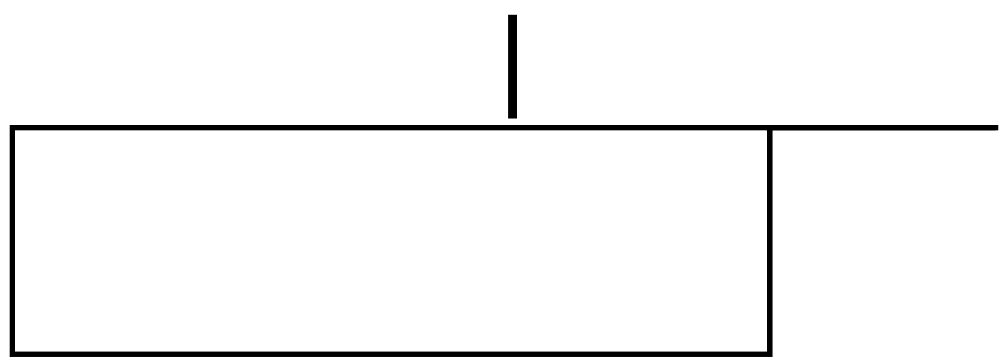
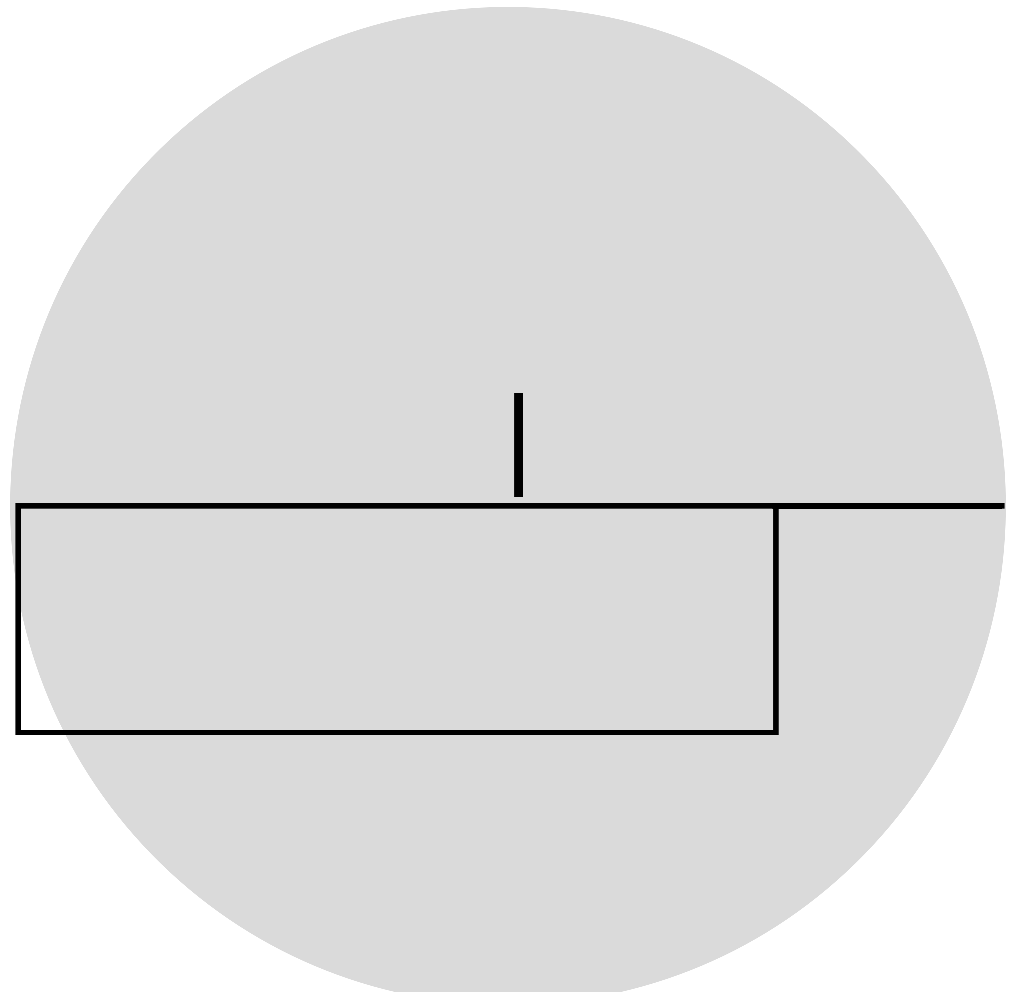
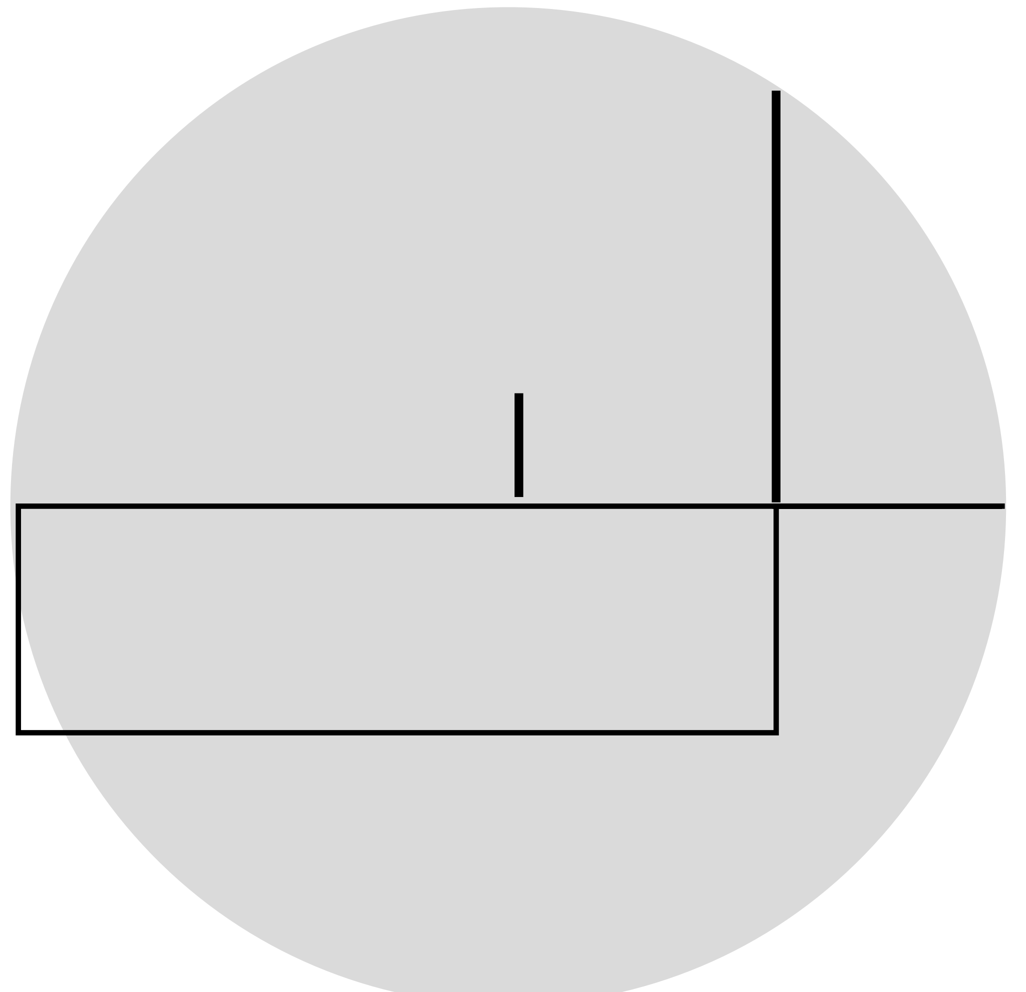
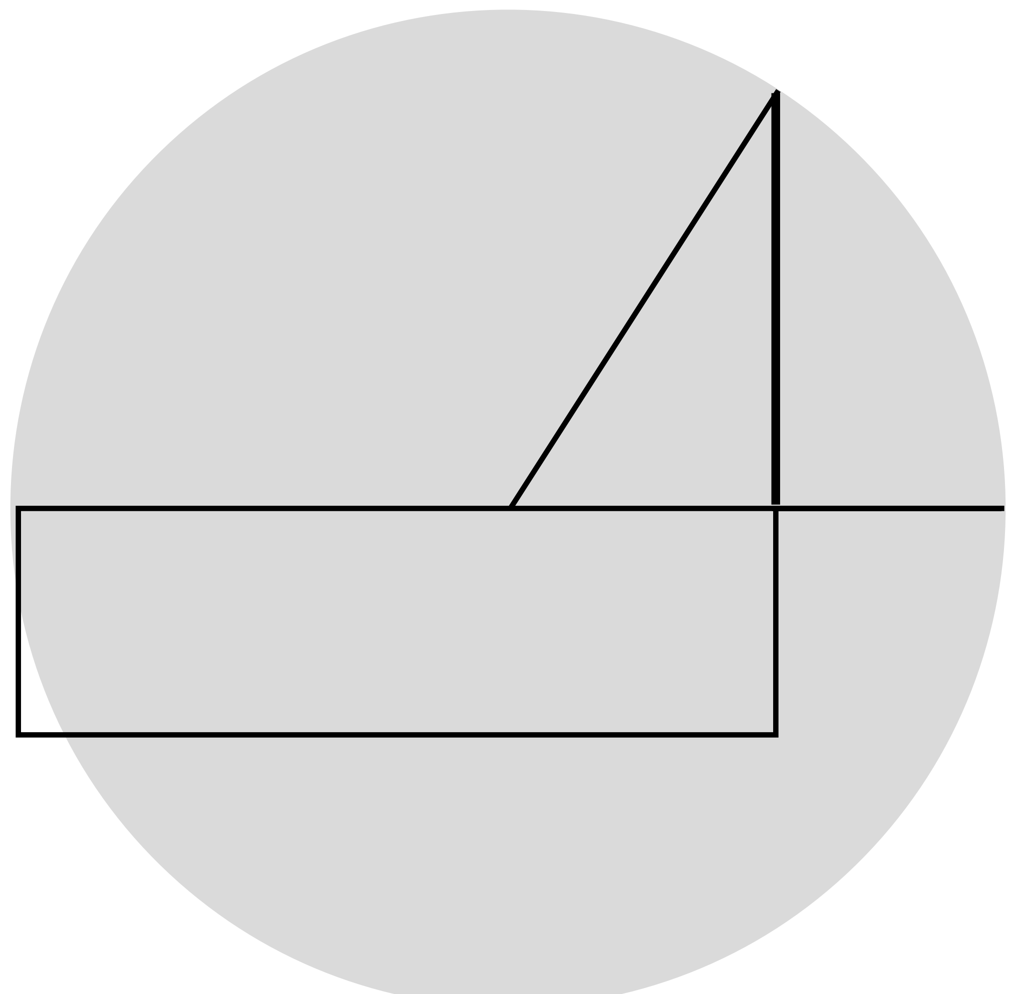
$$\left(\frac{x+y}{2}\right)^2 = xy + \left(\frac{x-y}{2}\right)^2$$

find sqrt









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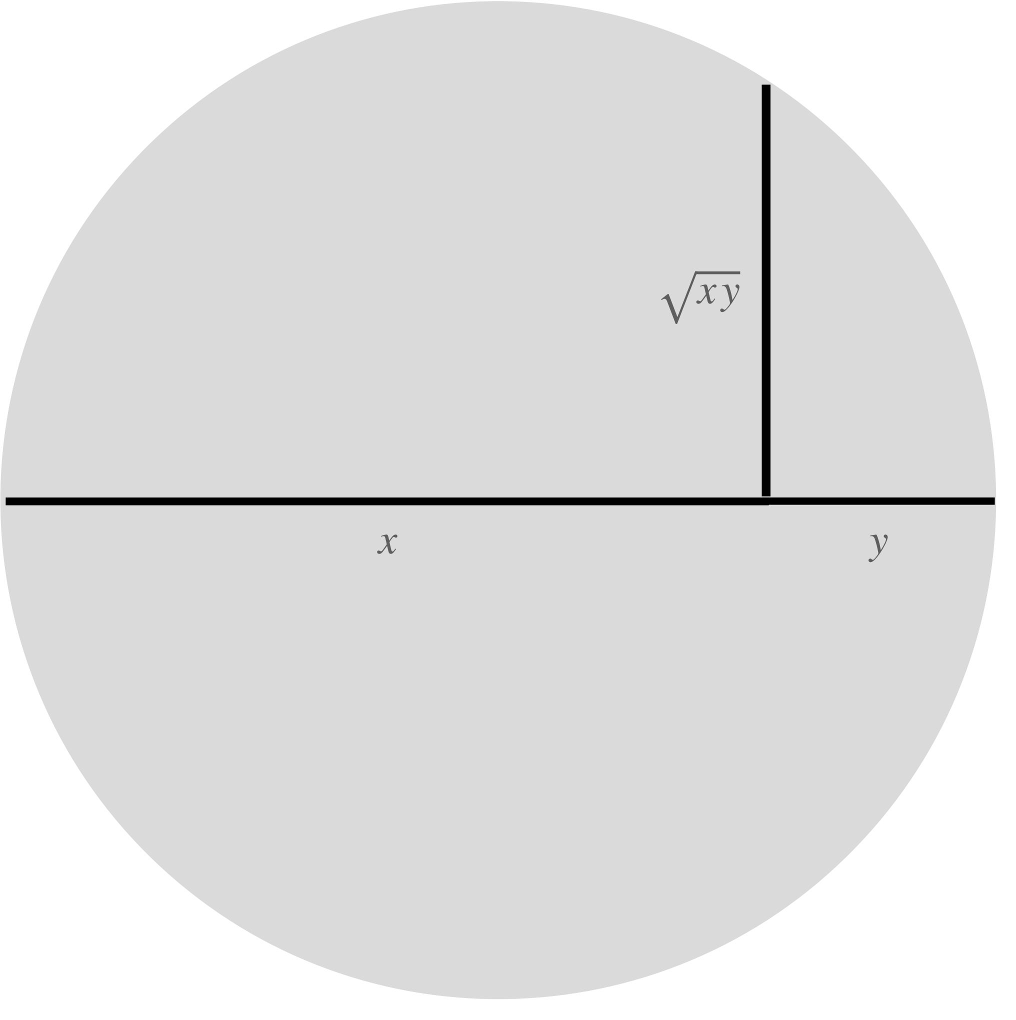
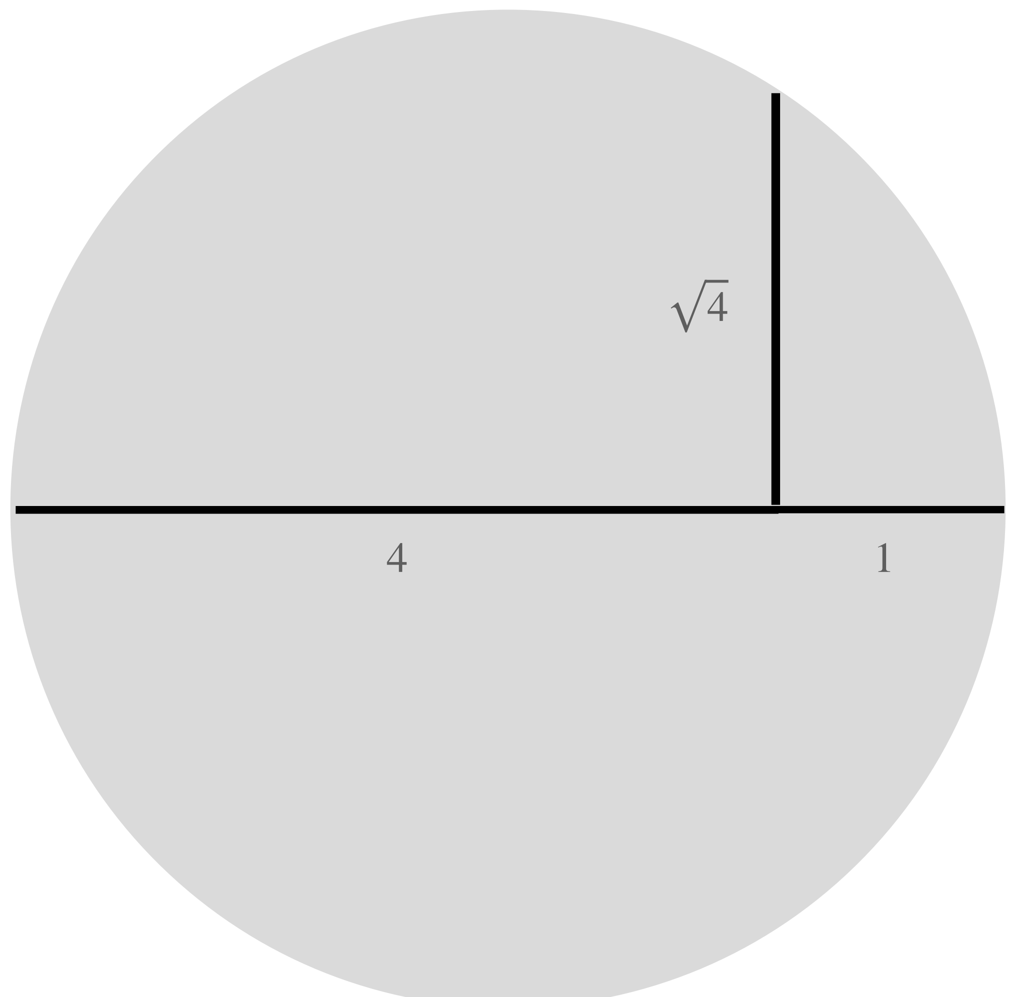
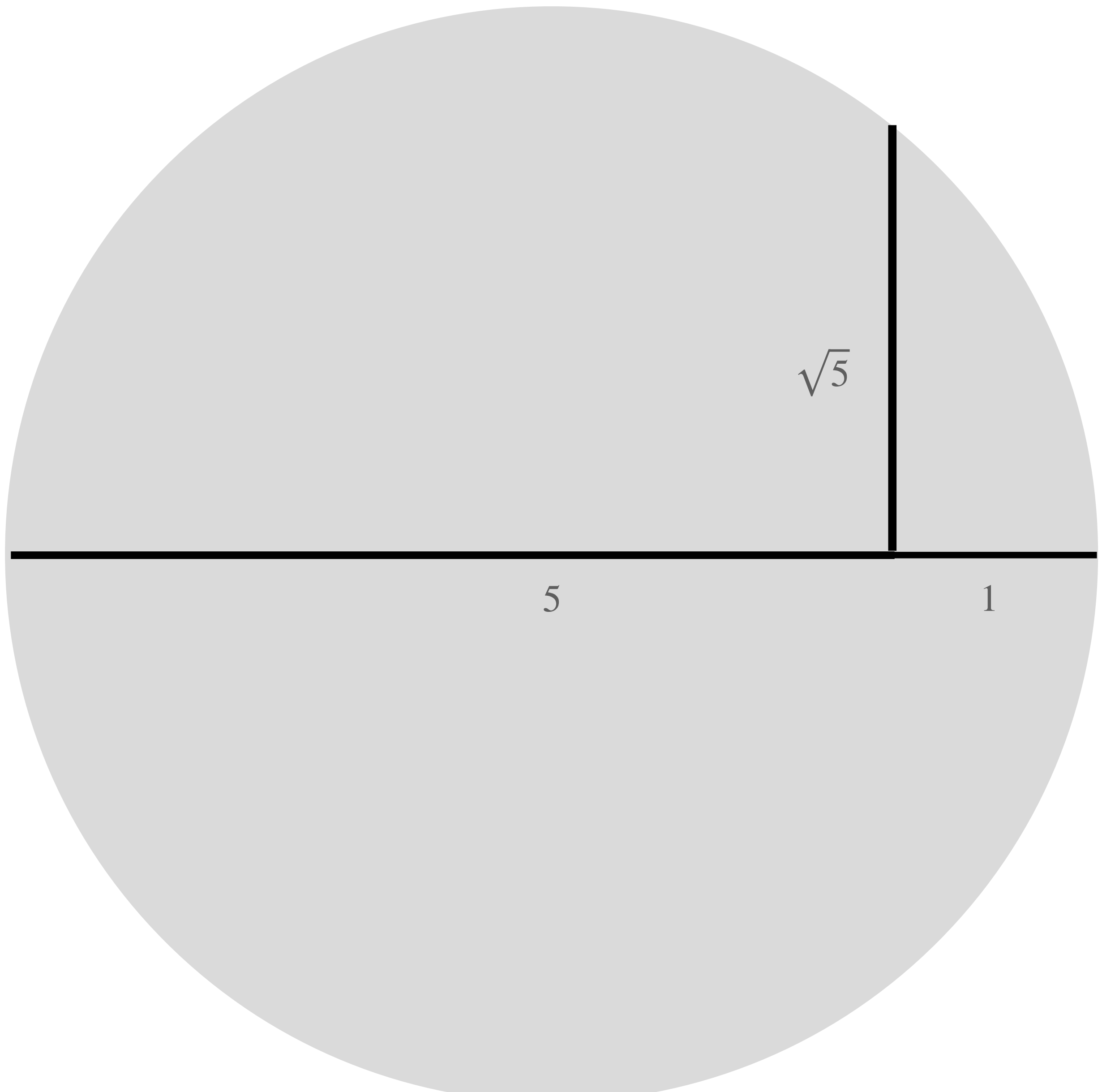


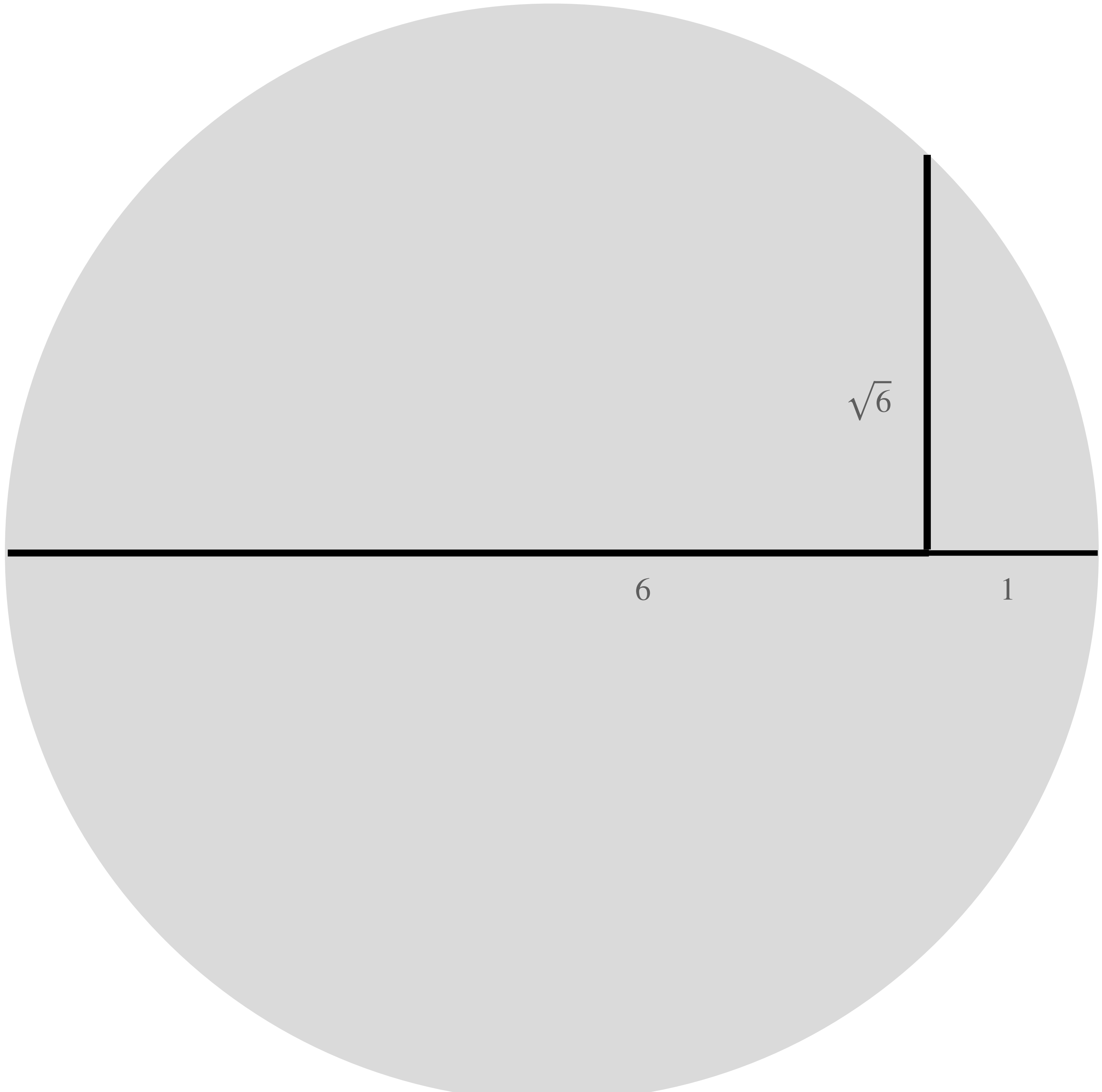
$\sqrt{xy}$
x
y



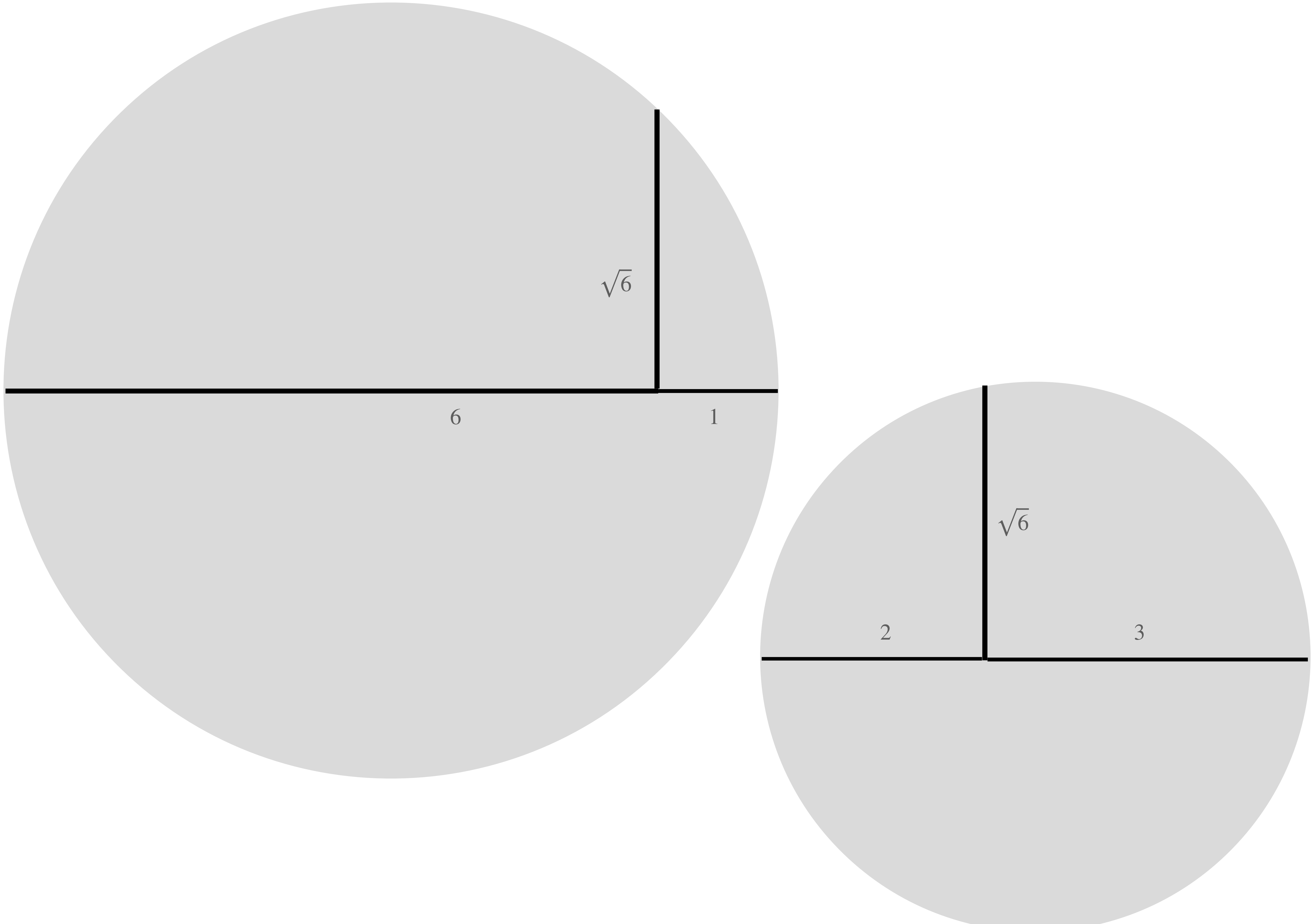
$\sqrt{4}$
4
1



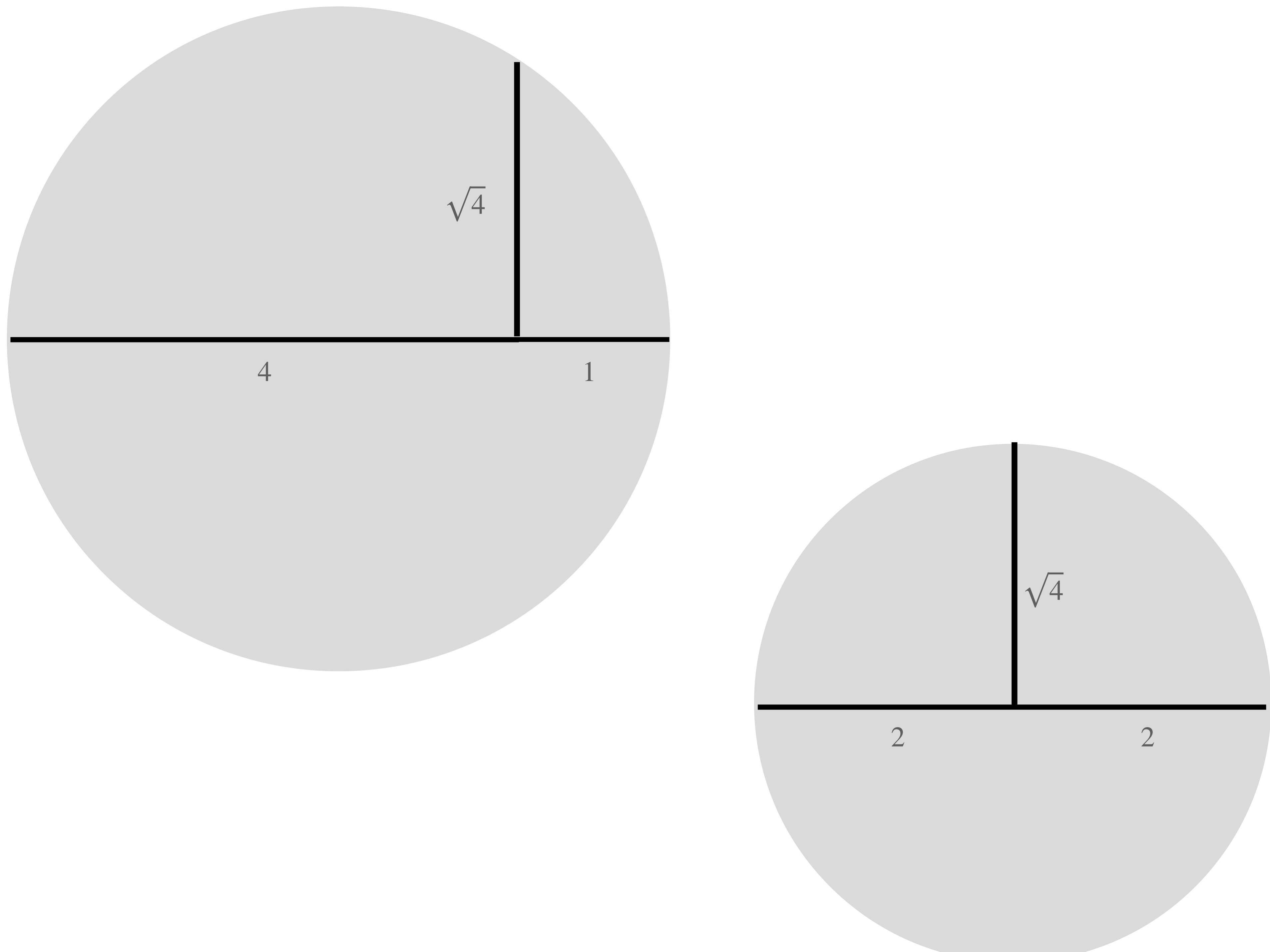
$\sqrt{5}$
5
1



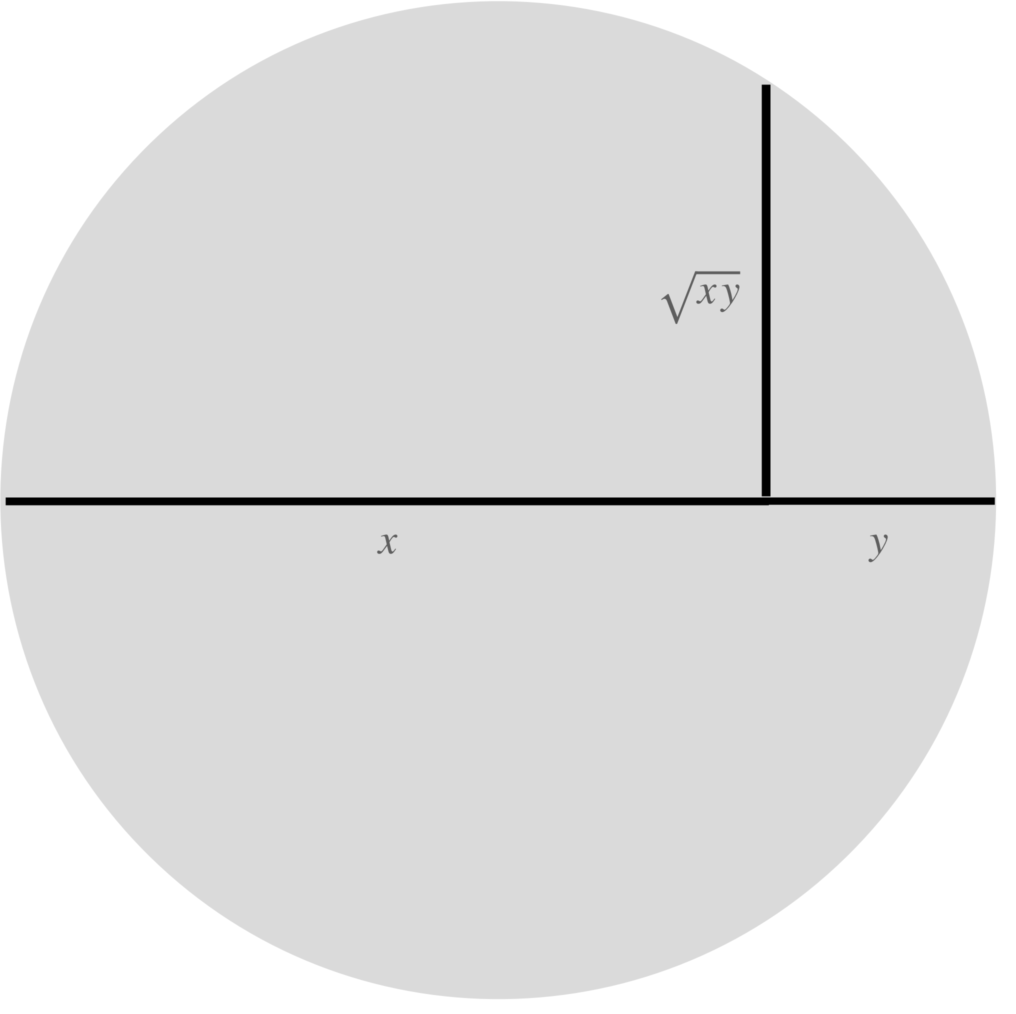
$\sqrt{6}$
6
1



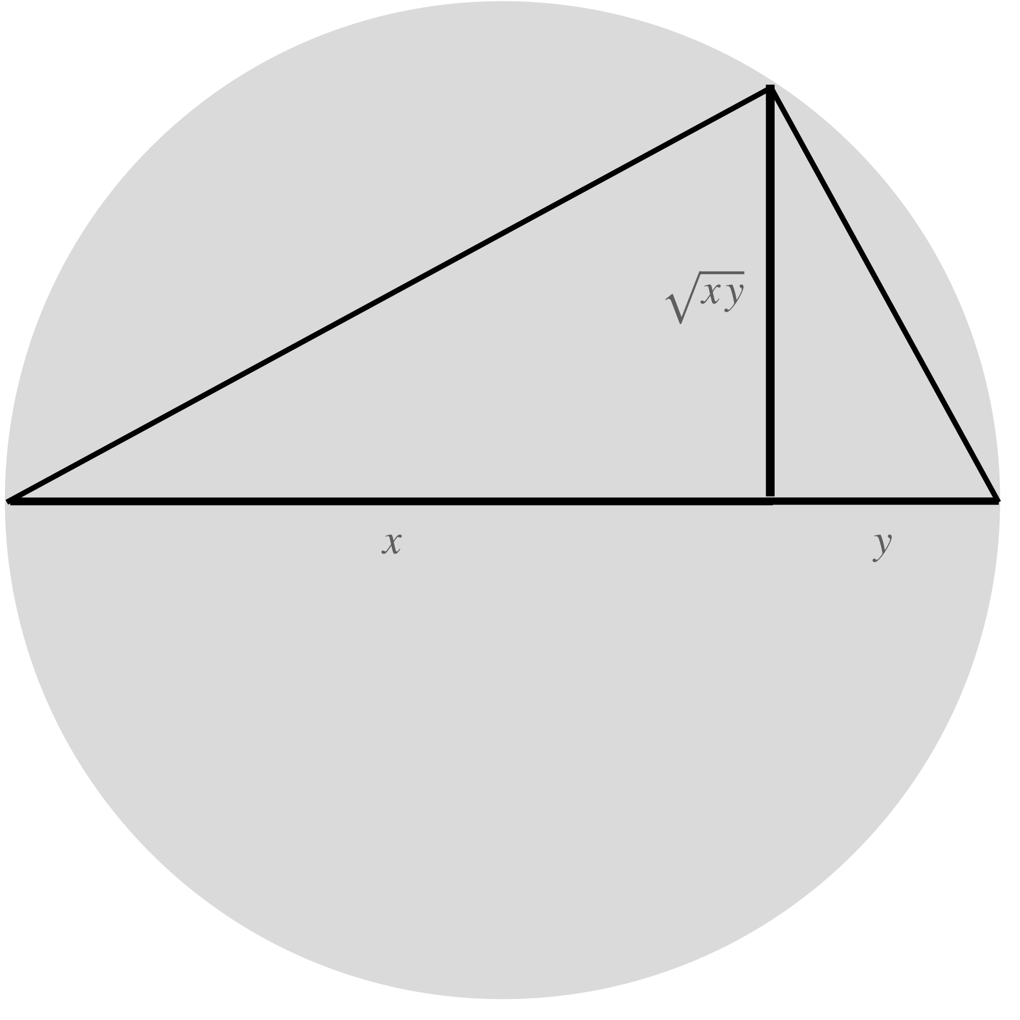
$\sqrt{6}$
6
1
$\sqrt{6}$
2
3



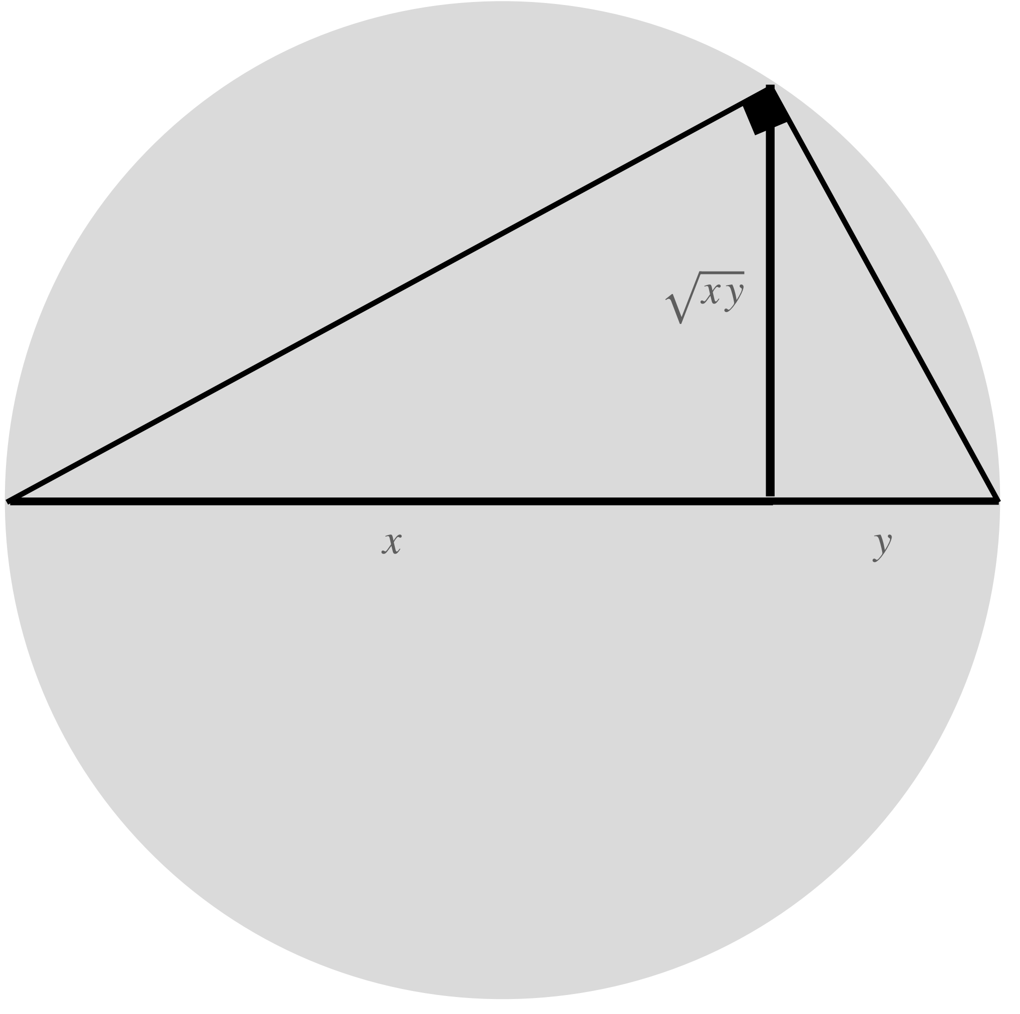
$\sqrt{4}$
4
1
$\sqrt{4}$
2
2



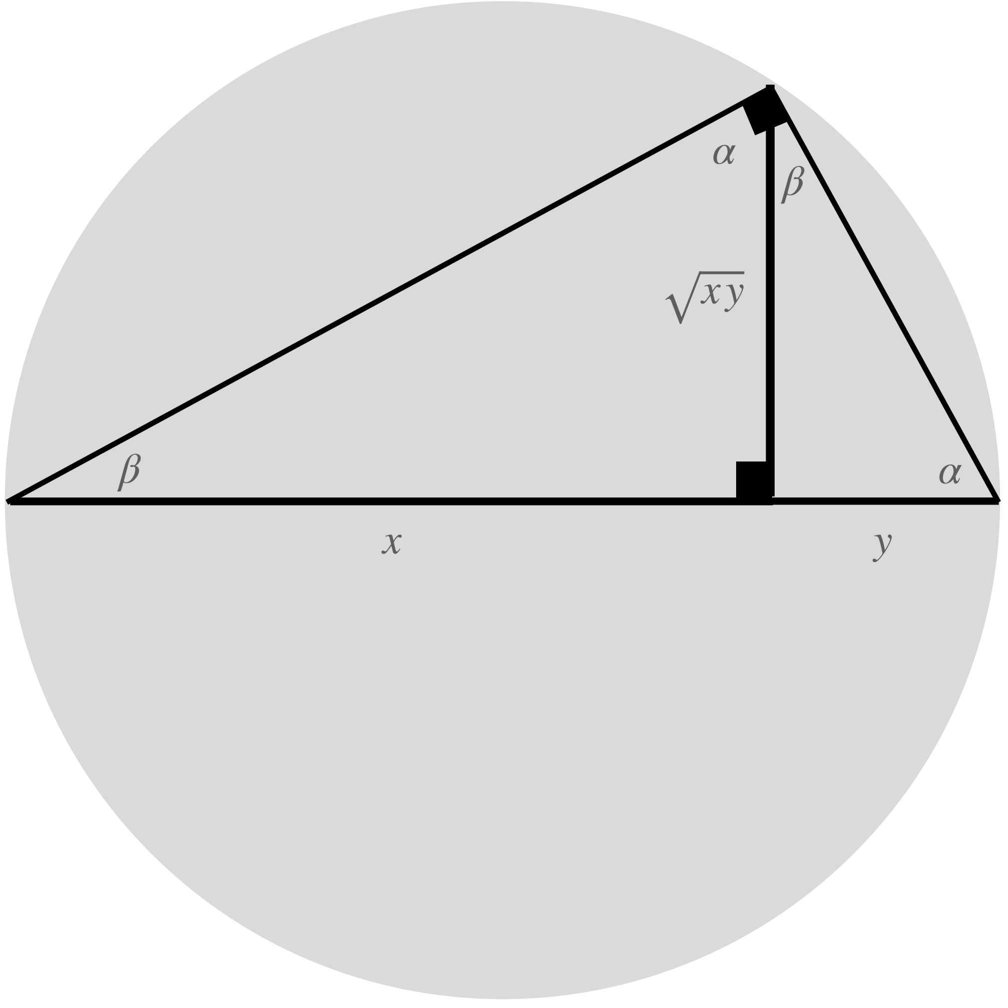
$\sqrt{xy}$
x
y



$\sqrt{xy}$
x
y

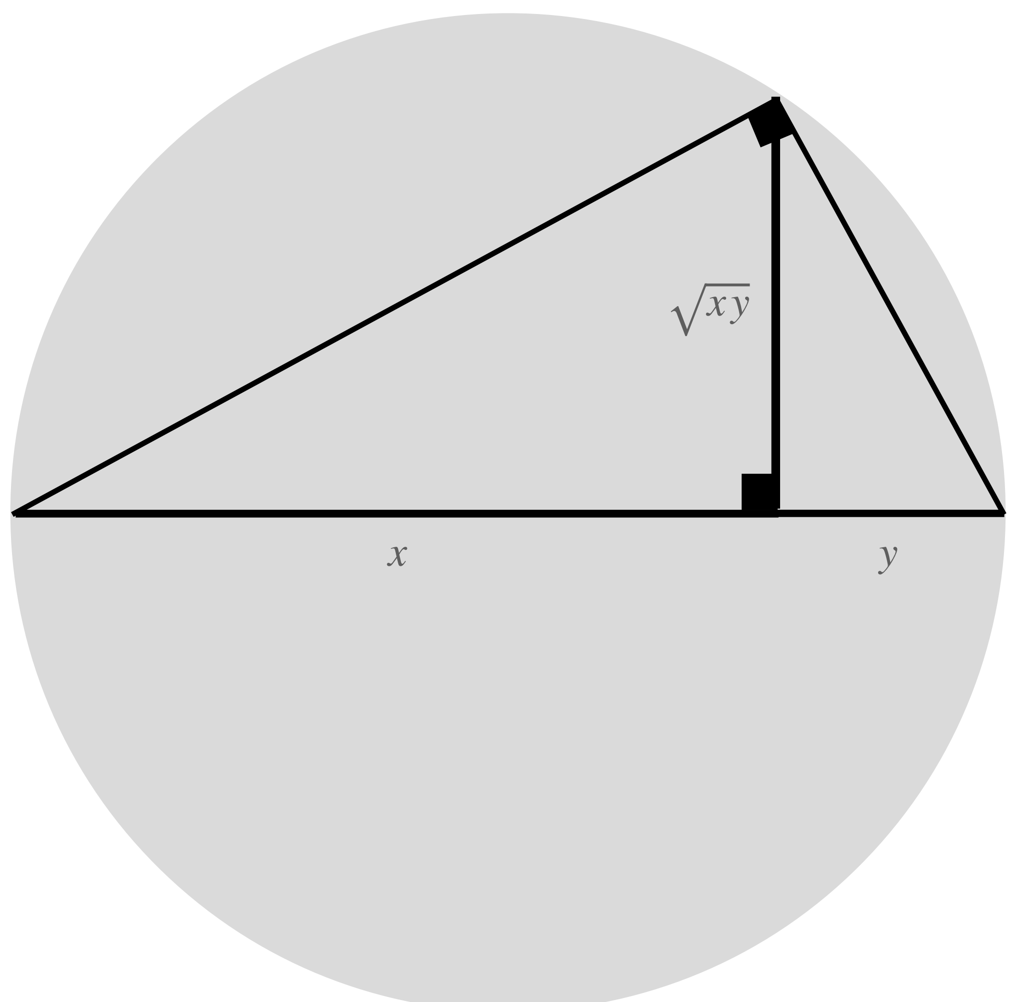


$\sqrt{xy}$
x
y



$\beta = 90 - \alpha$

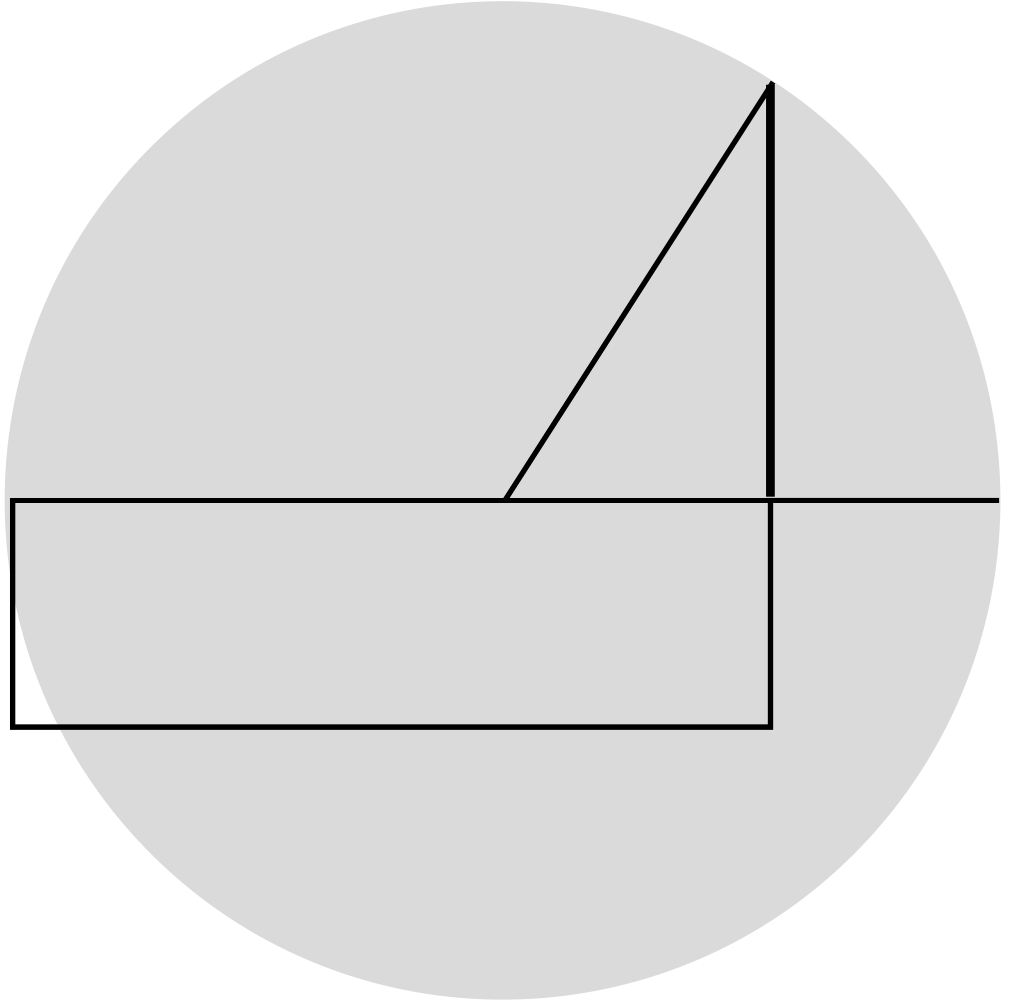
3 similar right triangles.

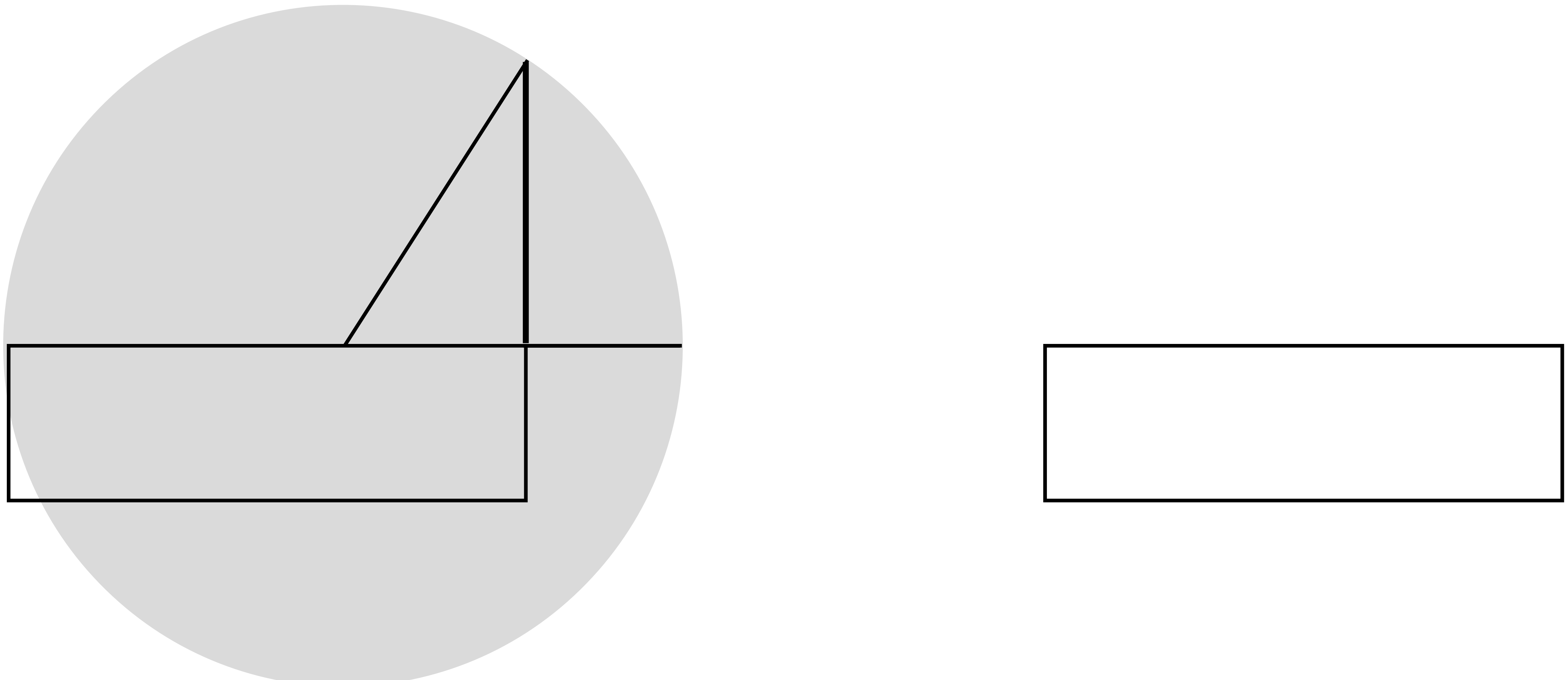


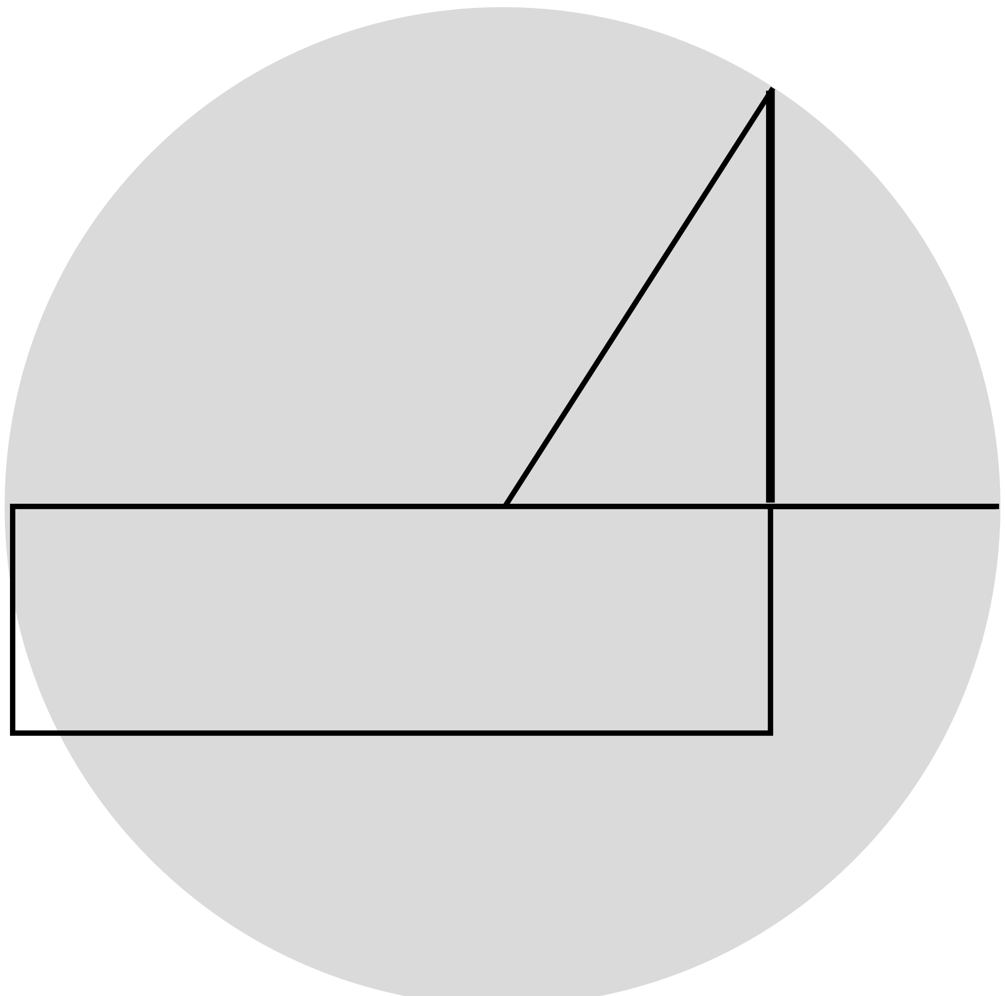
3 similar right triangles.

$$\frac{x}{\sqrt{xy}} = \frac{\sqrt{xy}}{y}$$

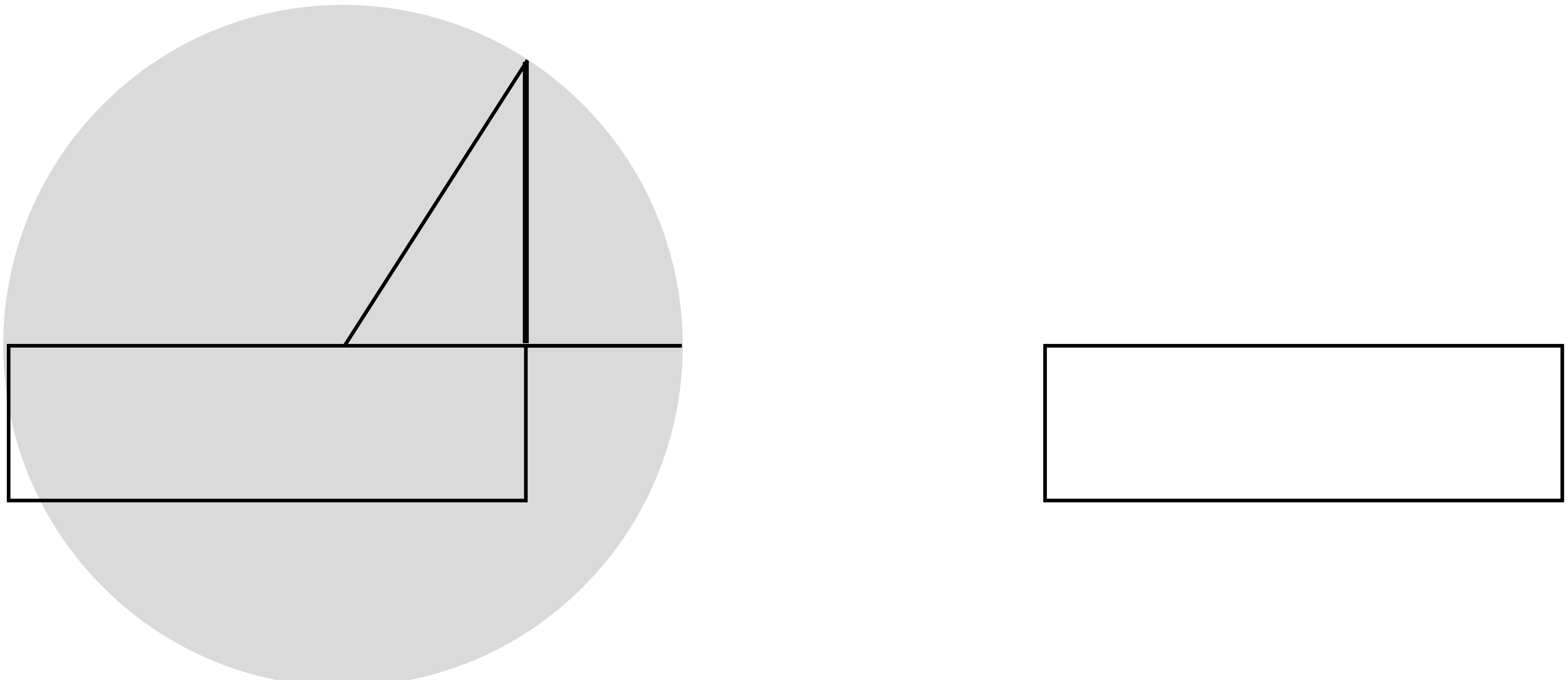
$$xy = \left(\sqrt{xy}\right)^2$$

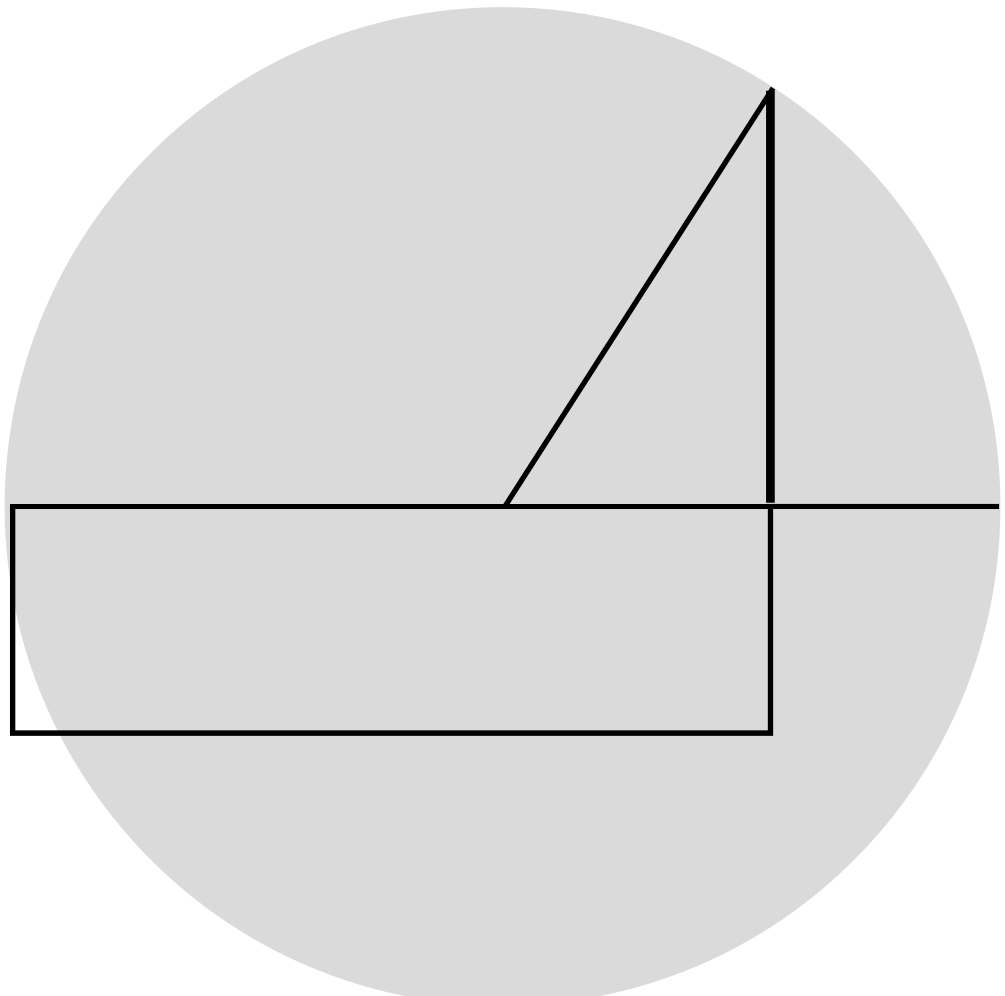


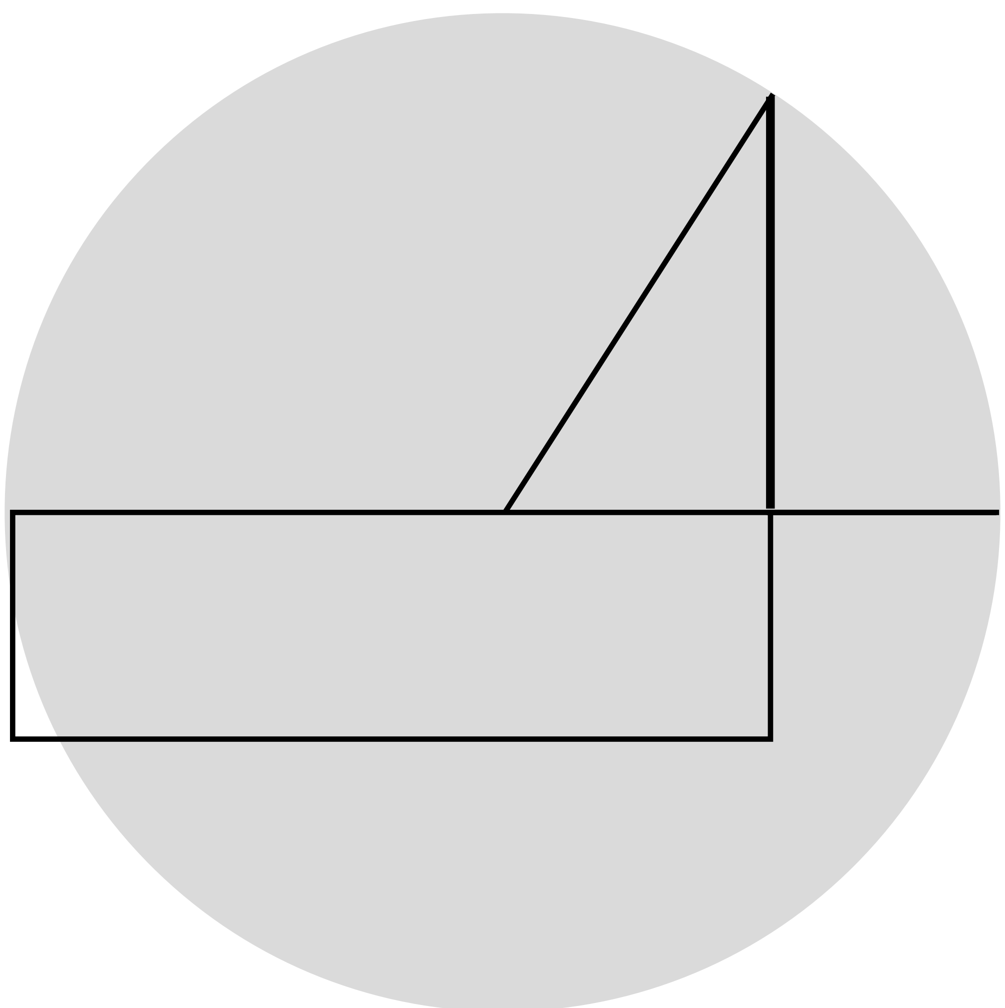


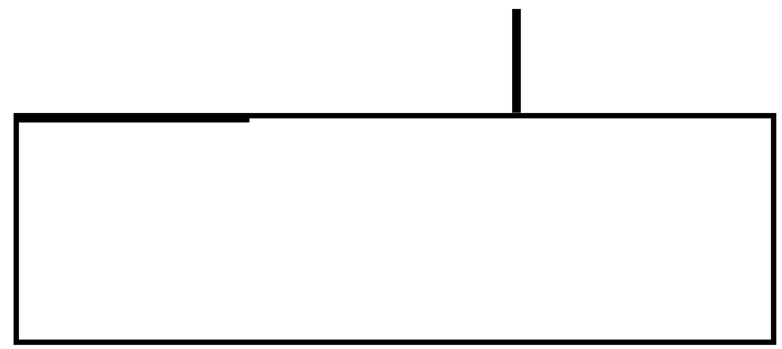


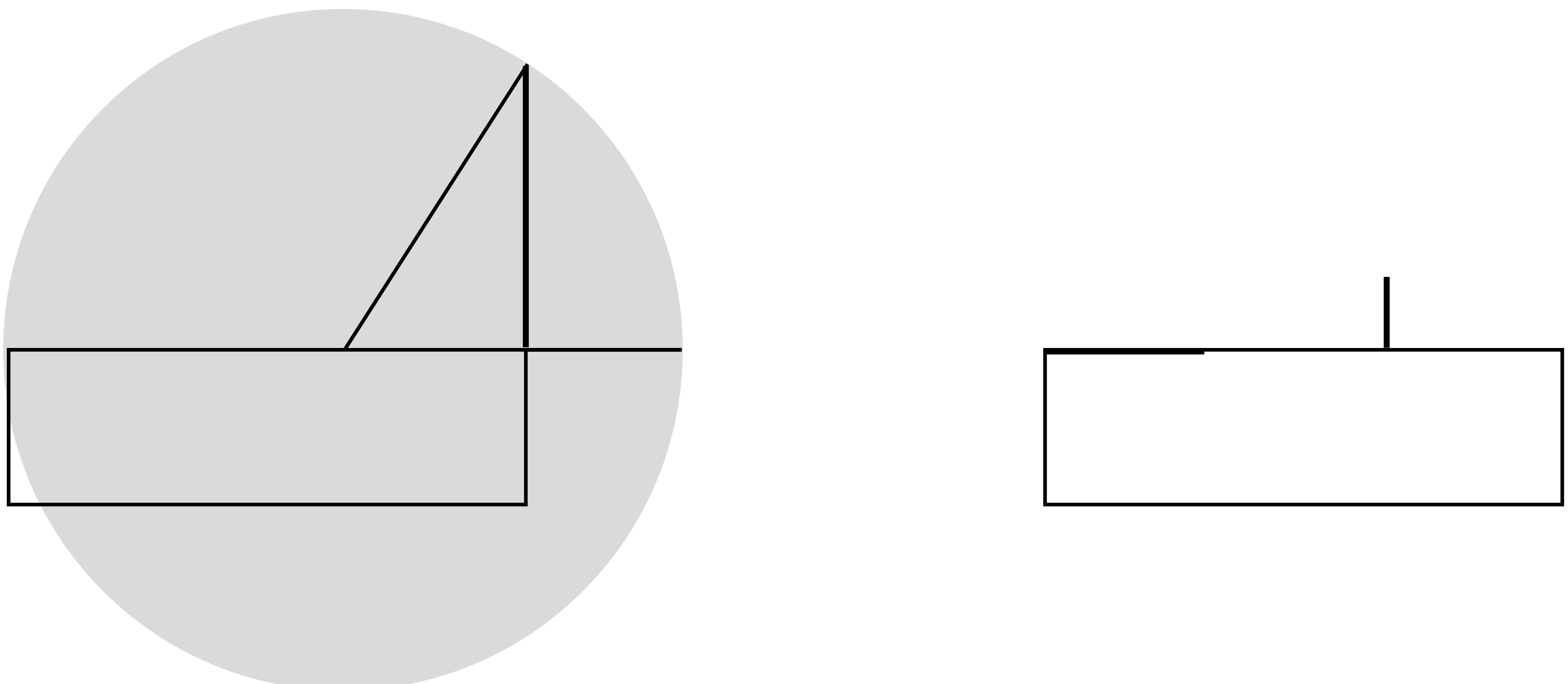
find sqrt











If a line be cut in equal parts, and to that another line be added,
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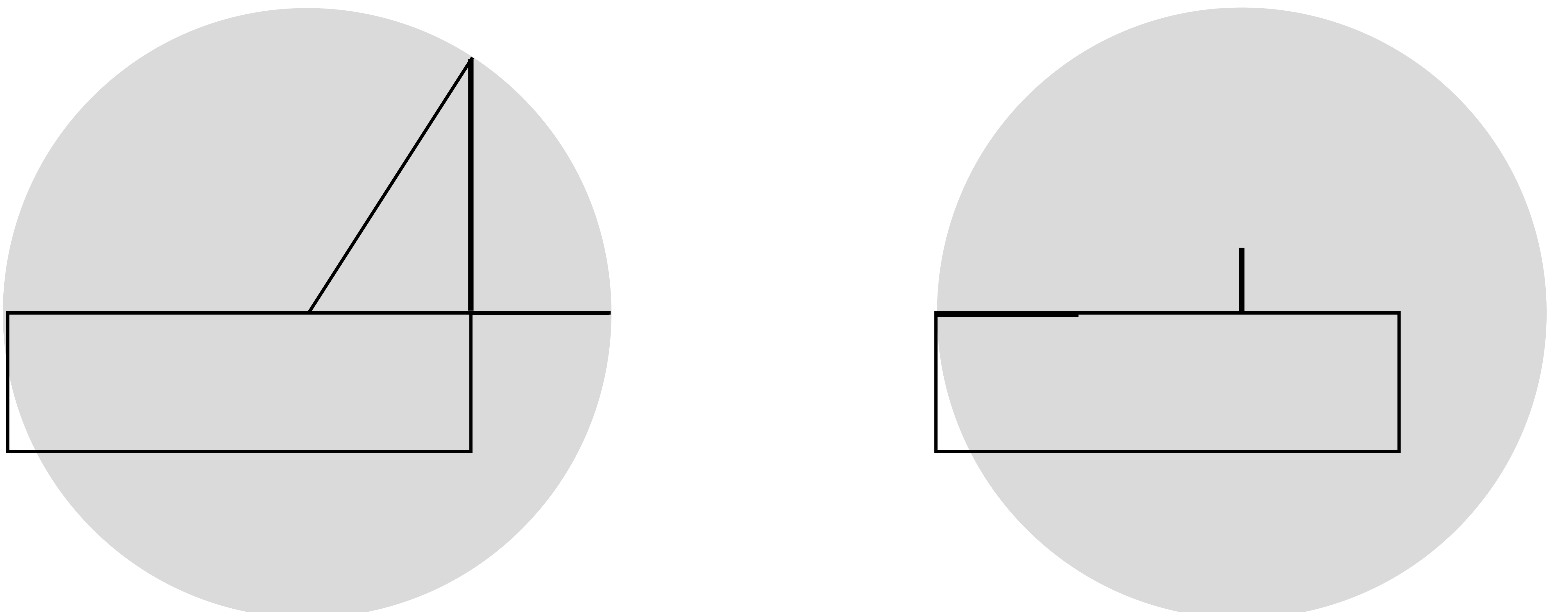

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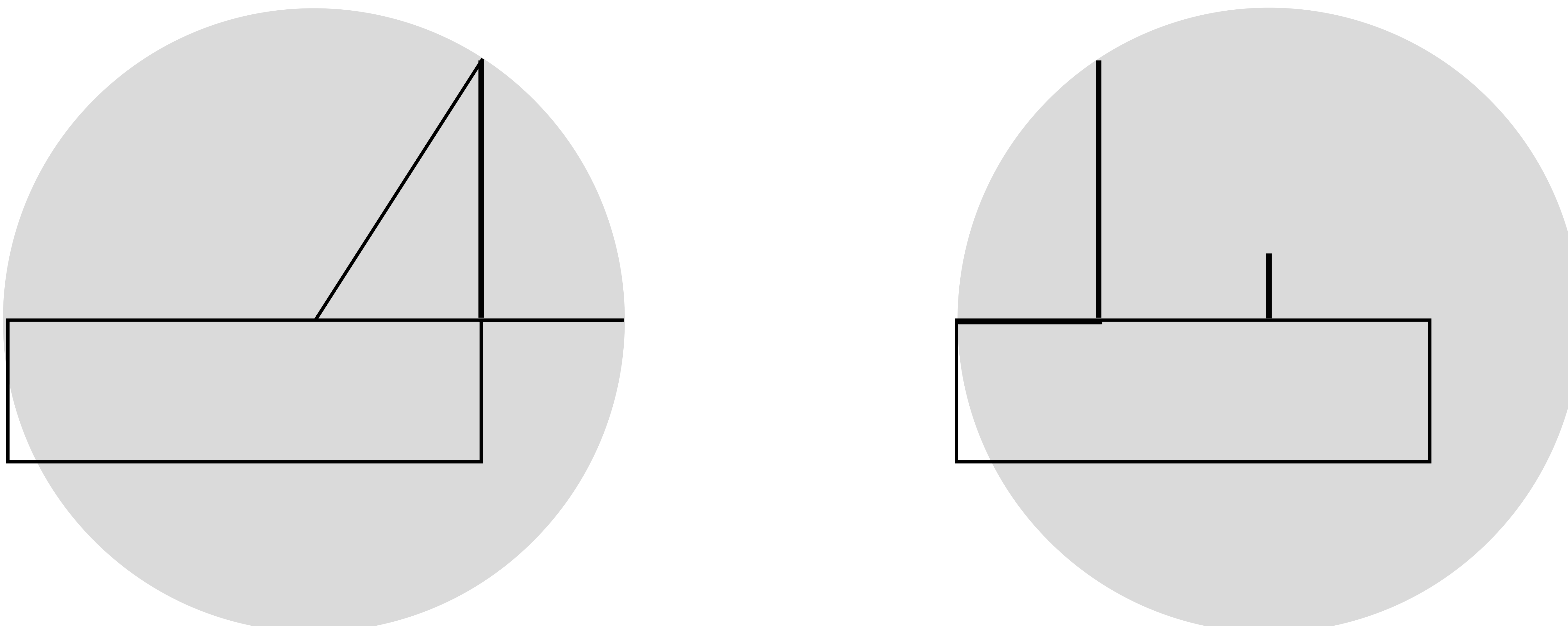

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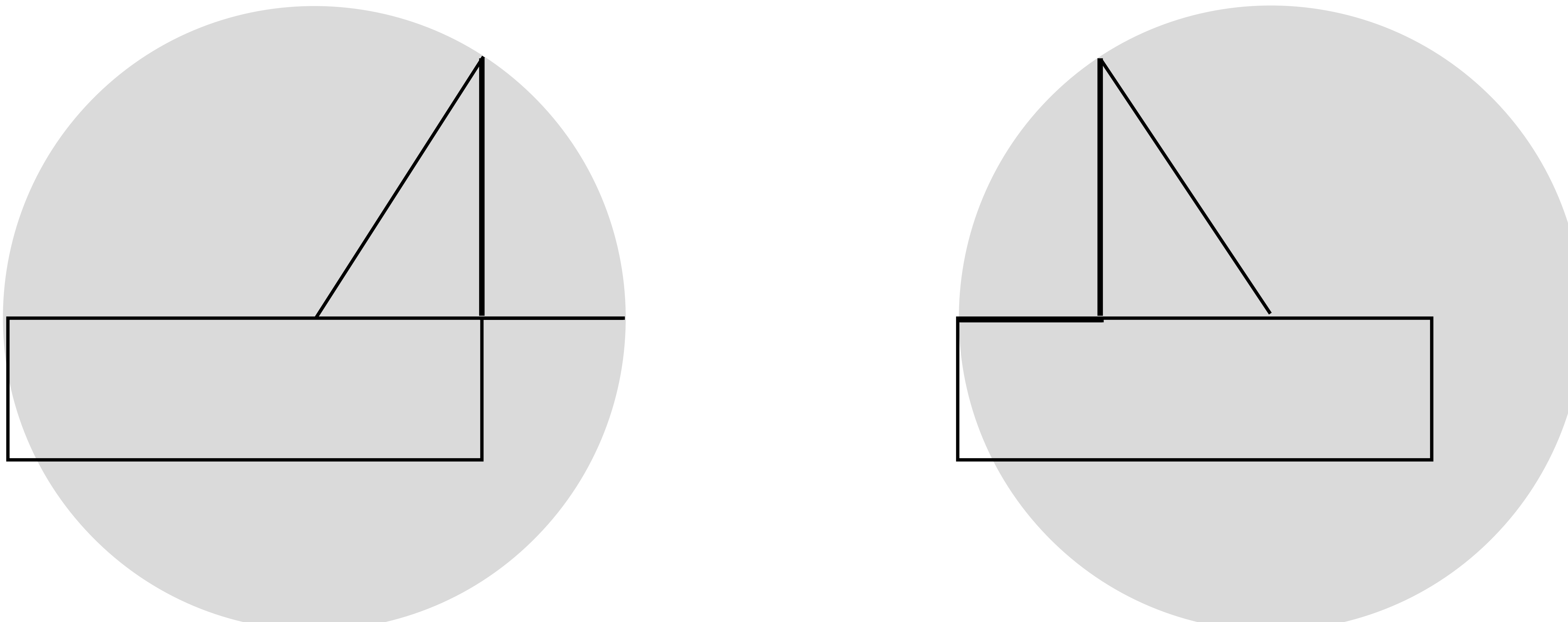

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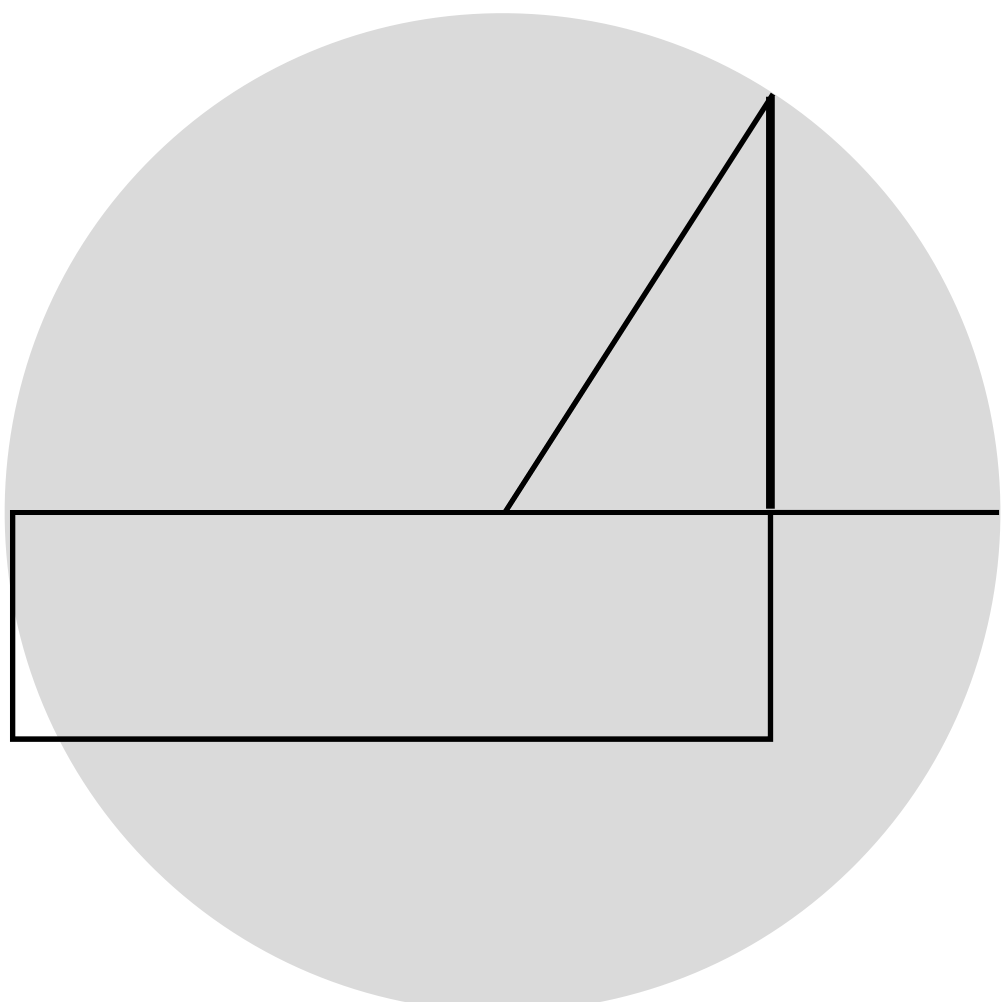
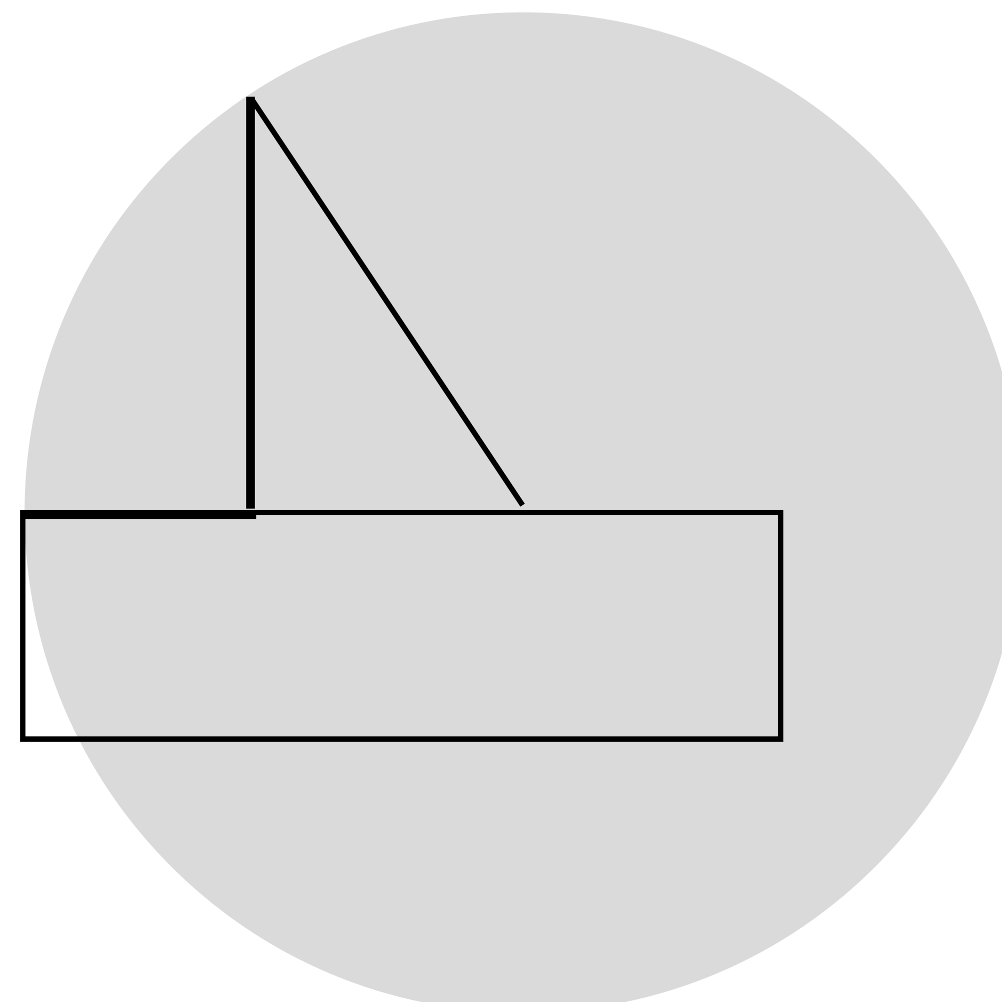
$$\left(\frac{x+y}{2}\right)^2 = xy + \left(\frac{x-y}{2}\right)^2$$

These are special cases of a more general truth…





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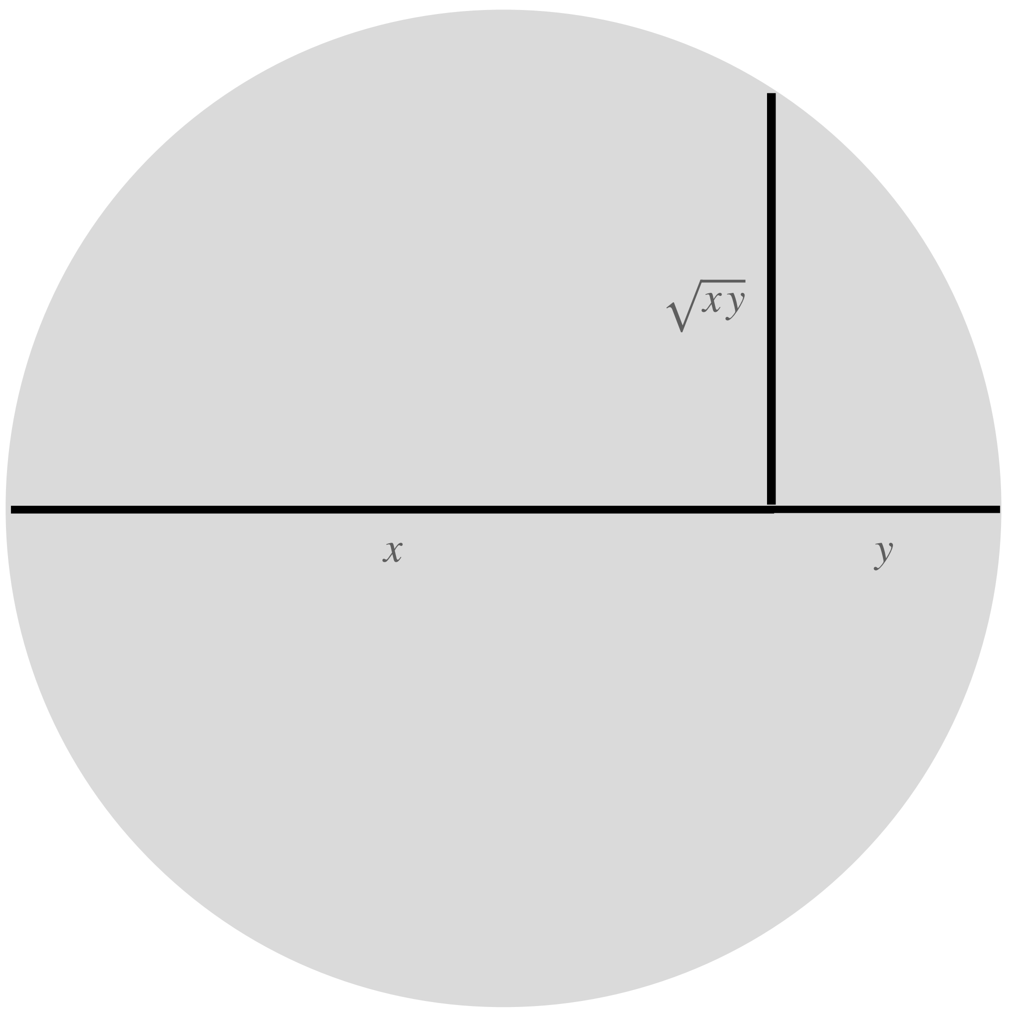
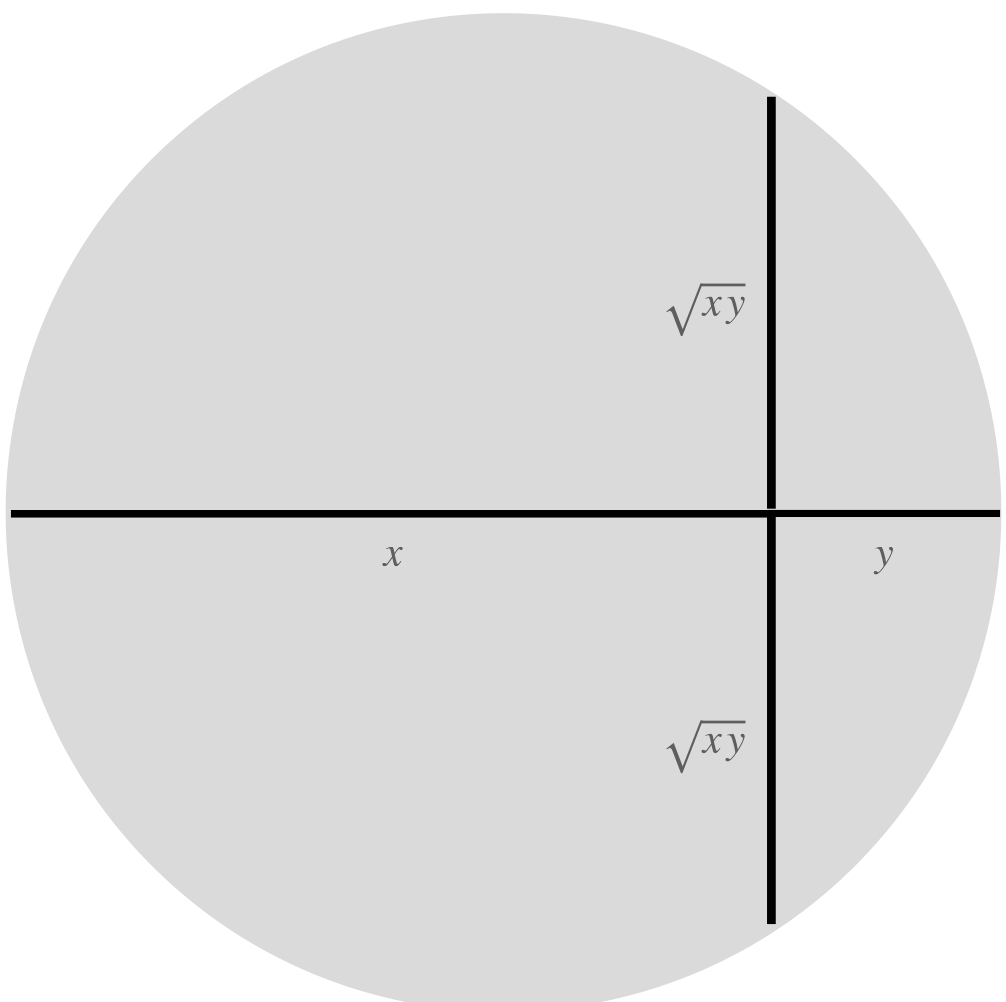
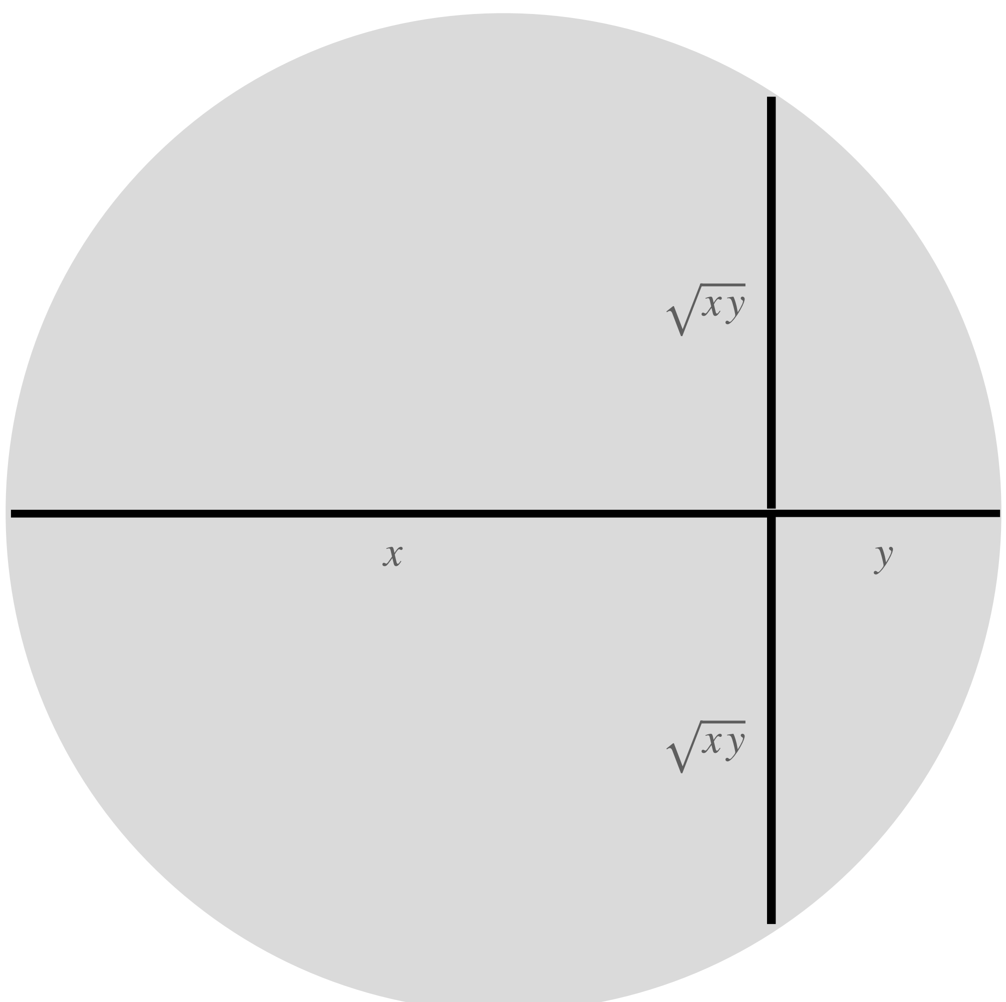
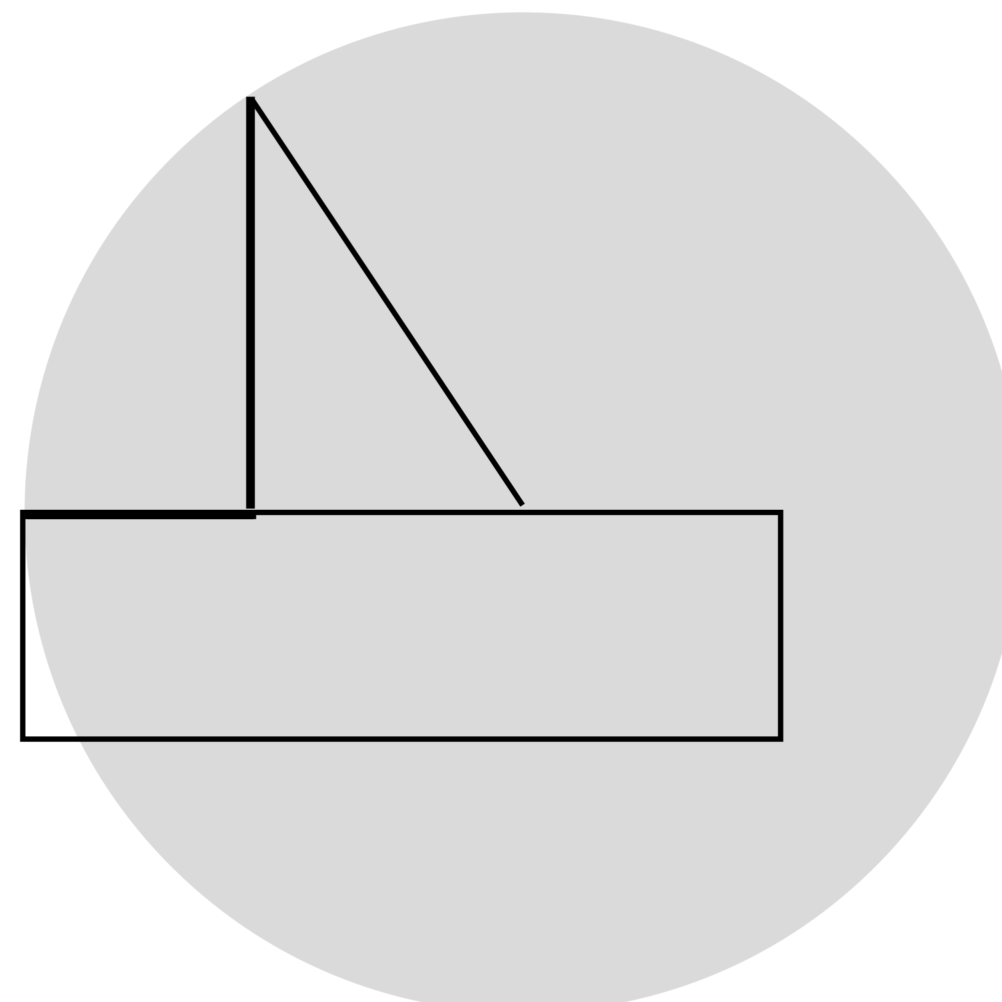


$\sqrt{xy}$
x
y



Two chords cut each other
such that
the rectangles on their segments are equal





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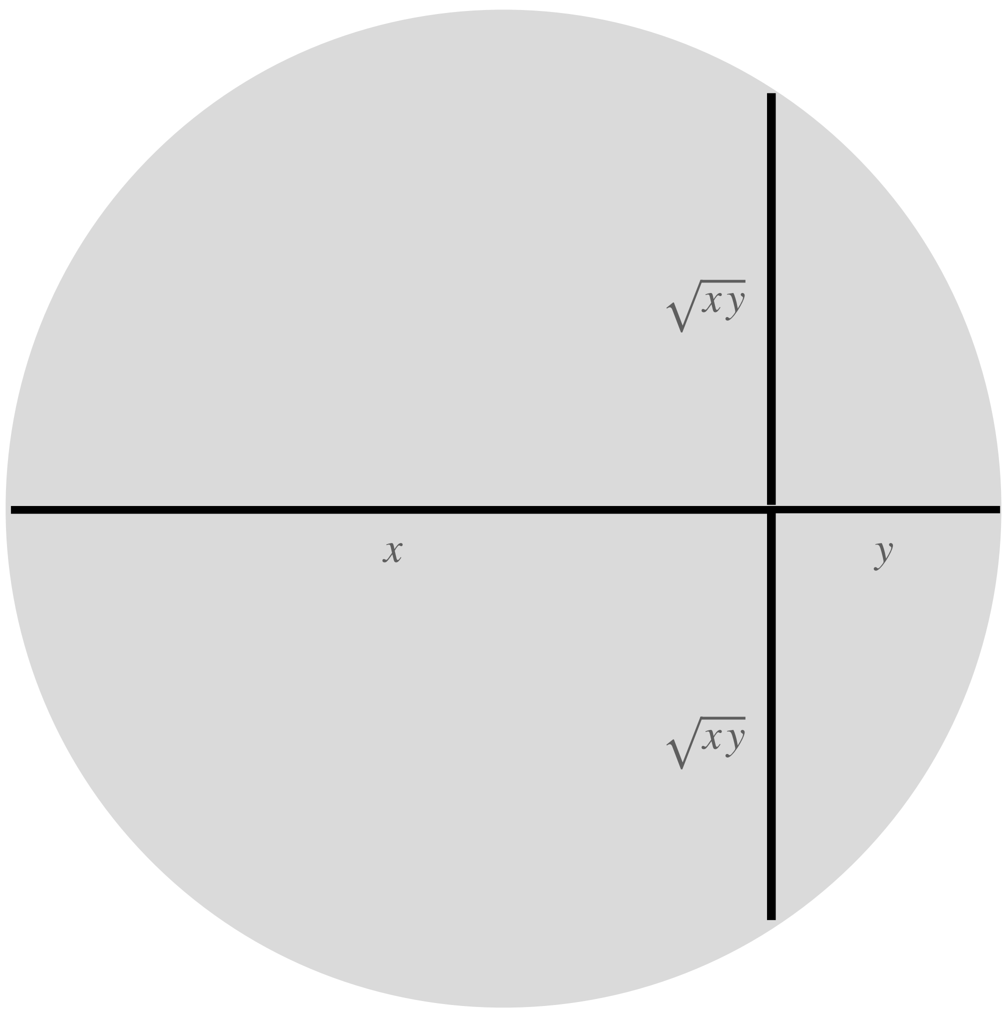

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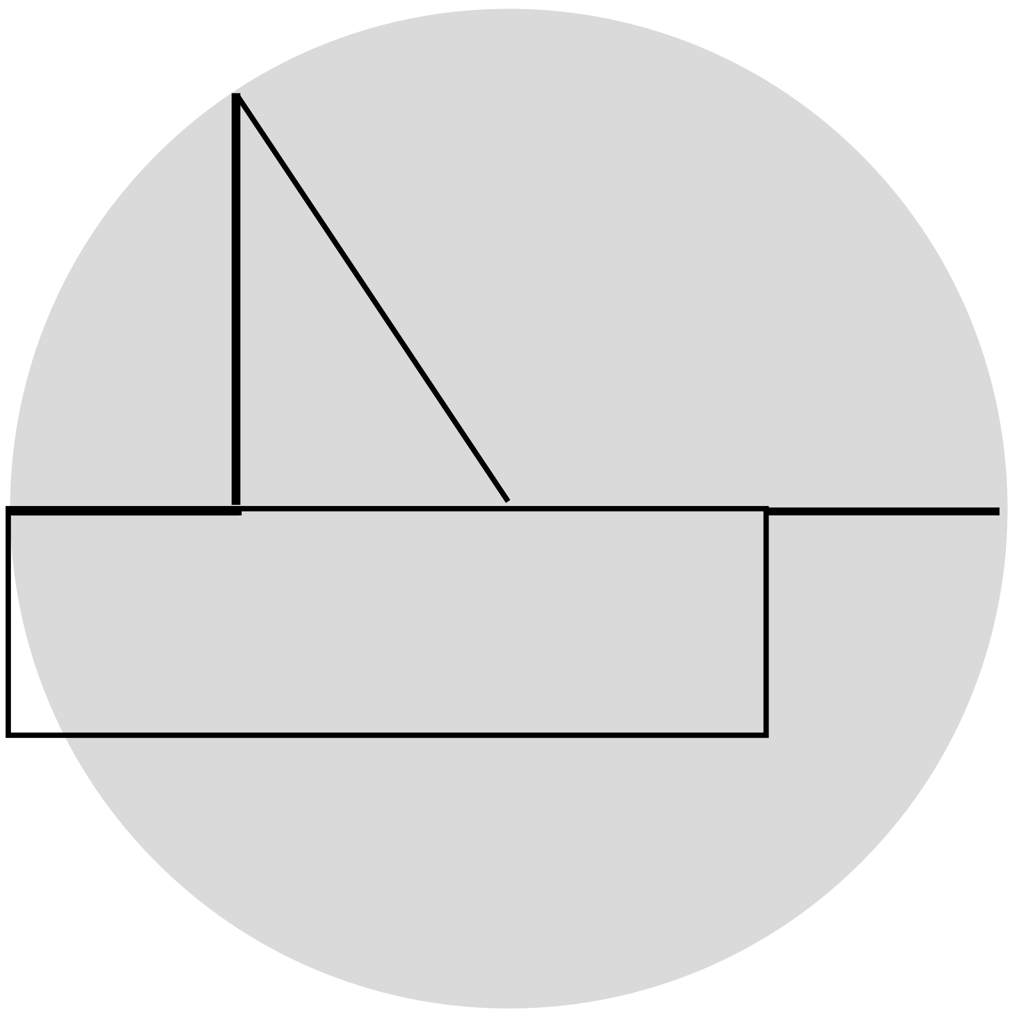

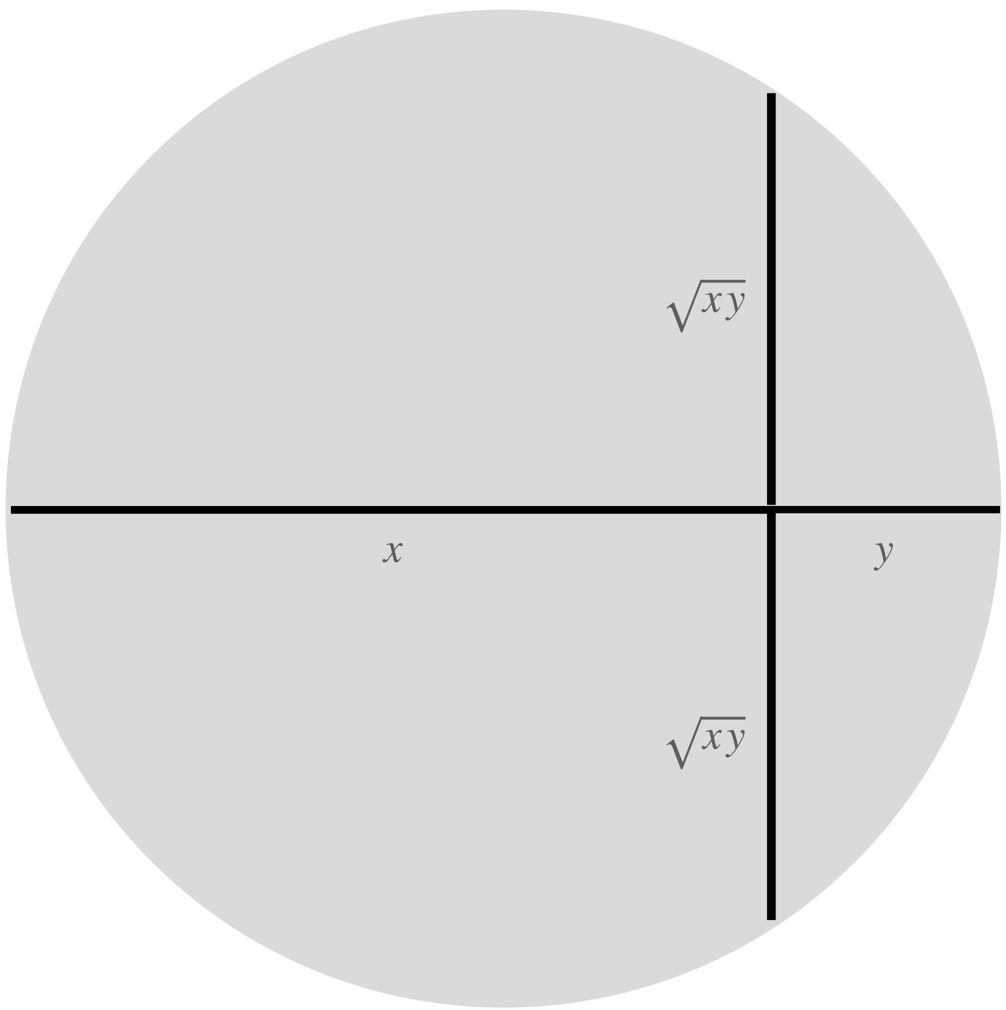

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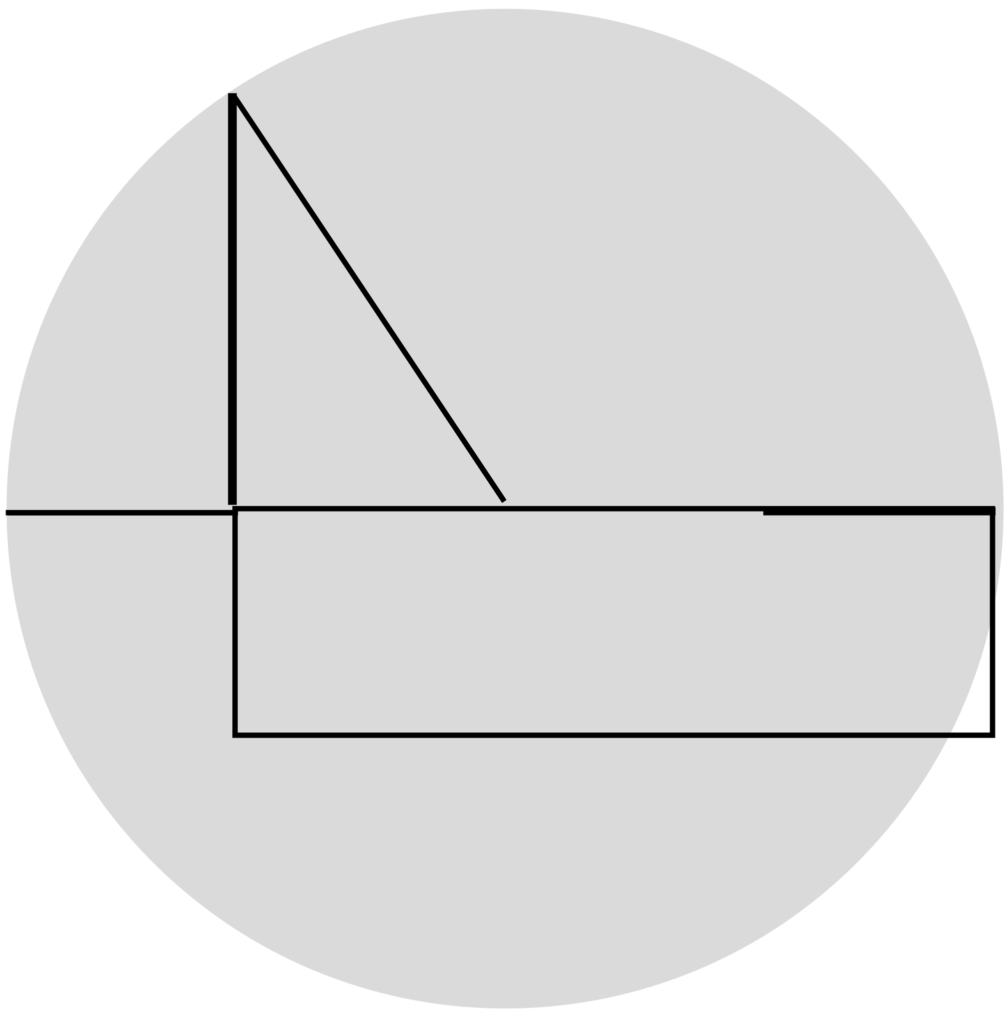

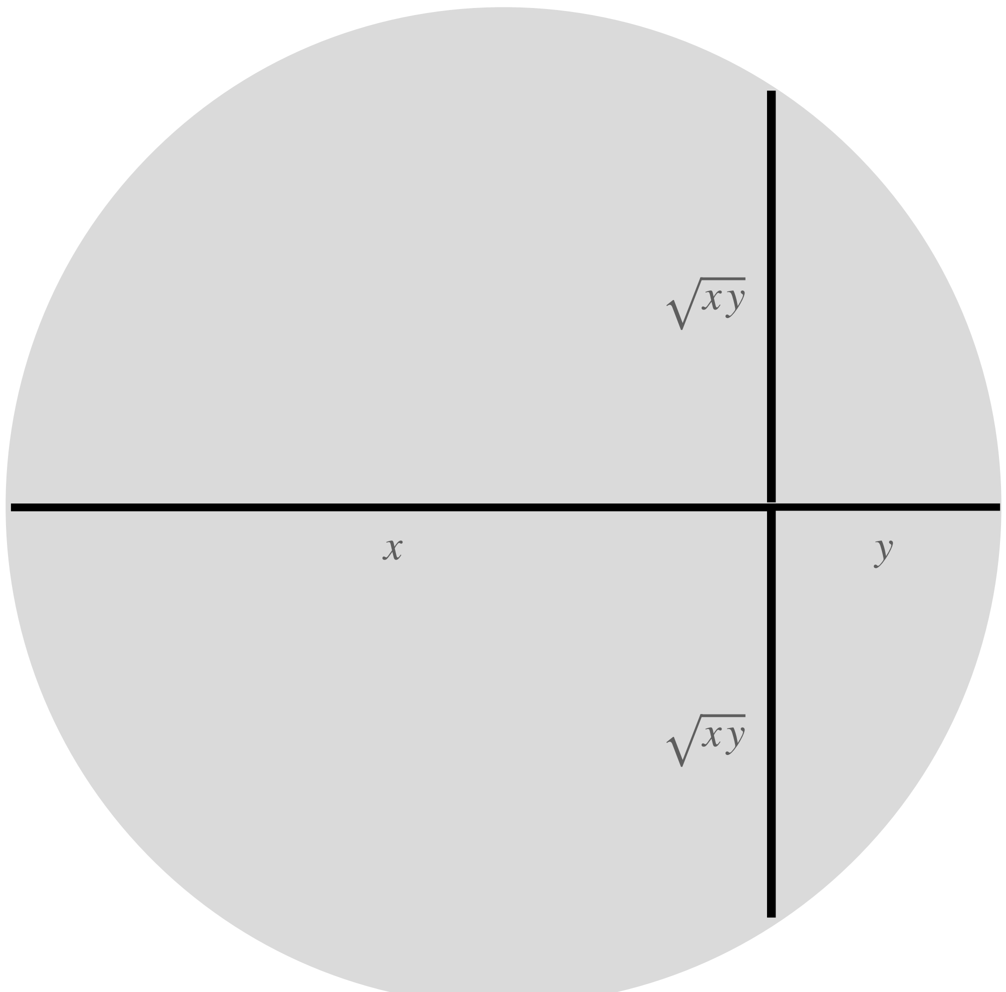

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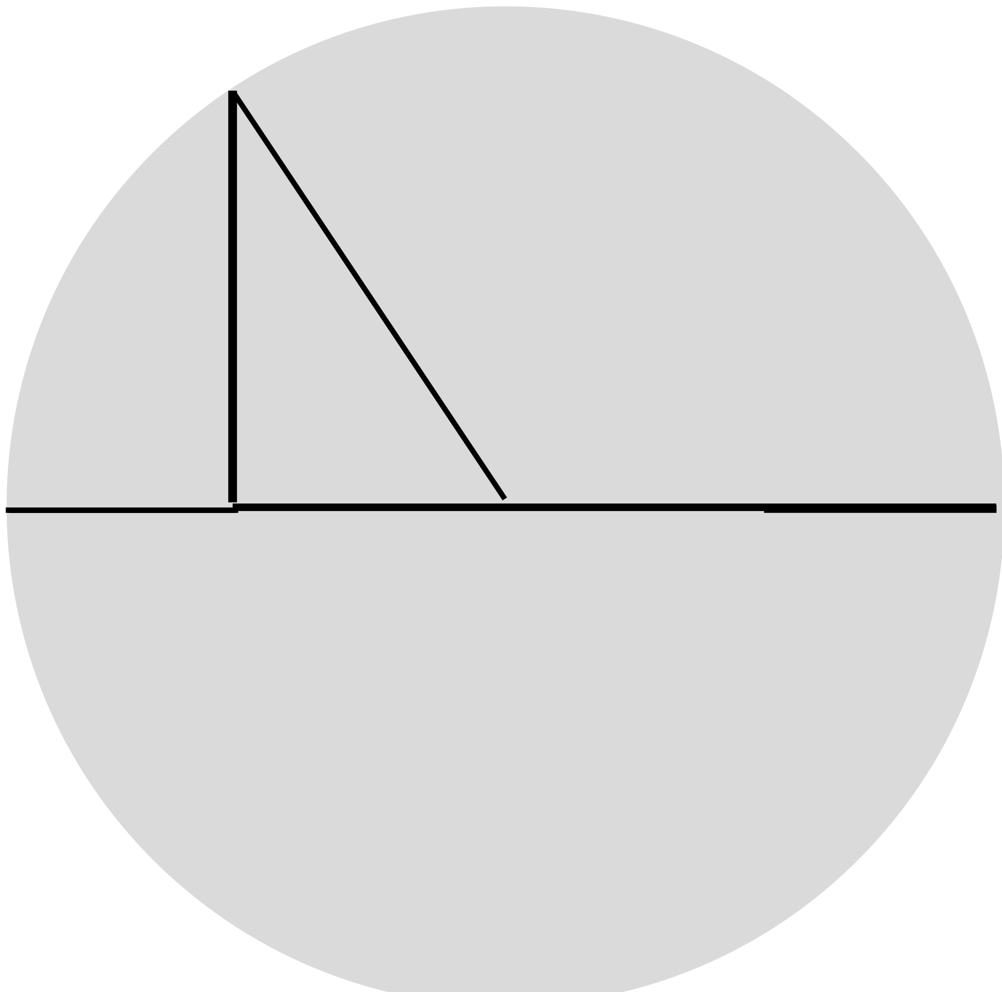
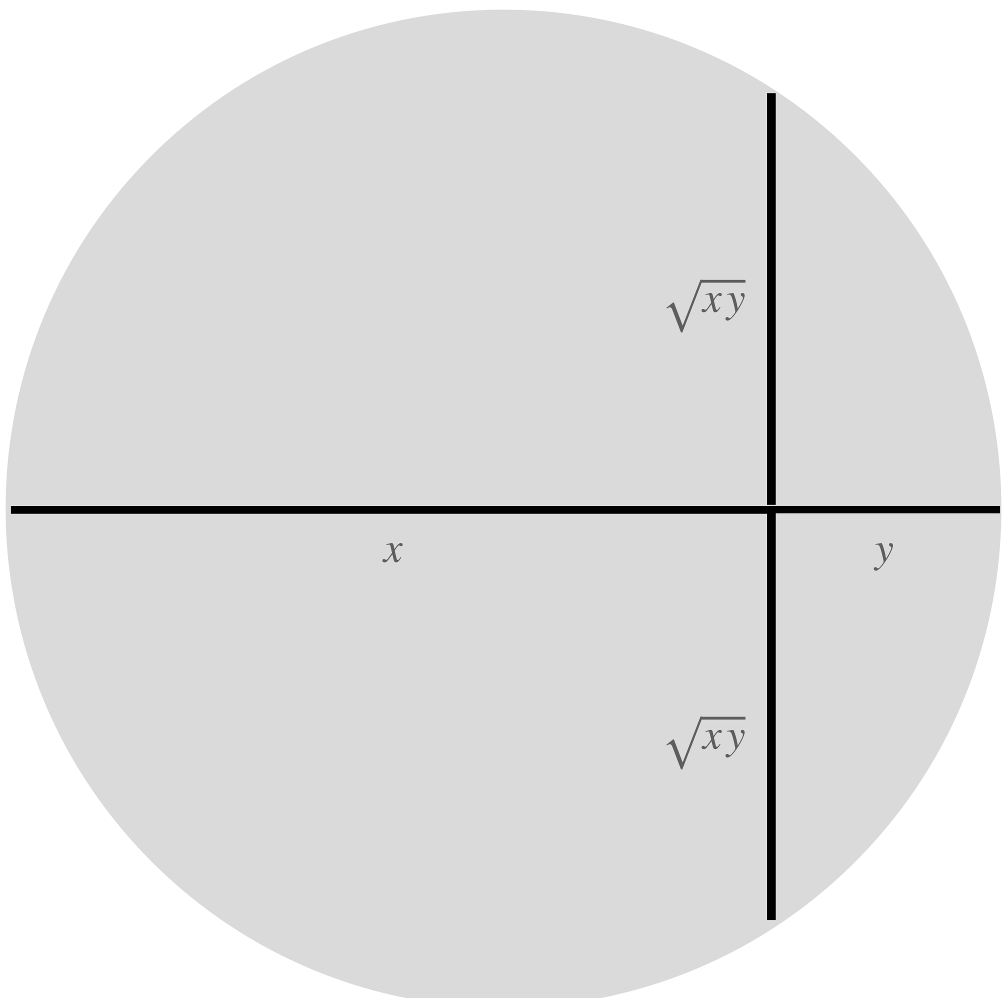
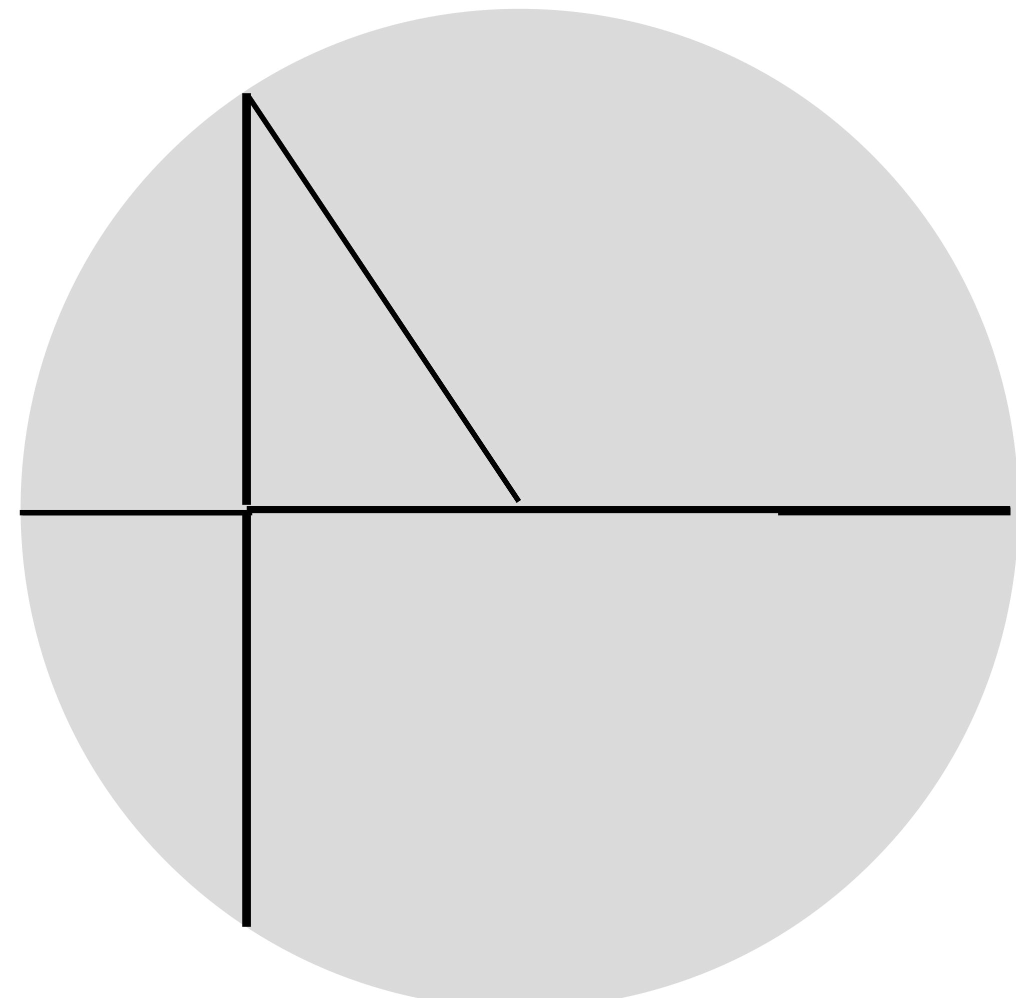
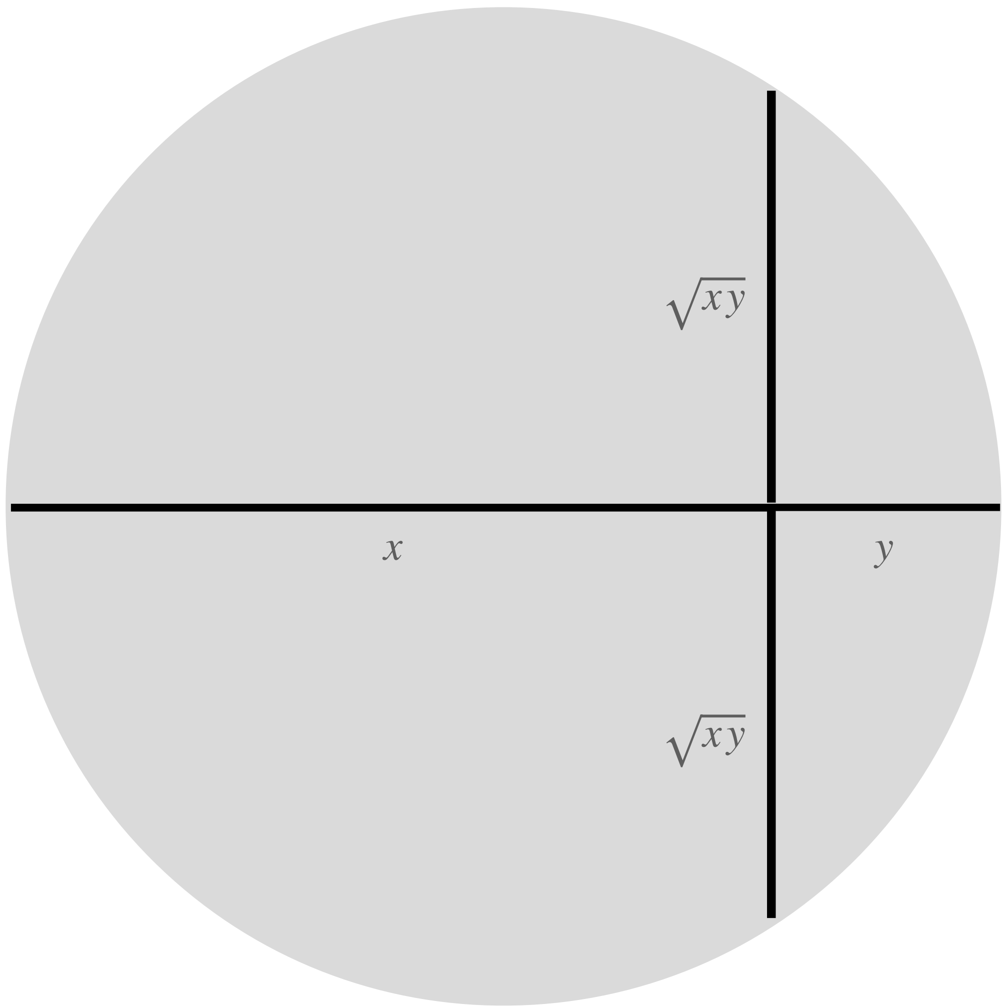
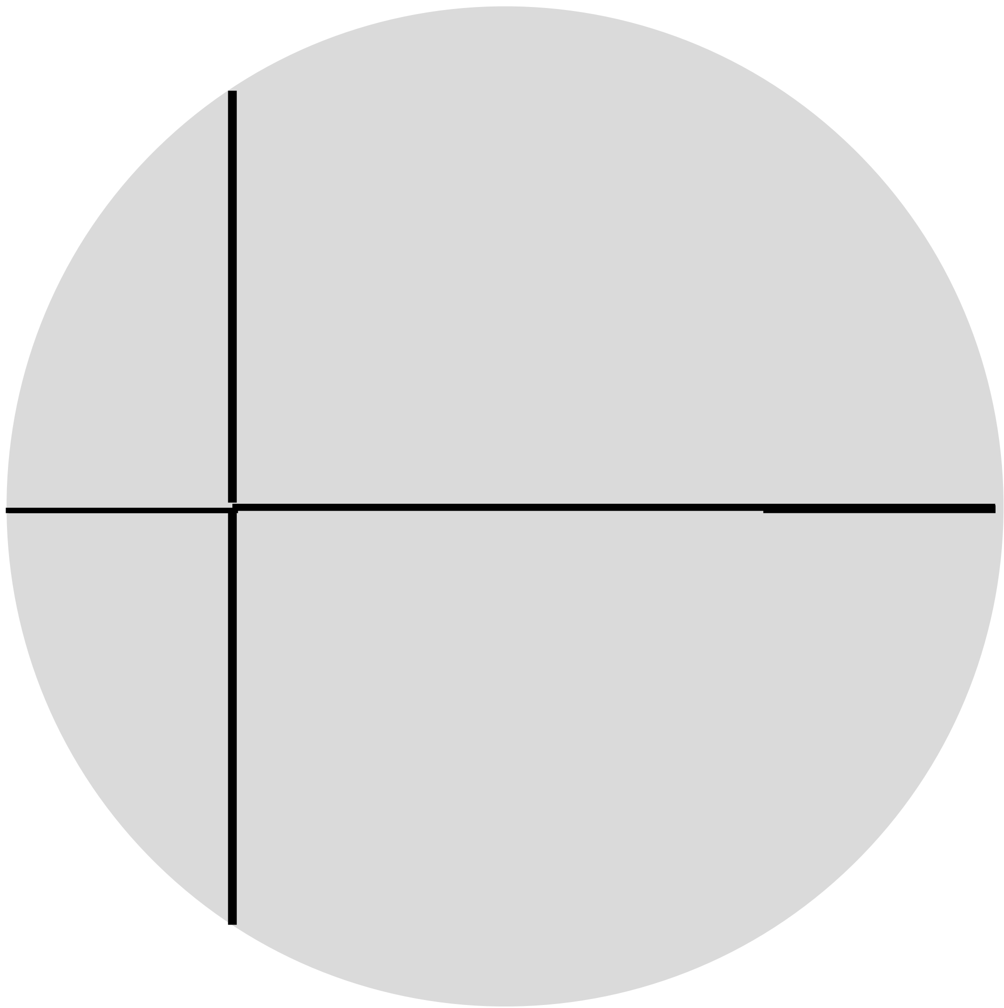






Two chords cut each other
such that
the rectangles on their segments are equal





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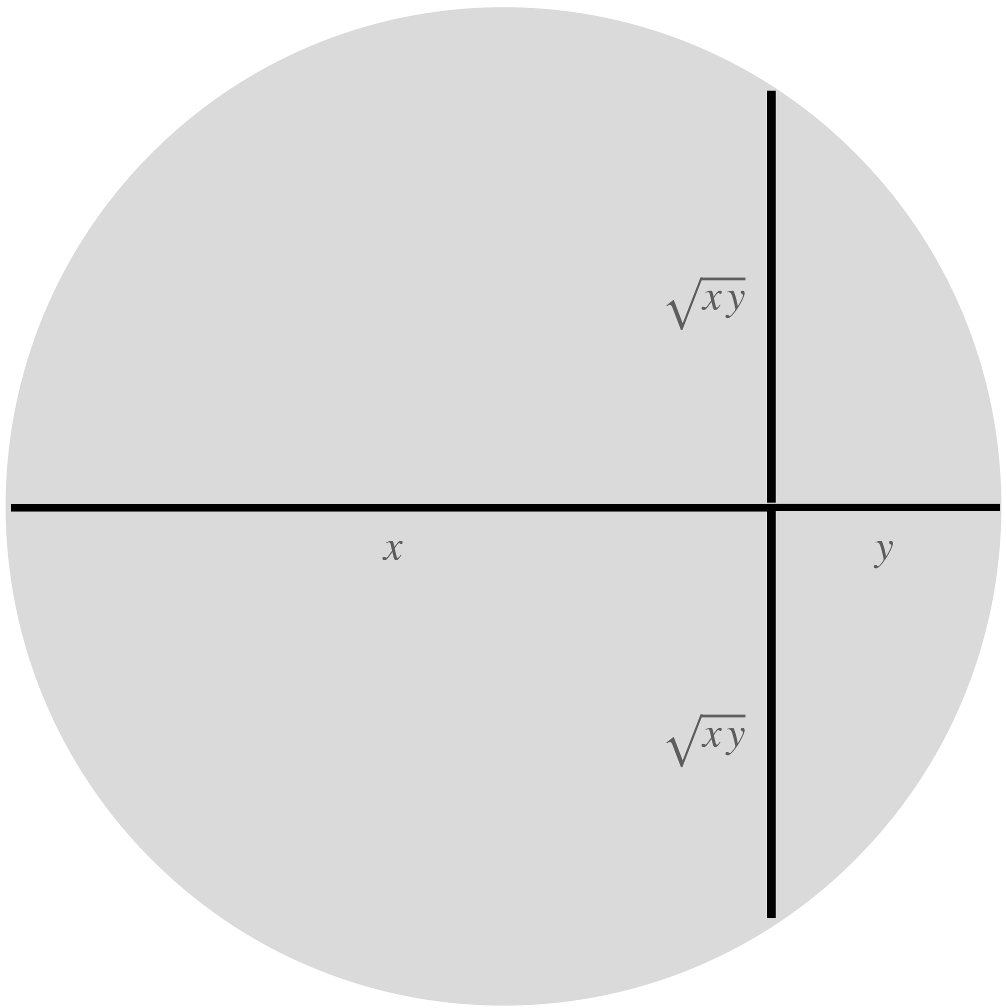

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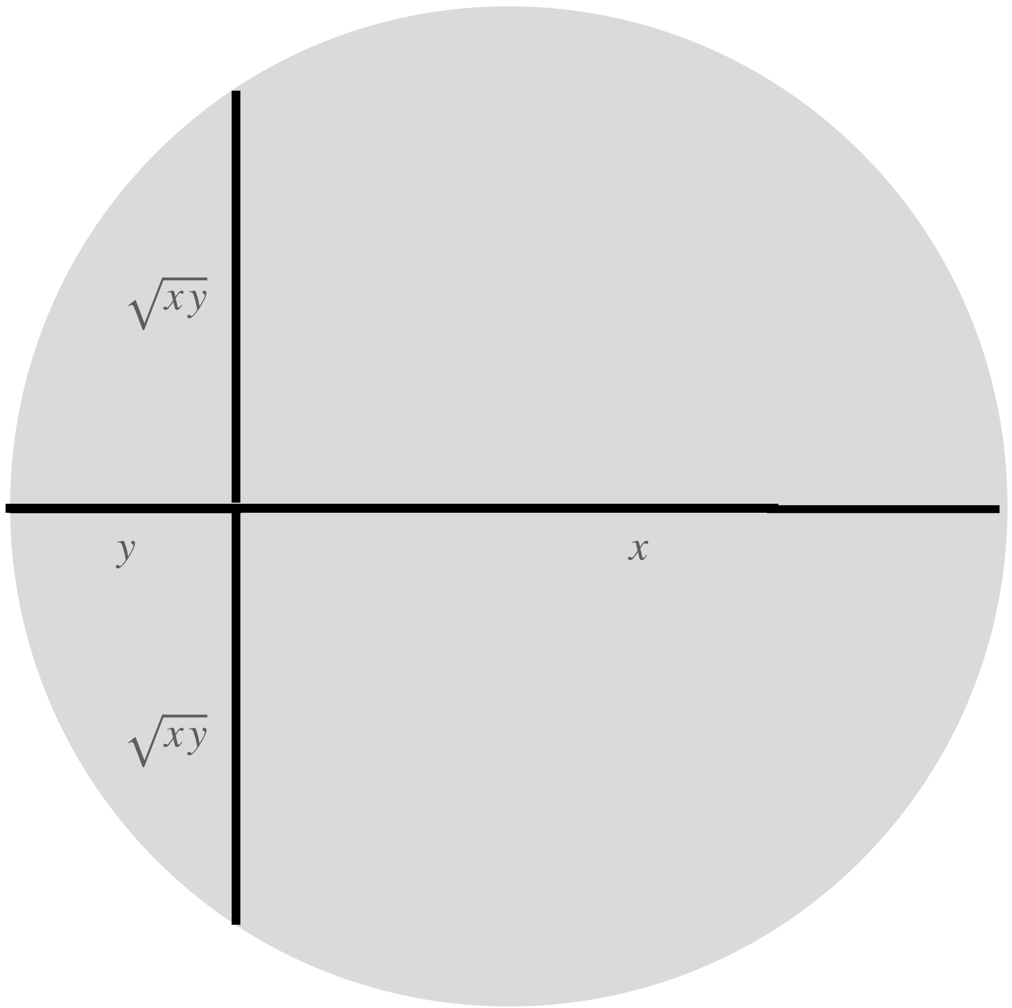

Two chords cut each other
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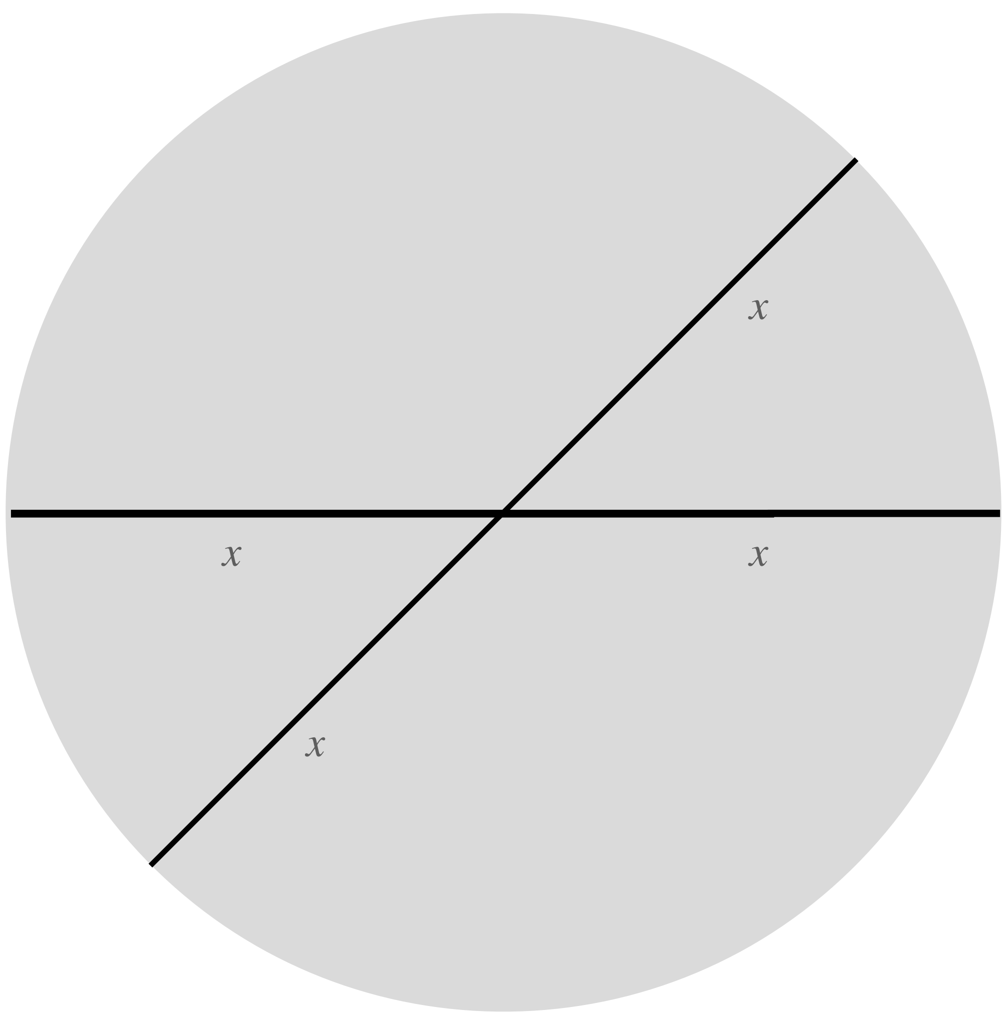
True of any and all chords of any length or intersecting angle?



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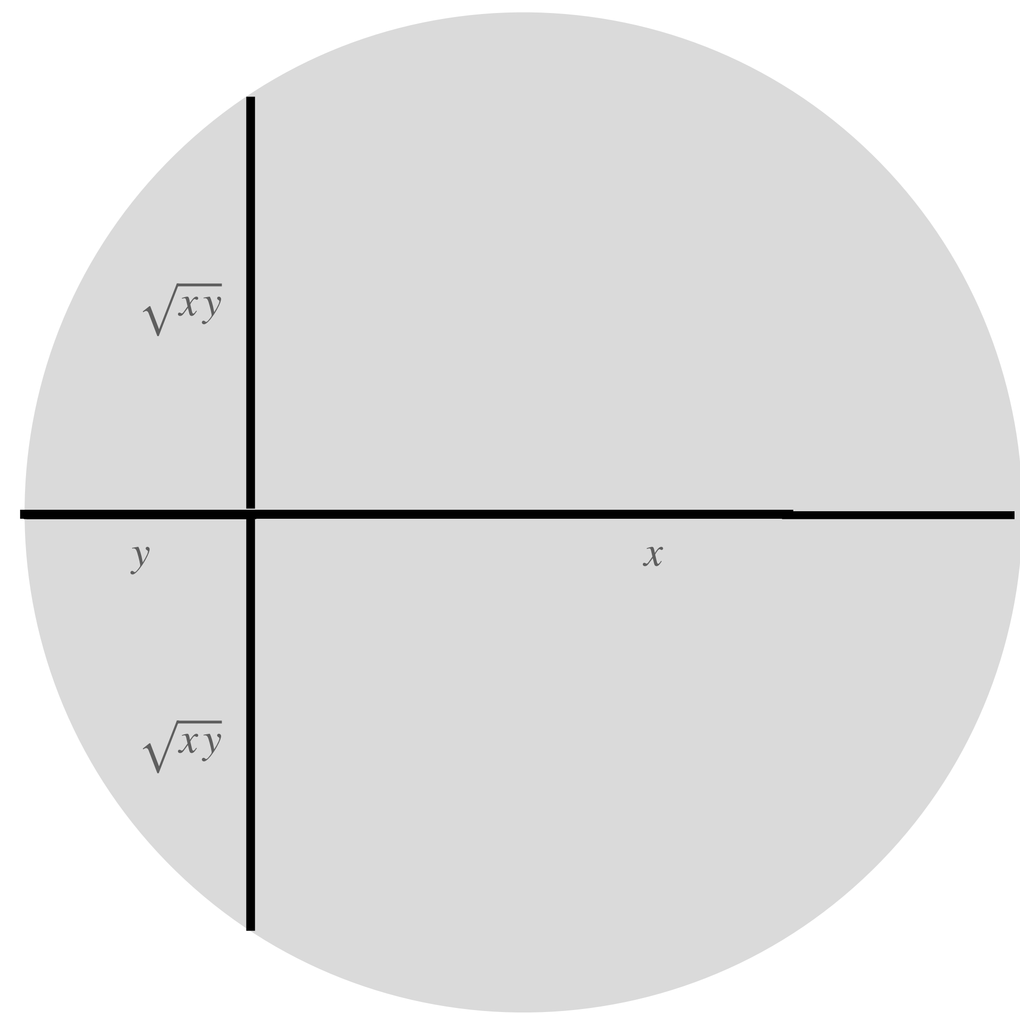

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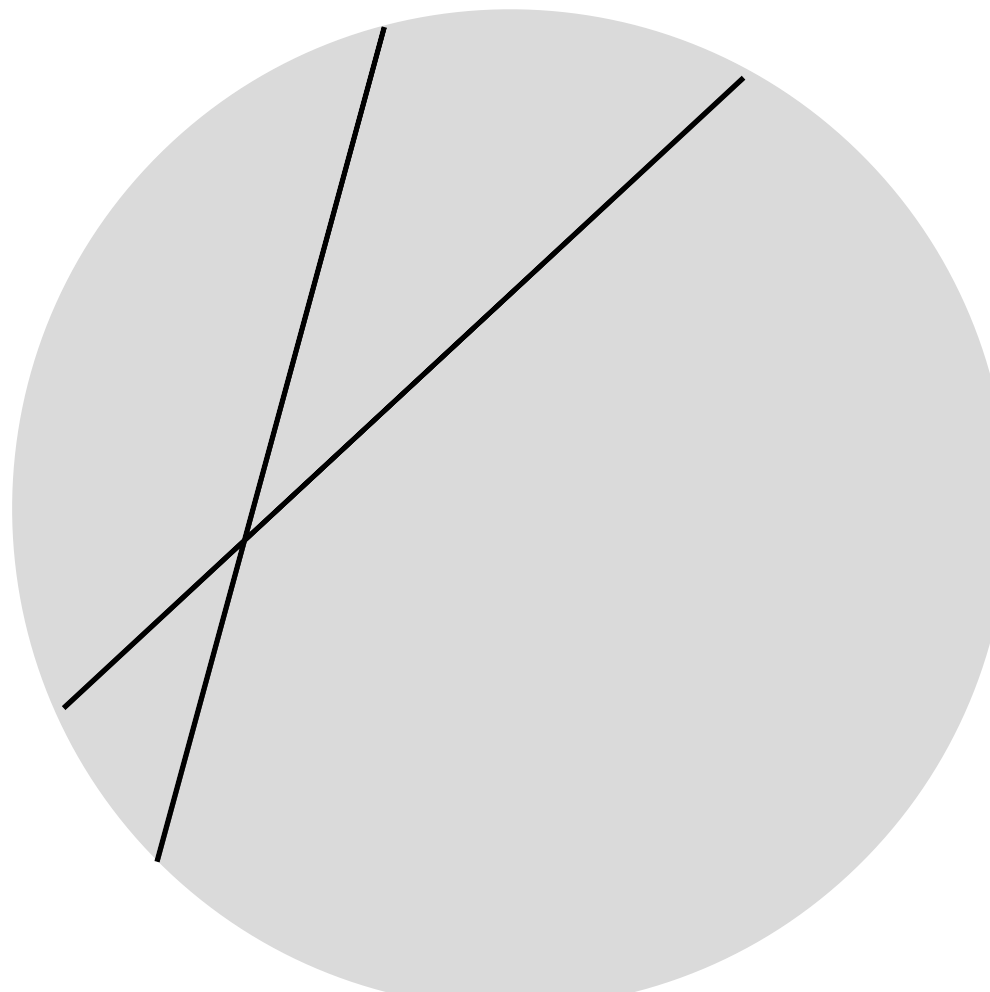
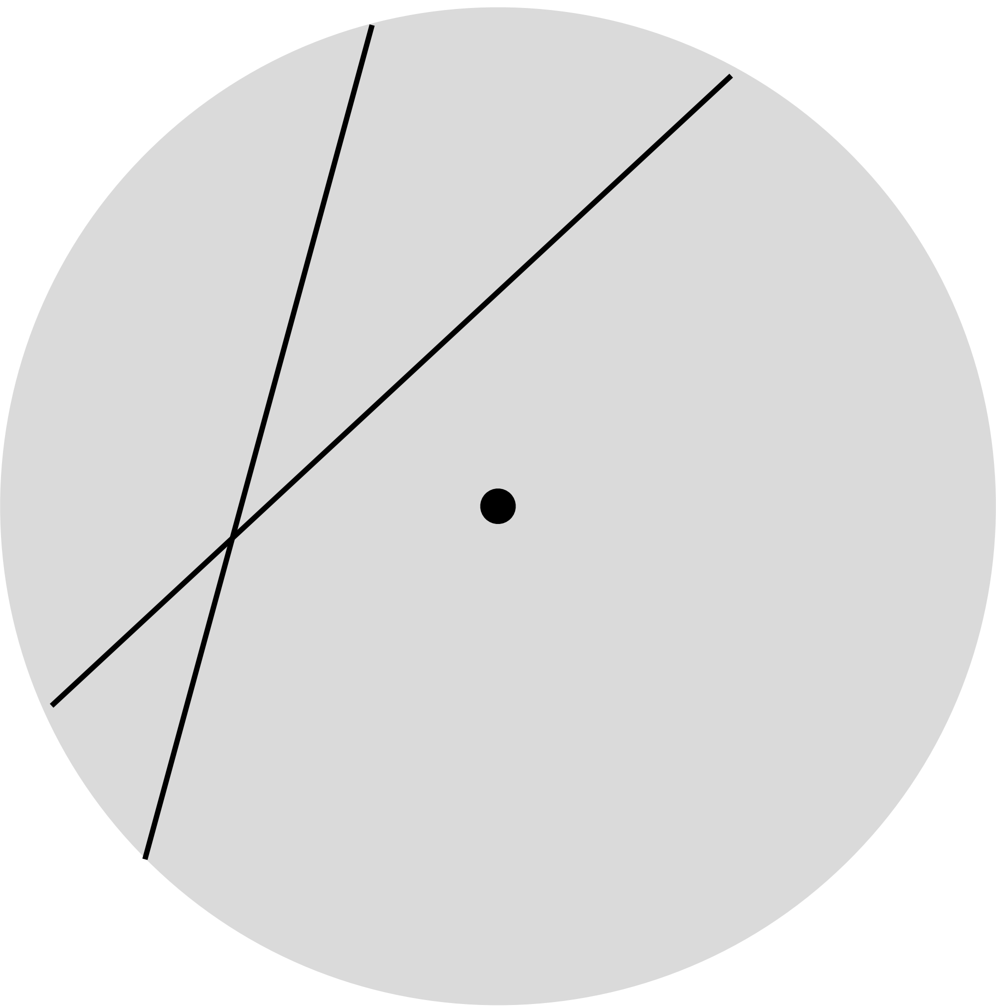




■ BC = ■ CK + □ BK*KA

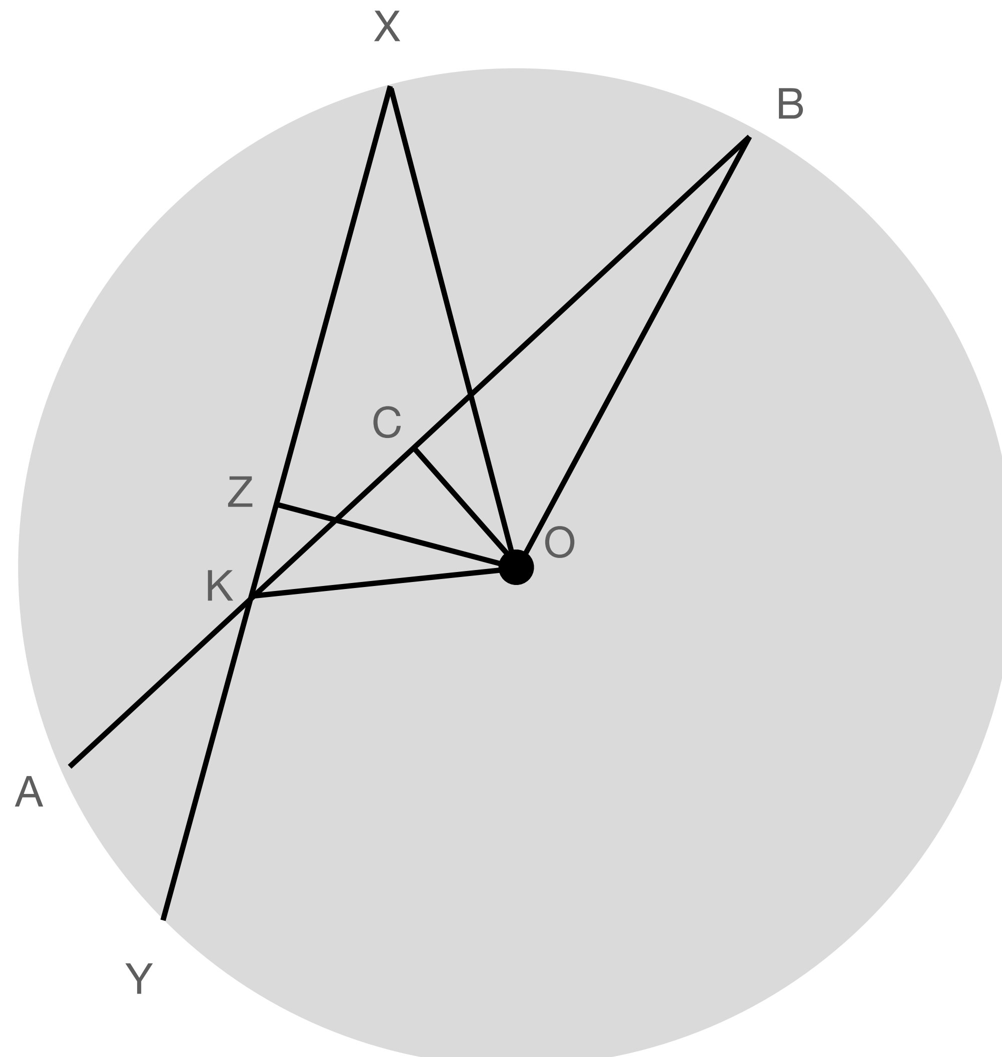
■ BC+ ■ CO = ■ CO + ■ CK + □ BK*KA

■ BO = ■ KO + □ BK*KA

■ XZ = ■ ZK + □ XK*KY

■ XZ+ ■ ZO = ■ ZO + ■ ZK + □ XK*KY

■ XO = ■ KO + □ XK*KY



BO = XO

■ BO = ■ XO

■ BO = ■ KO + □ BK*KA

■ XO = ■ KO + □ XK*KY

□ BK*KA = □ XK*KY

■BC = ■CK + ▯BK*KA

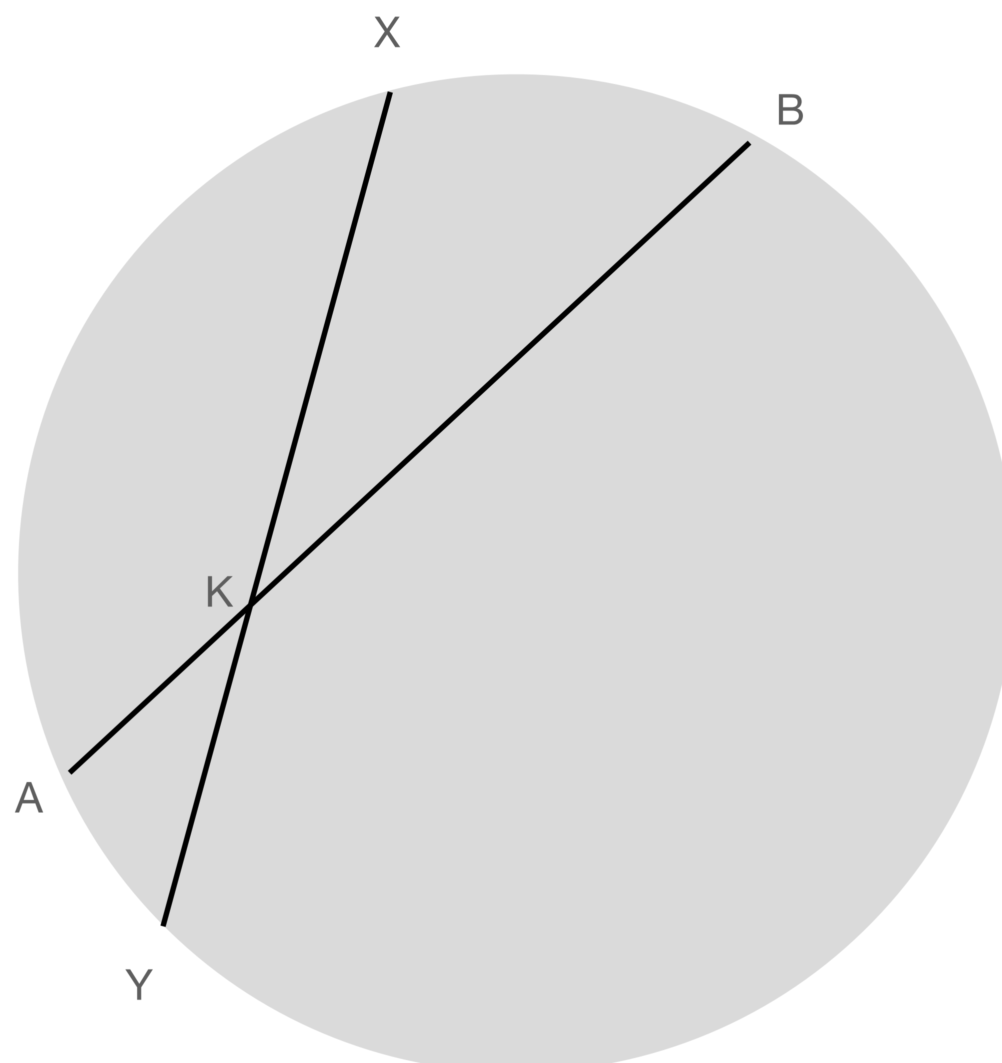
■BC+■CO = ■CO + ■CK + ▯BK*KA

■BO = ■KO + ▯BK*KA

■XZ = ■ZK + ▯XK*KY

■XZ+■ZO = ■ZO + ■ZK + ▯XK*KY

■XO = ■KO + ▯XK*KY



BO = XO

■BO = ■XO

■BO = ■KO + ▯BK*KA

■XO = ■KO + ▯XK*KY

▯BK*KA = ▯XK*KY

Two chords cut each other
such that
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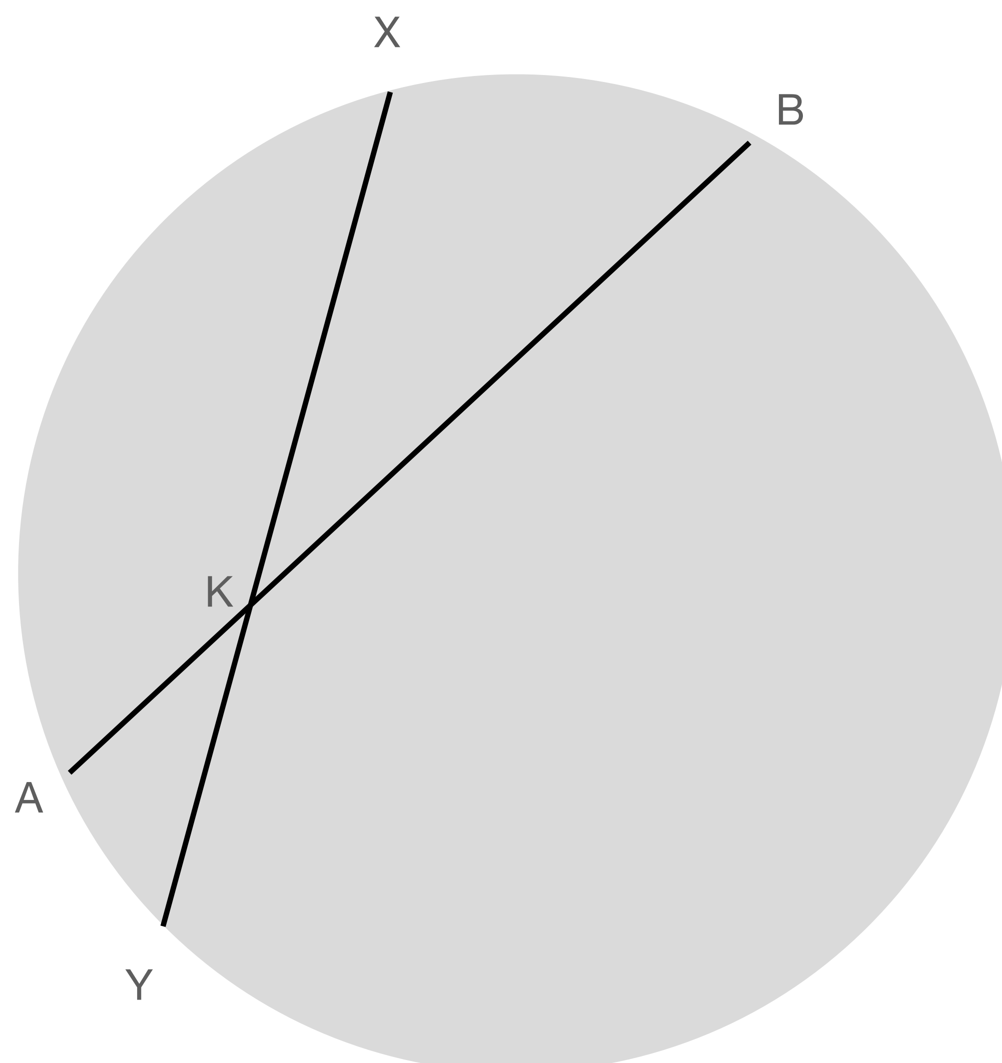


Recipe for sqrt(N):

N=pq (could be 1,N)

Draw diameter p+q, and its circumference

Mark p

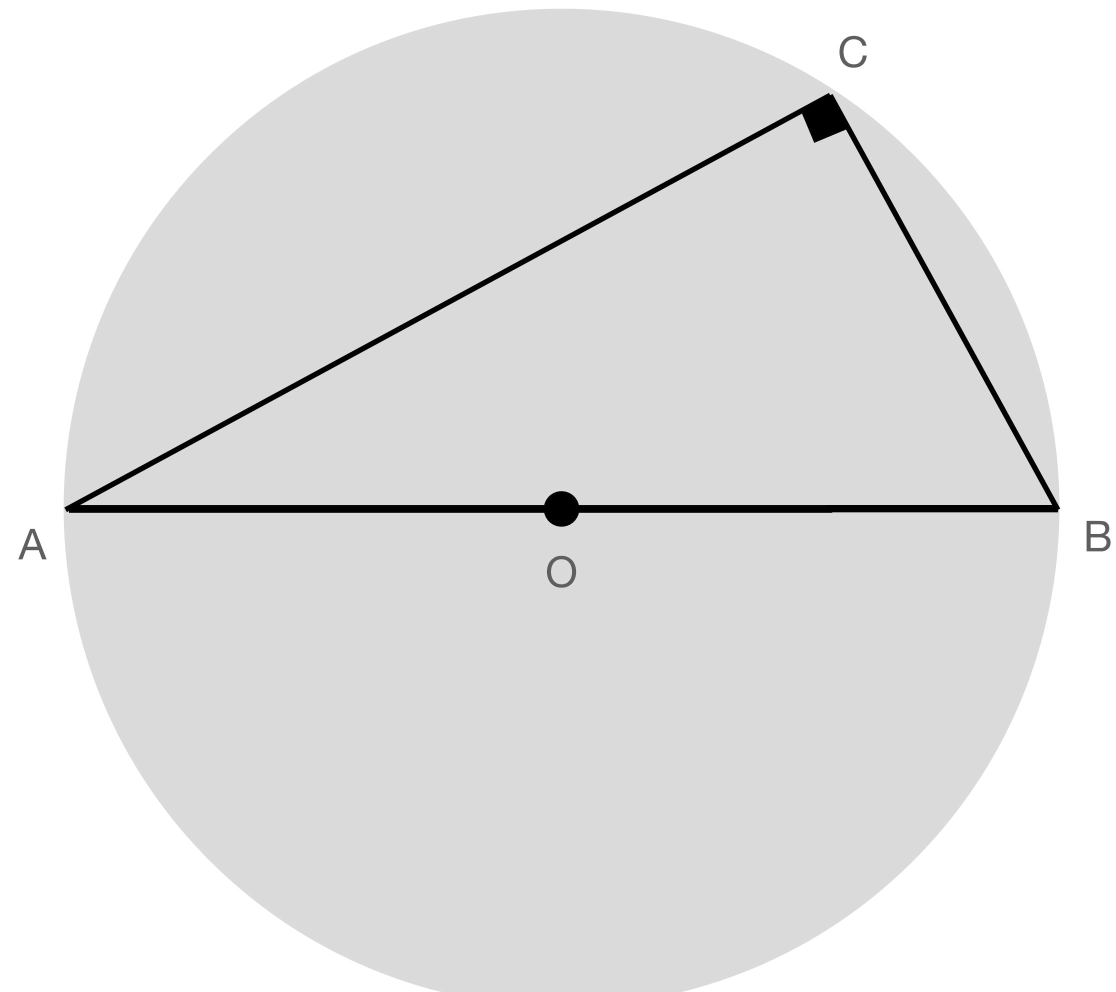
At p draw perpendicular h to circumference

(diameters bisect perpendicular chords)

h*h = p*q = N by above theorem

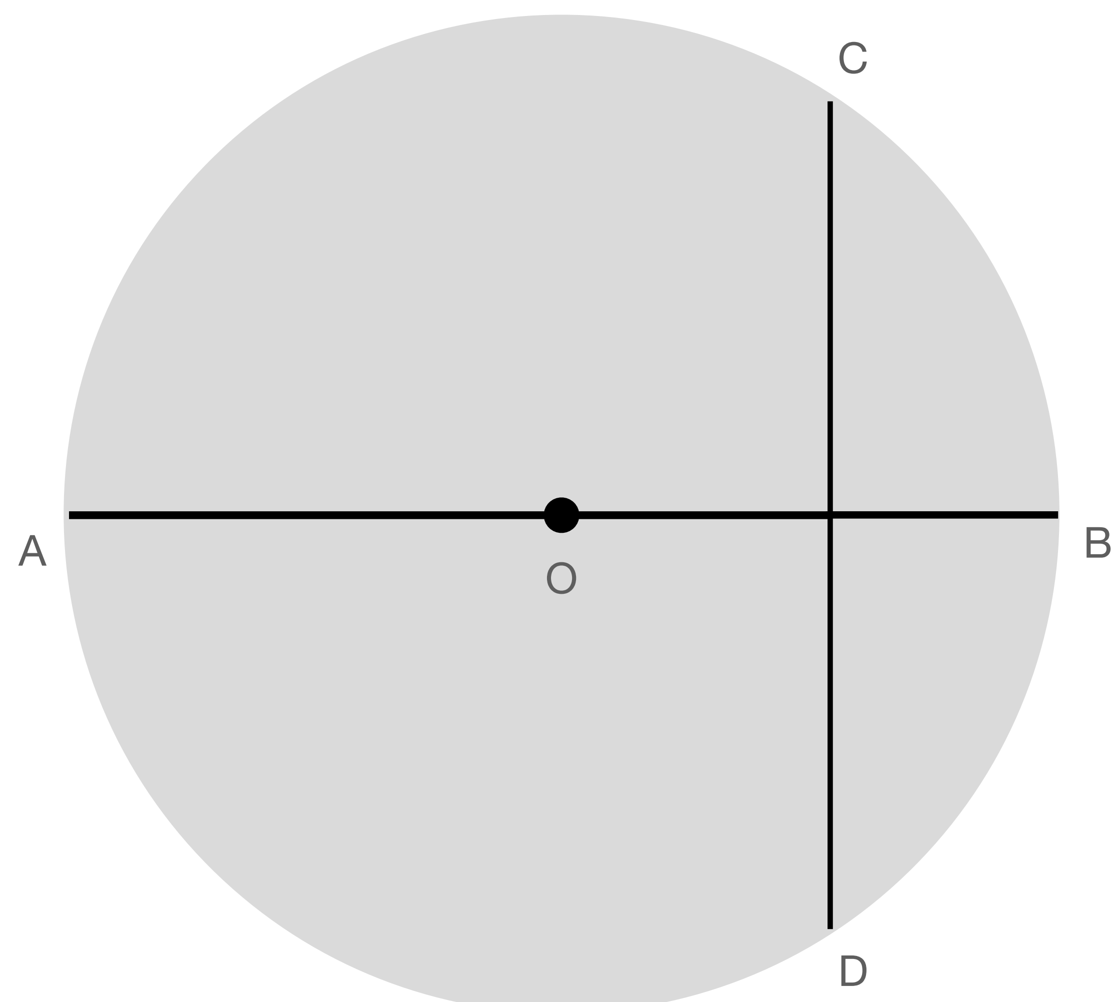
Questions

I claimed without proof that triangles drawn on diameters are right. Are they? Why?



Hint: start by drawing CO and identifying isosceles triangles

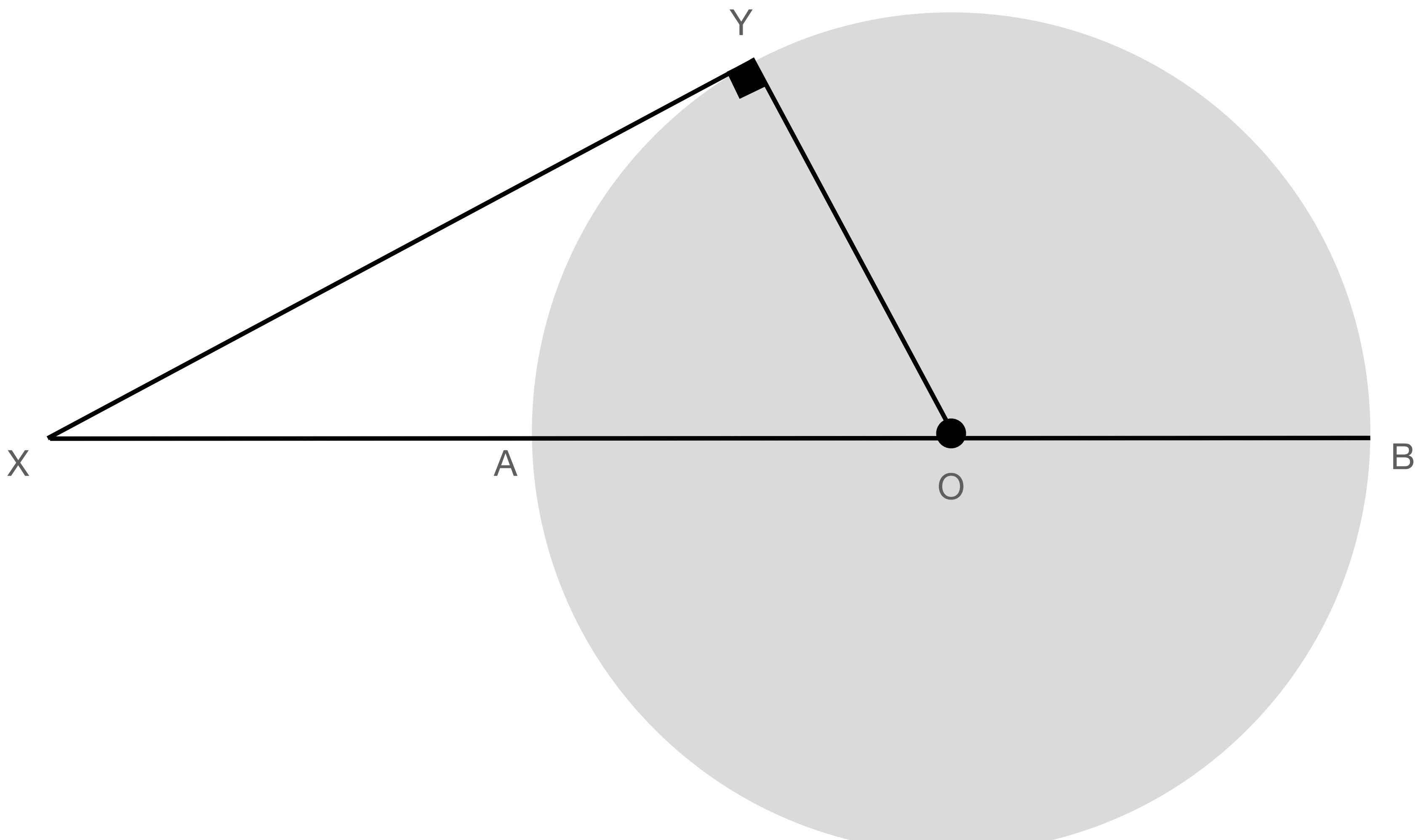
I also claimed without proof that a diameter cutting a chord at right angles is a bisection of the chord. (I could also have said: a diameter that bisects a chord does so at right angles). Why is all that true?



Questions

Consider tangent XY and the statement ■XY = ▯ XA*XB. Why is this true?

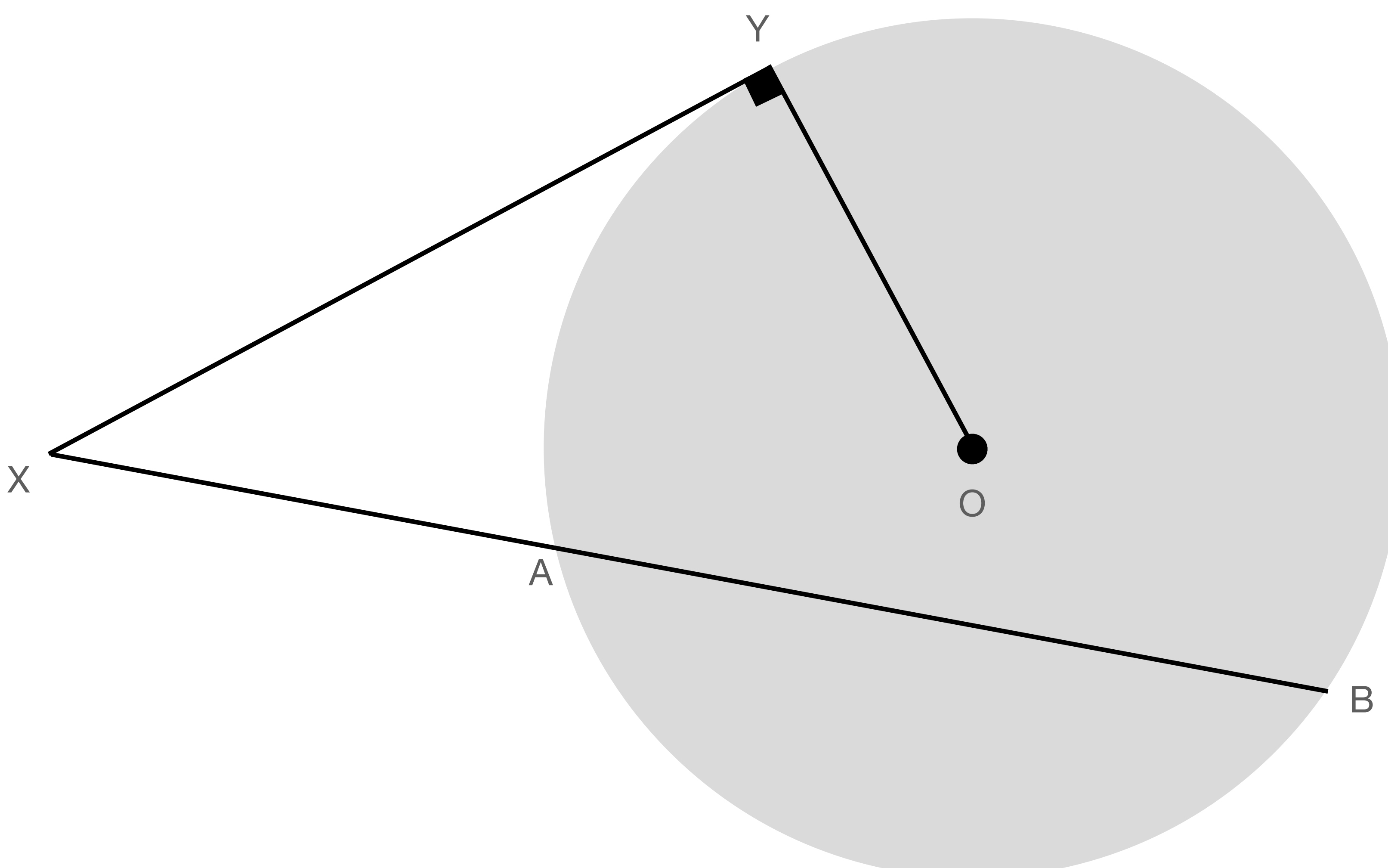
This is another method to derive square roots.



Hint: consider the line XB, in which AB is divided into equal sections and XA added to it

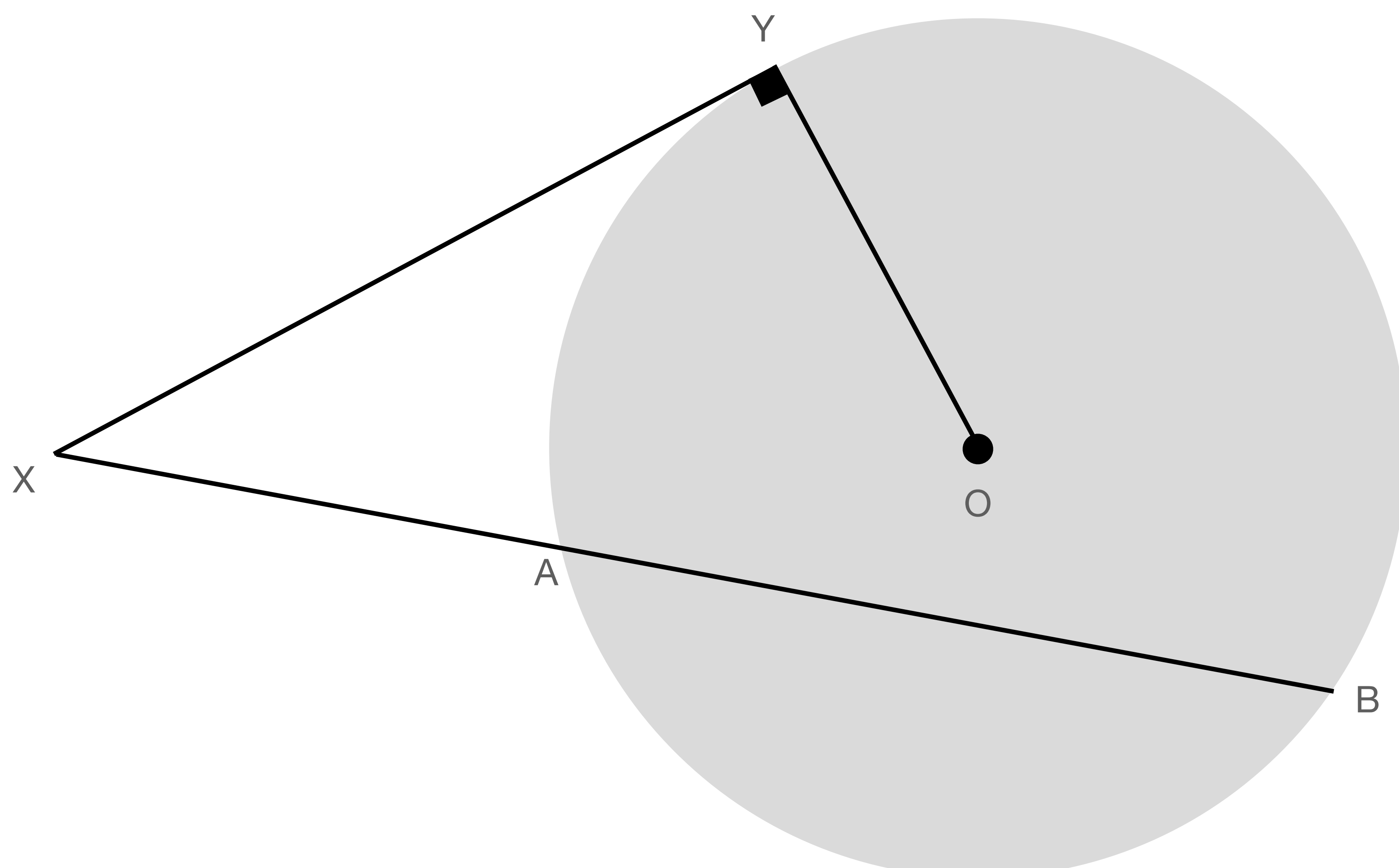
Questions

Consider ■XY = ▯XA*XB. Is this true when XB does NOT include the diameter?



Hint: yes, but why?

Does the size of the circle matter to the truth of the construct?
What constraint on the solution space does the circle represent?



Questions

Reverse the construct: suppose I have a square root (XY), and I want to know how to reapportion its square area into a rectangle (XA*?).