Mental exercise:

Rejecting the null hypothesis

What is a null hypothesis?

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Typically a dummy argument

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E.g. you measure height in girls and boys

- you suspect they differ
- null hypothesis (H0): boys and girls have the same height

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WHY THIS TORTURED LANGUAGE?

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I'll show 2 examples from 400 BC.

Euclid's Elements

I.1: given a line segment, construct an equilateral triangle

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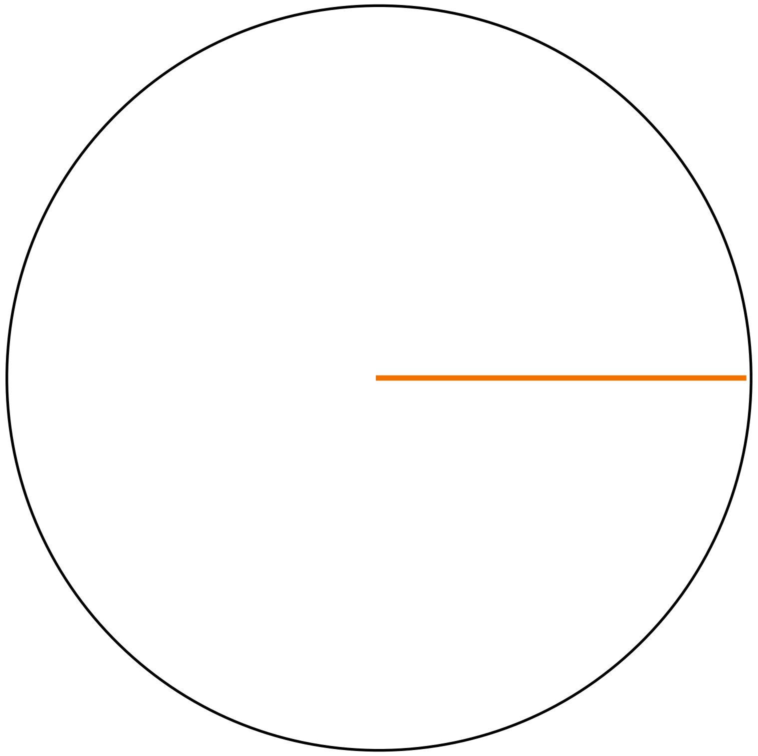

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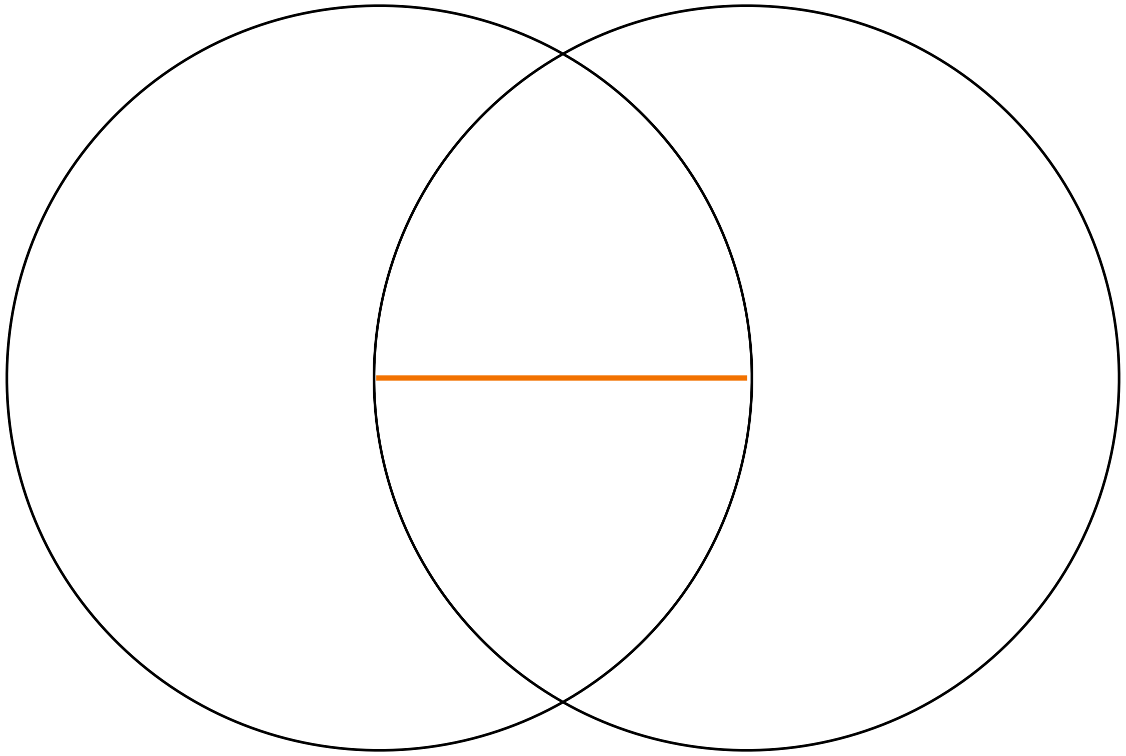

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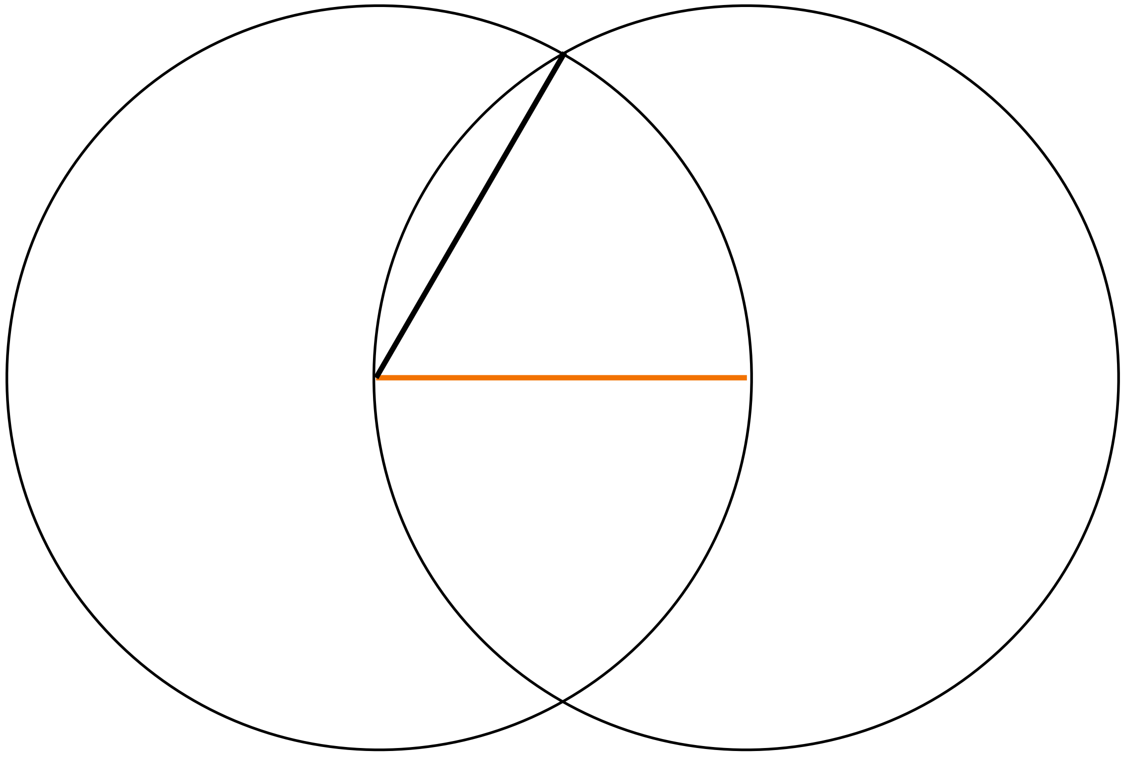

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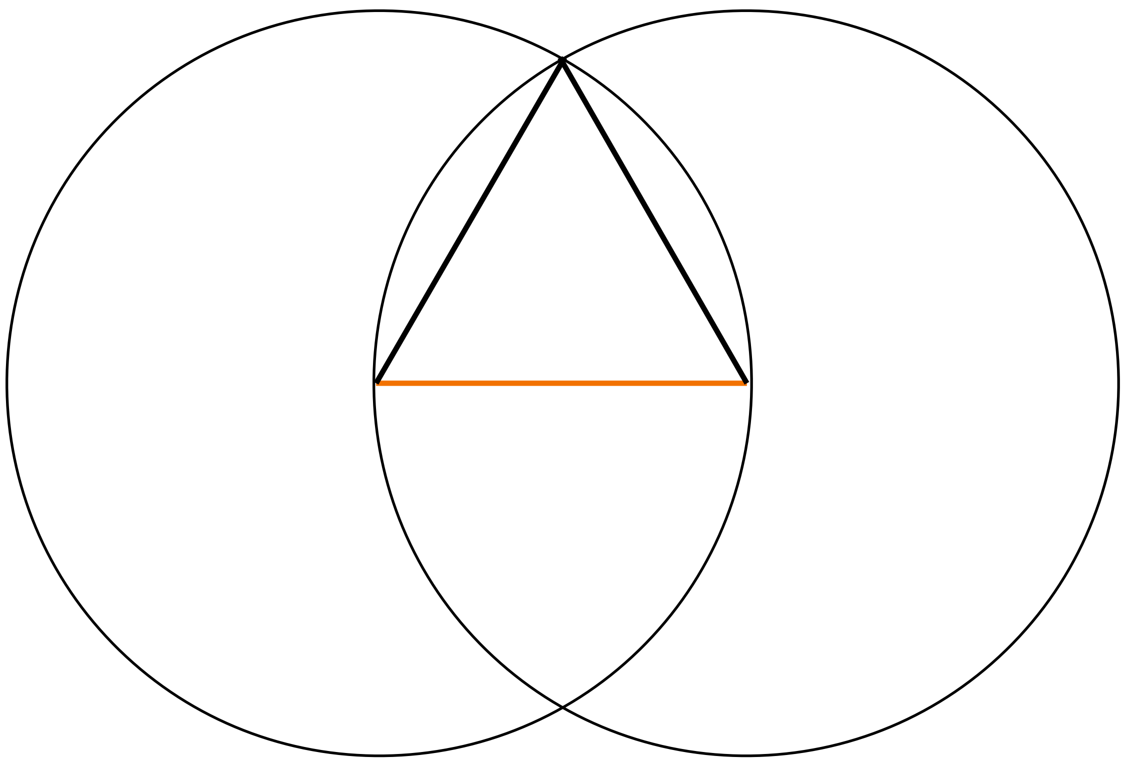

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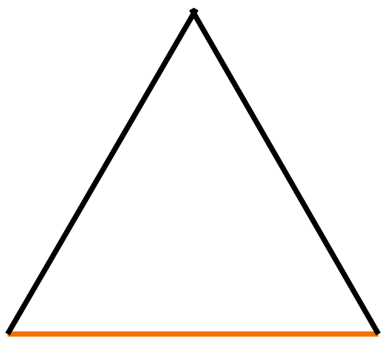

Euclid's Elements

I.2: move a line segment somewhere else

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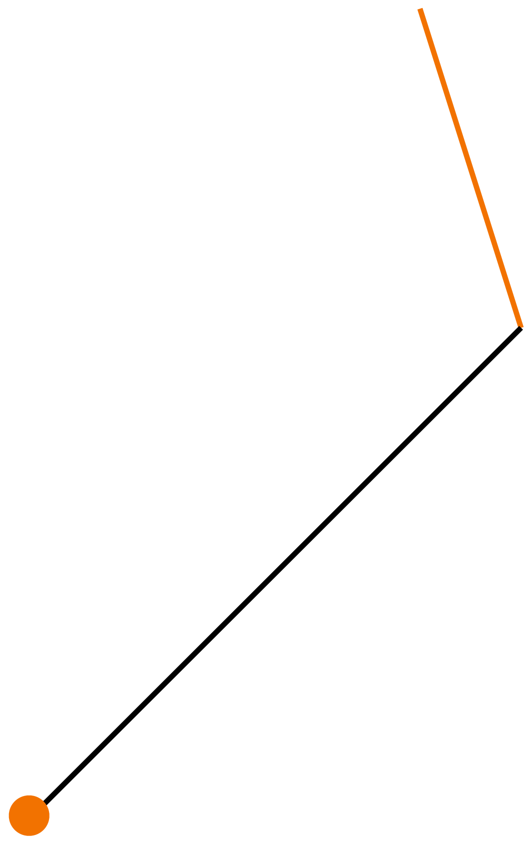

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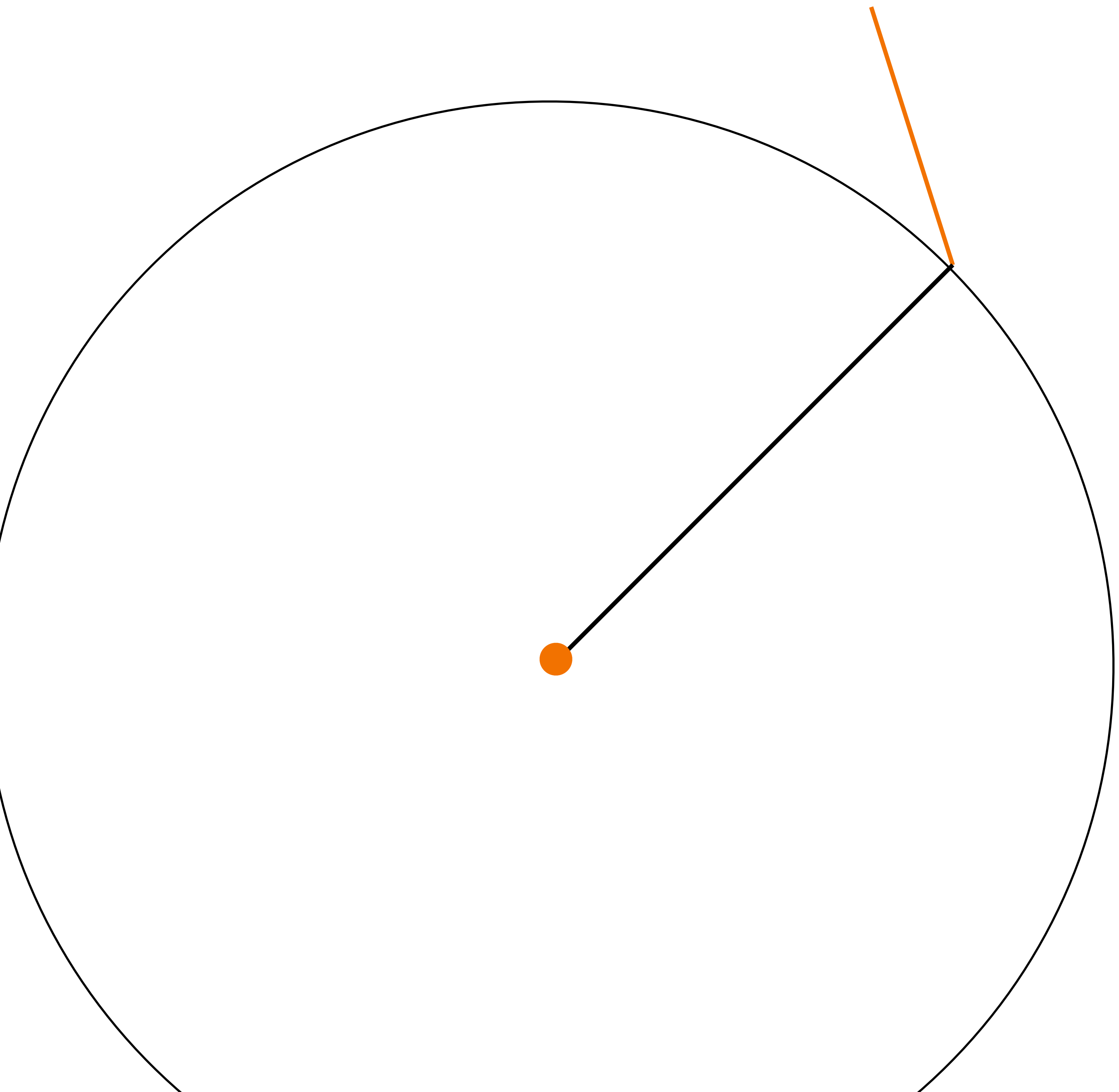

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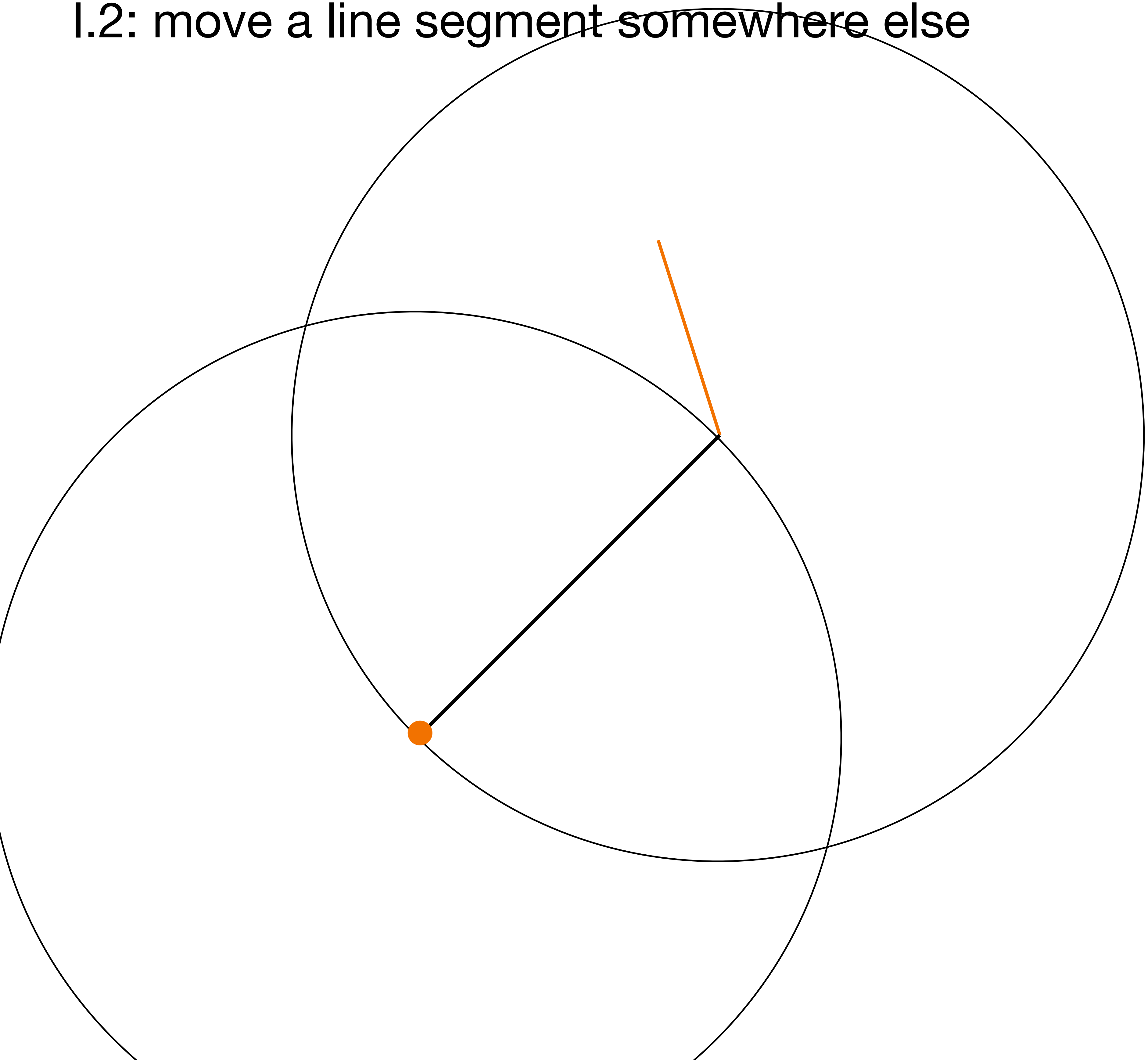

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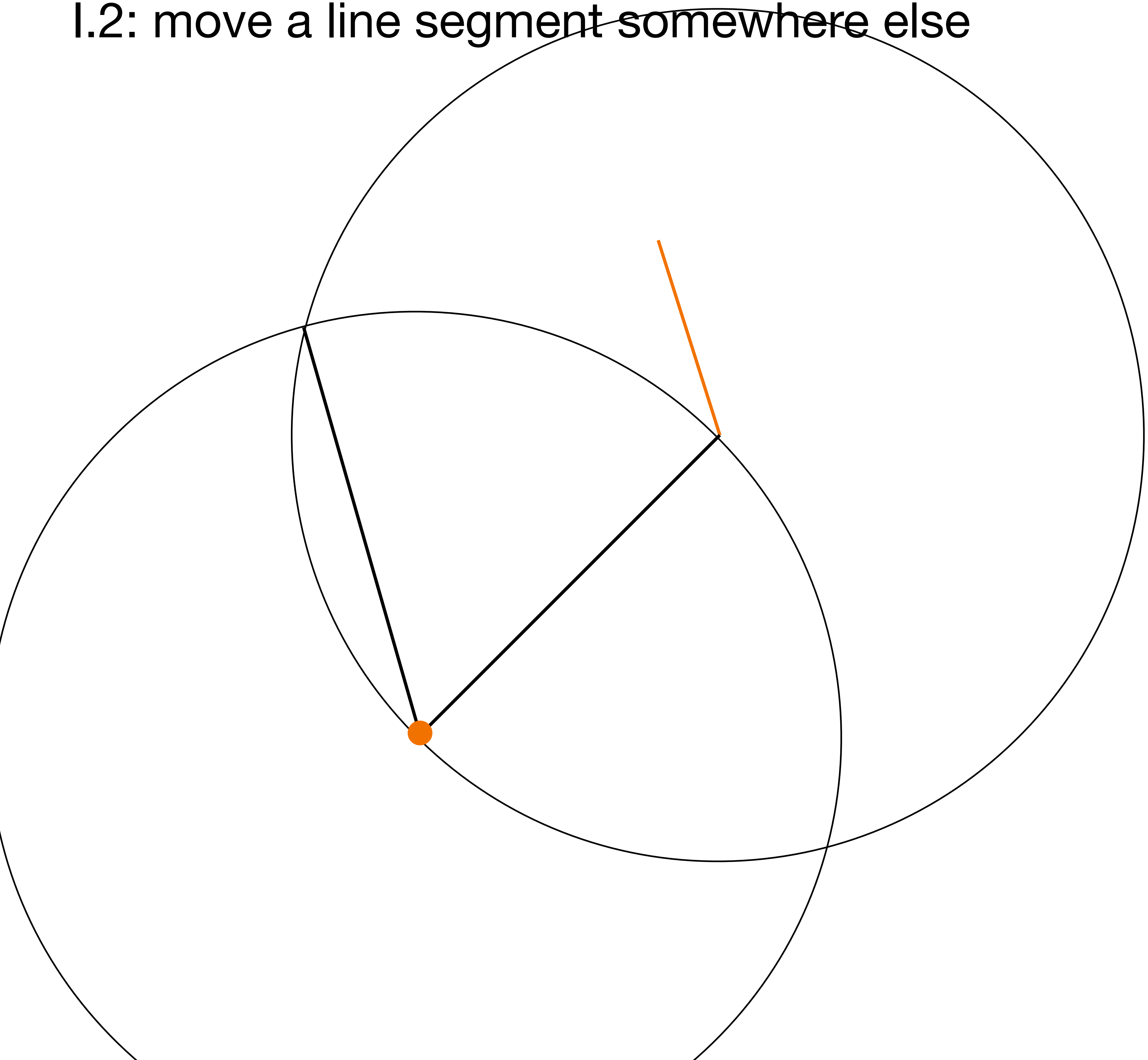

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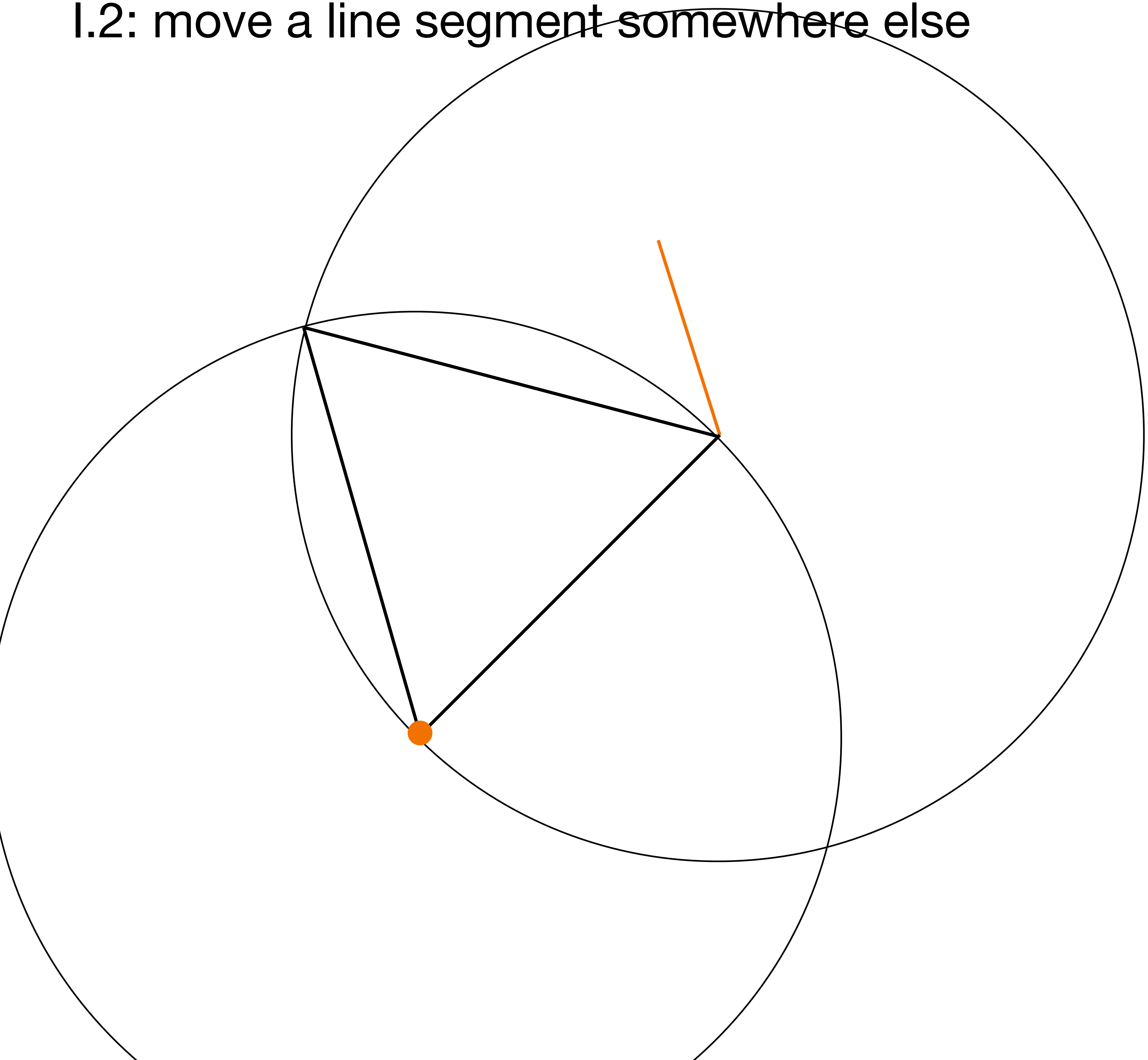

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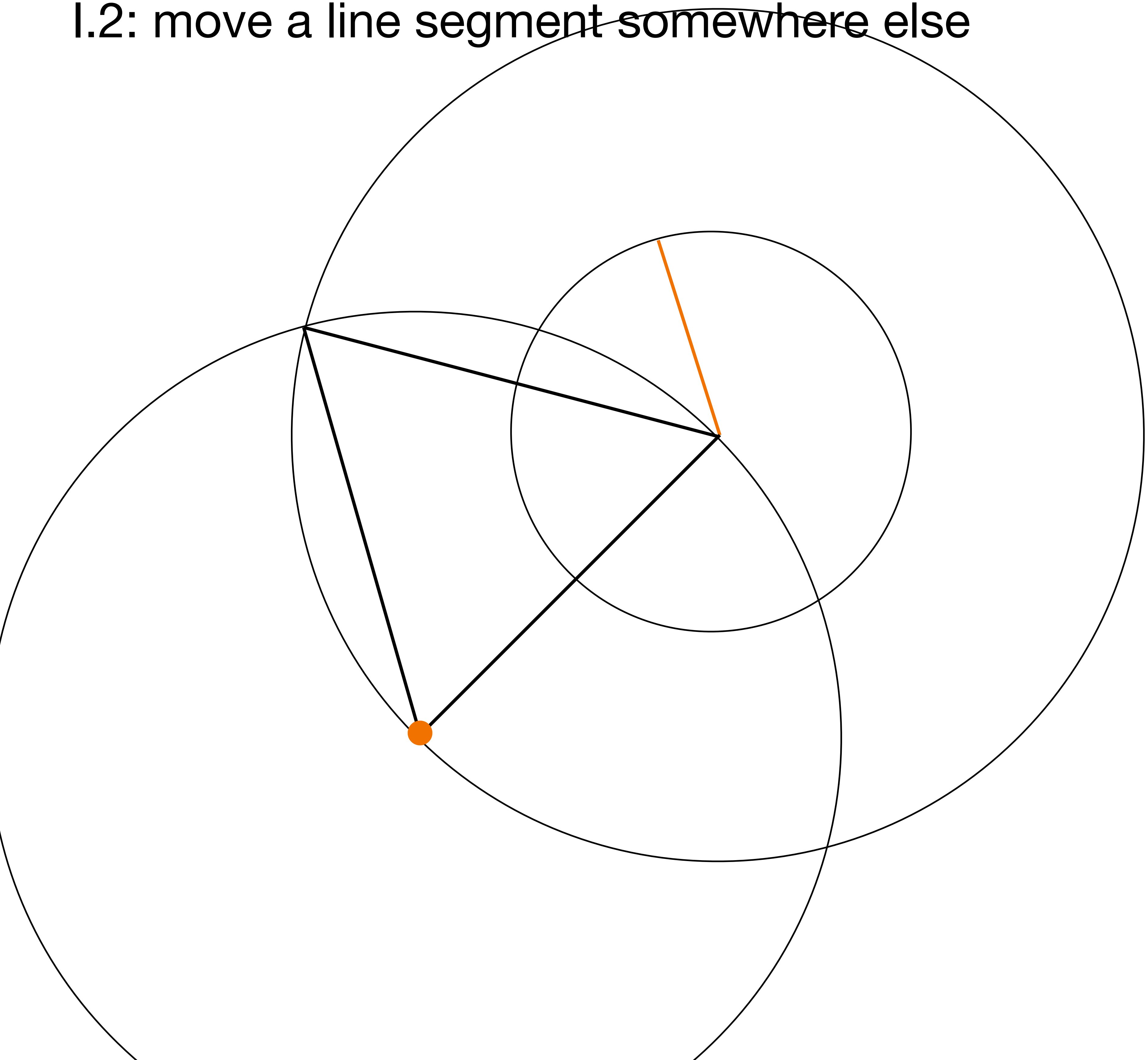

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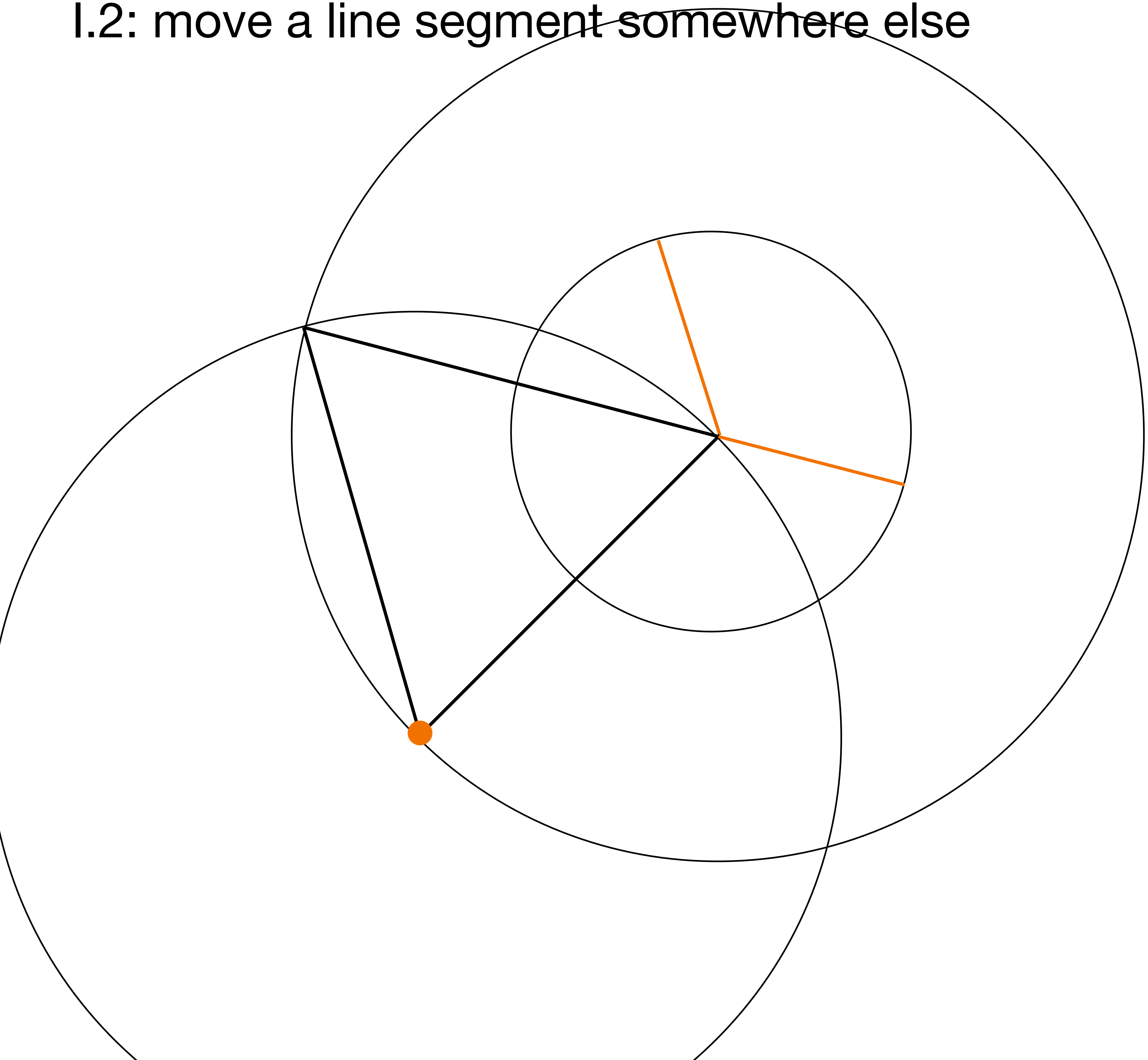

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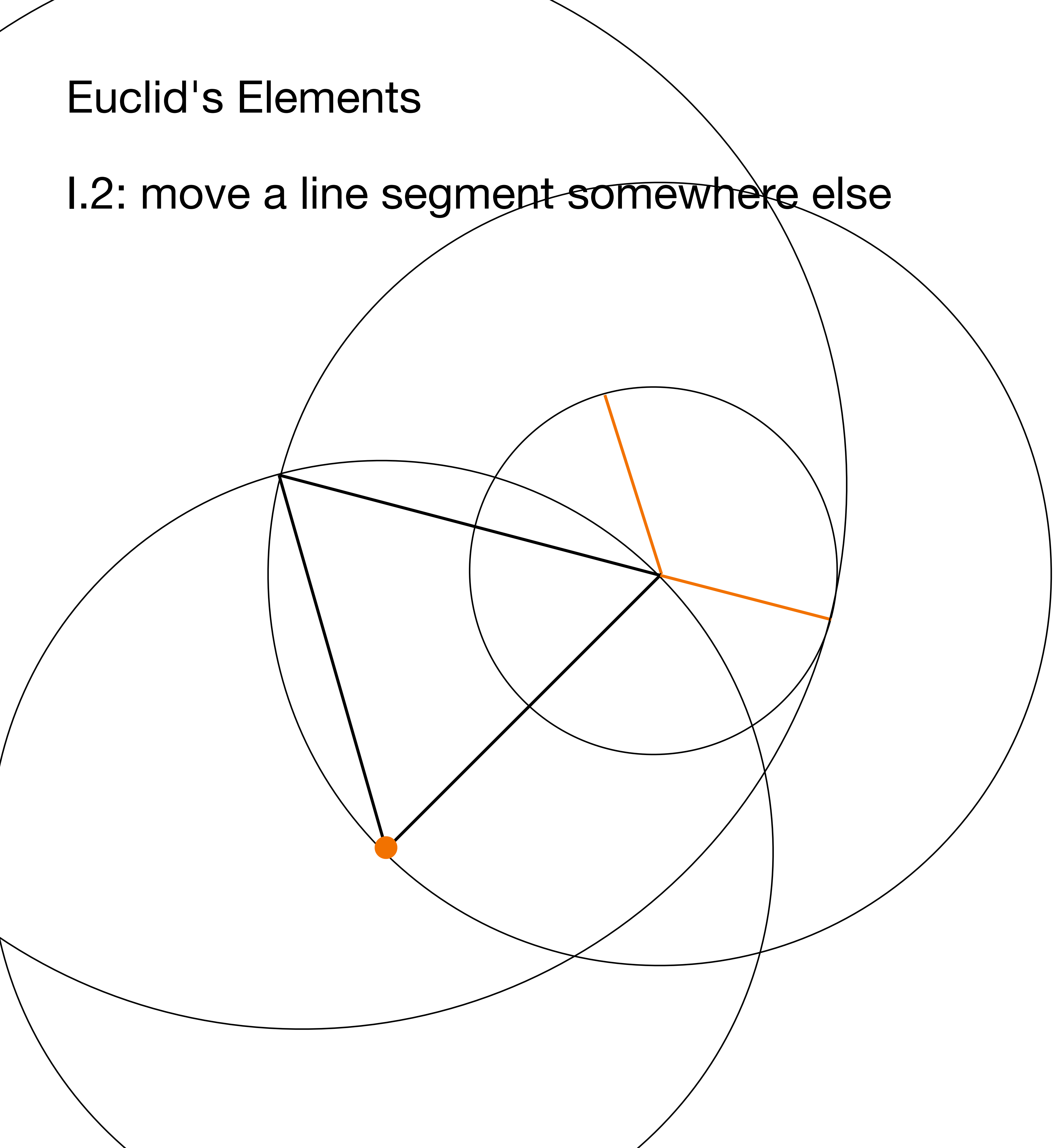

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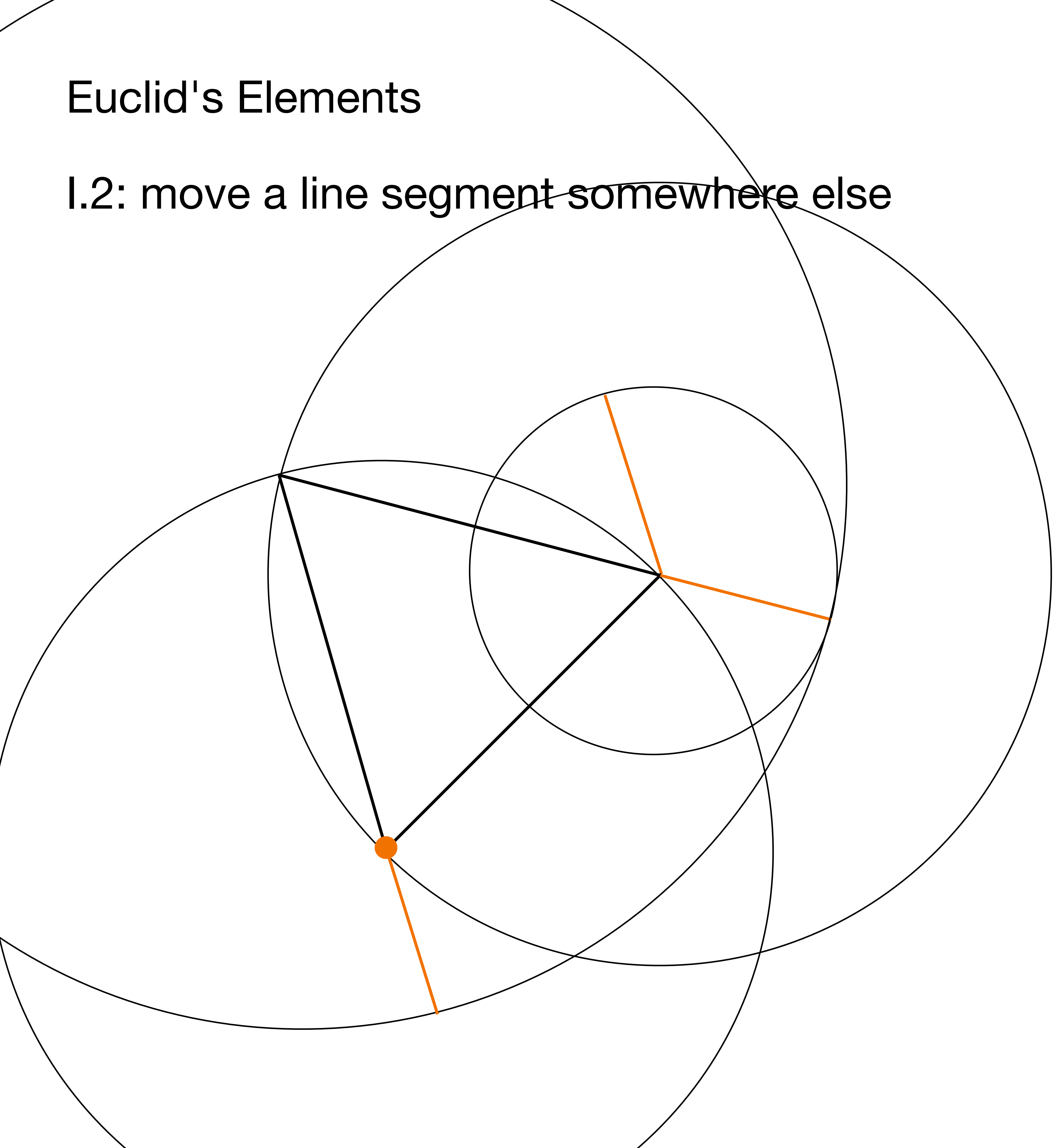

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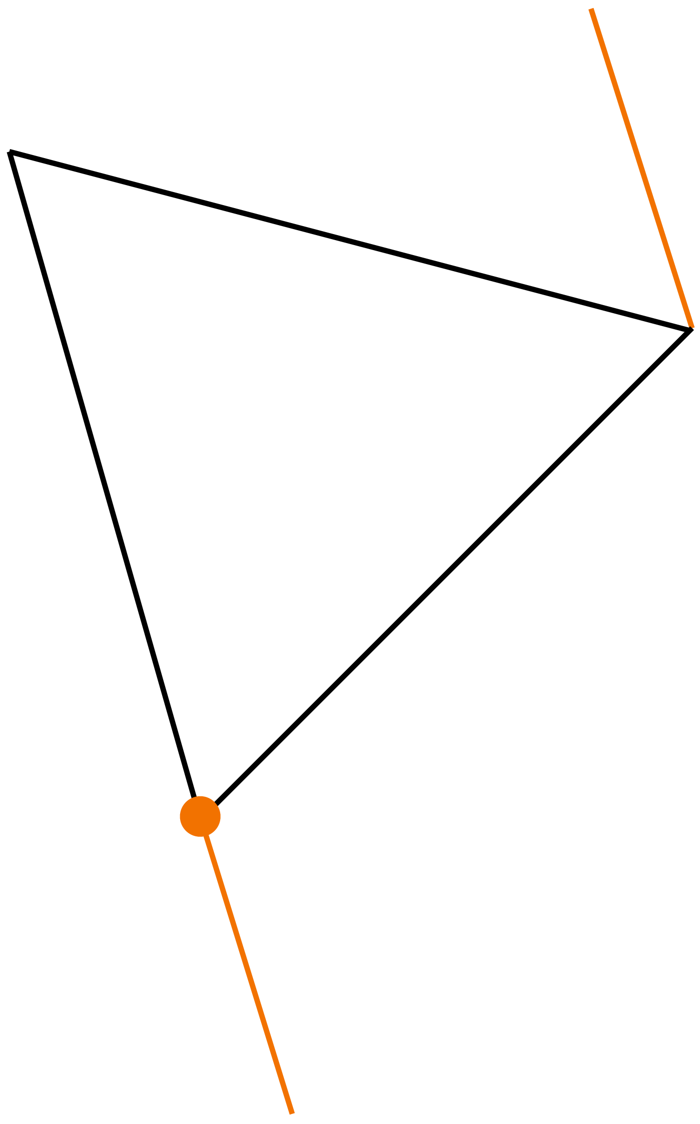
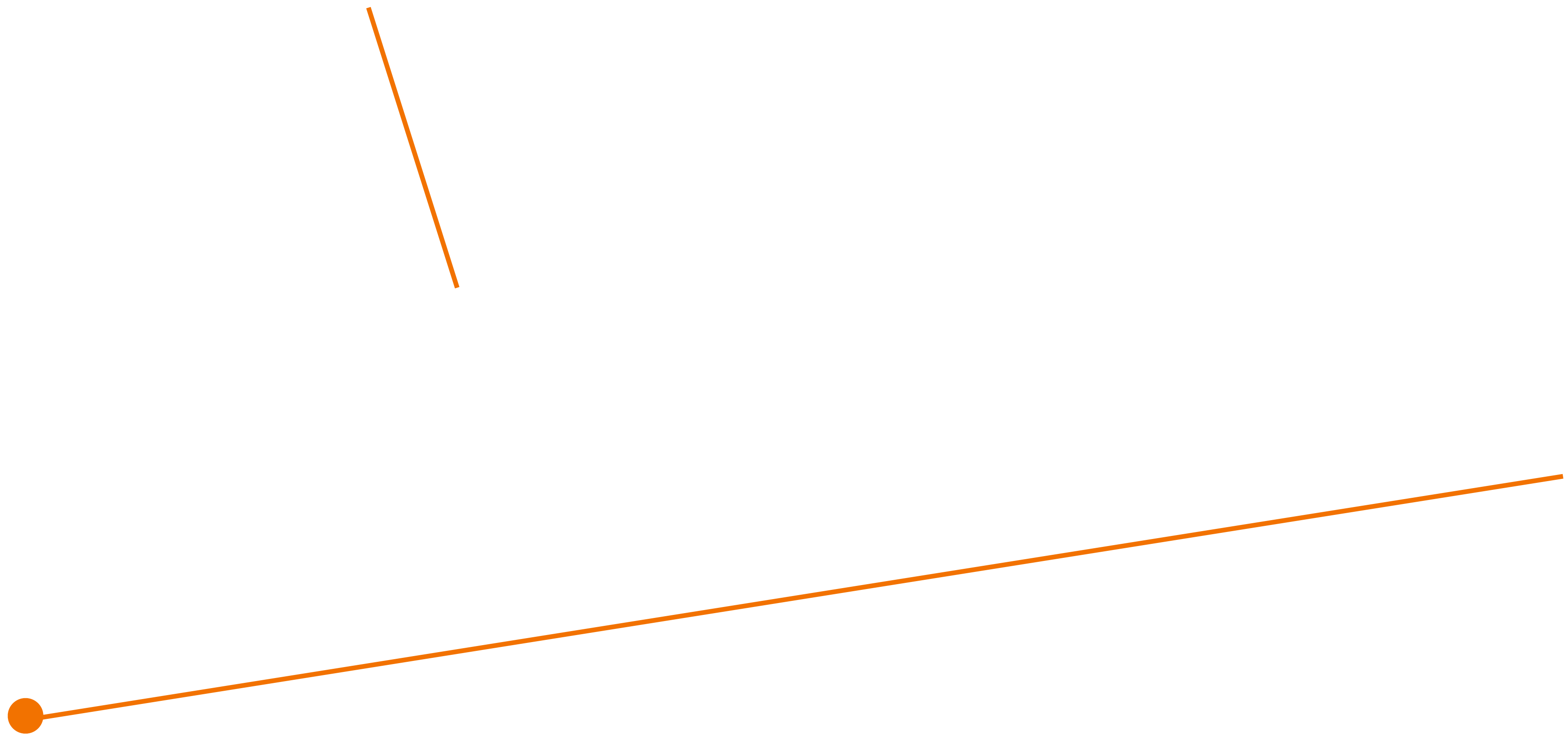


Euclid's Elements

I.3: to cut off a segment of a line at a point, equal to a given segment



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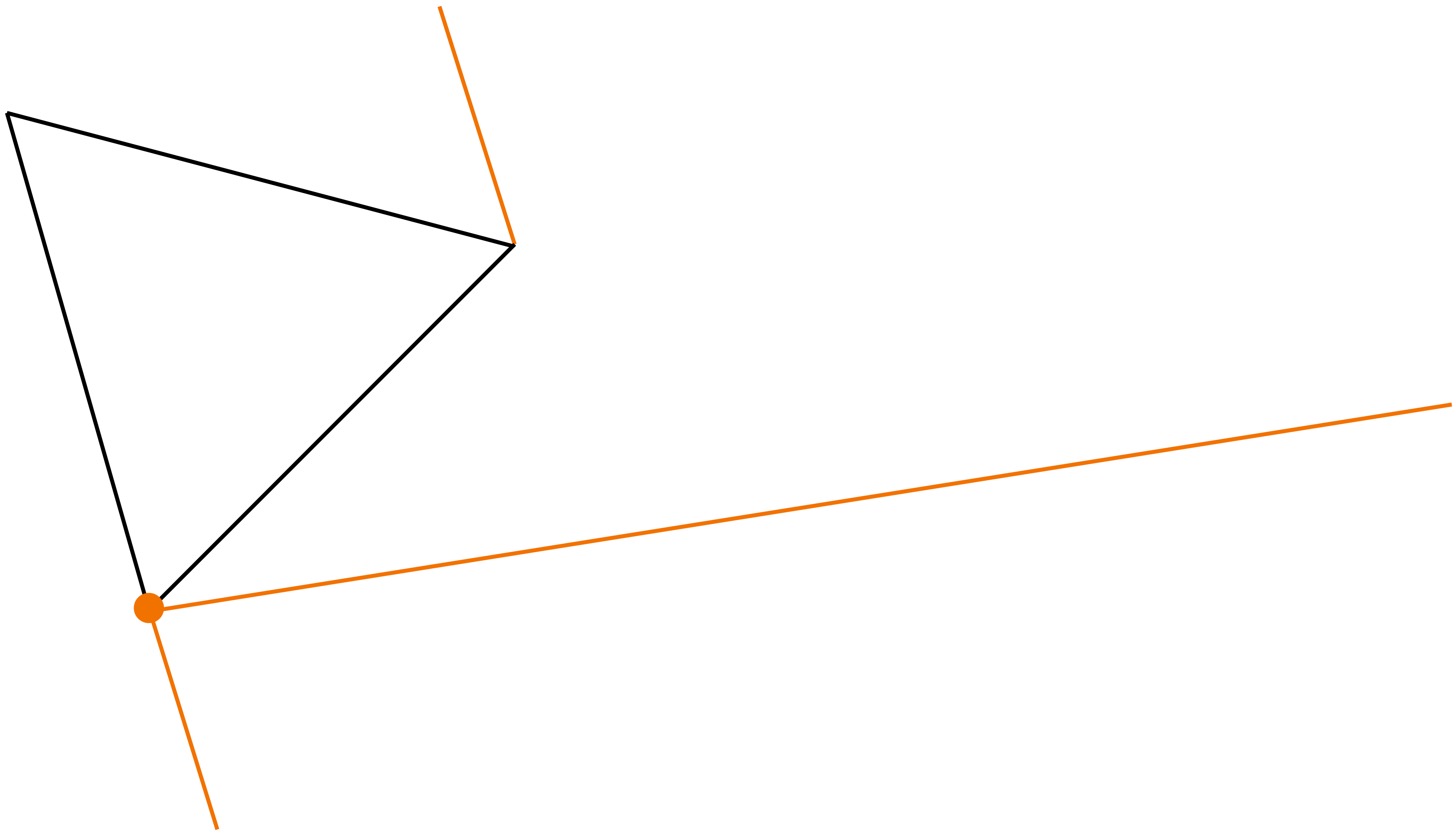

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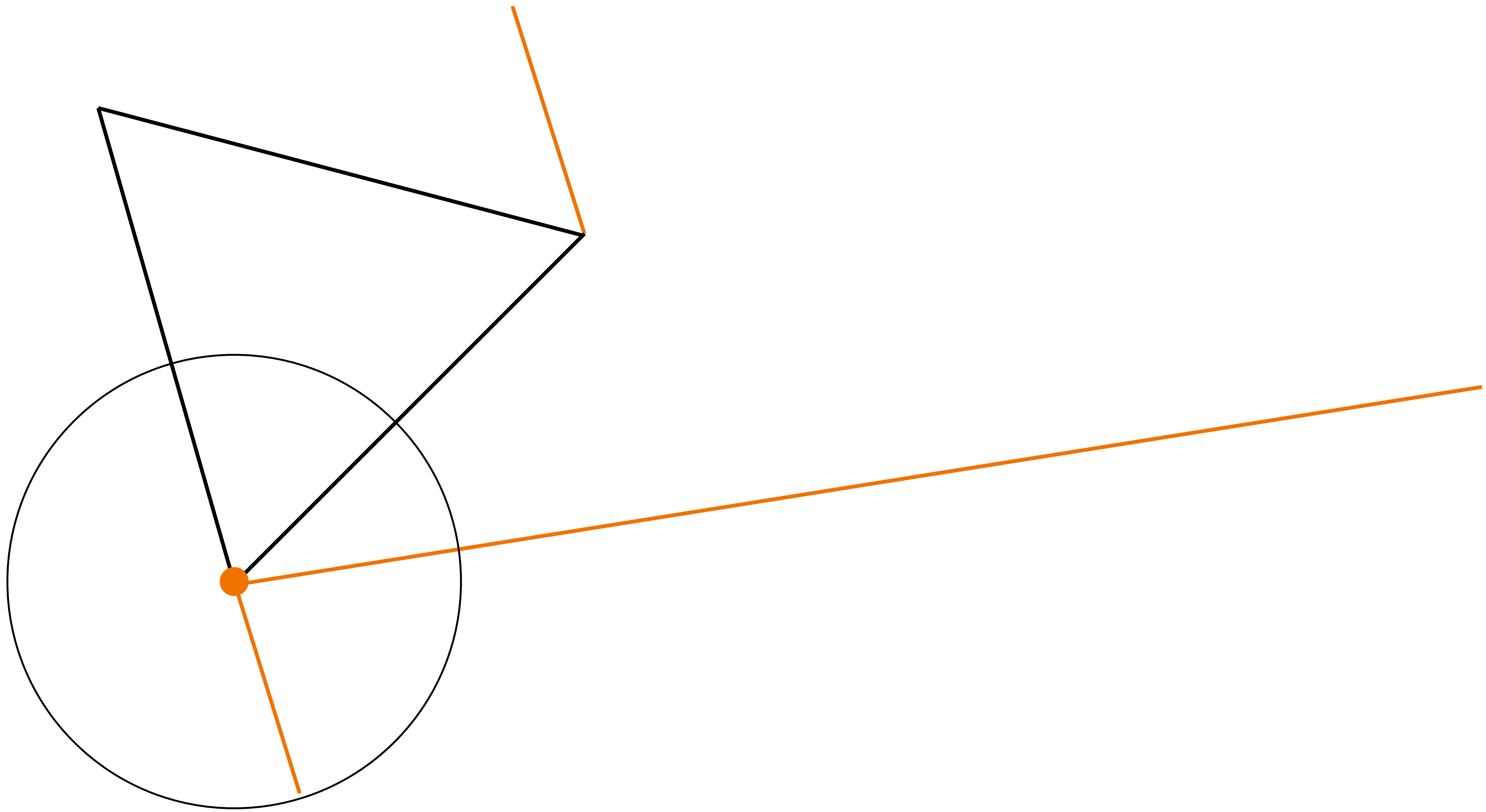

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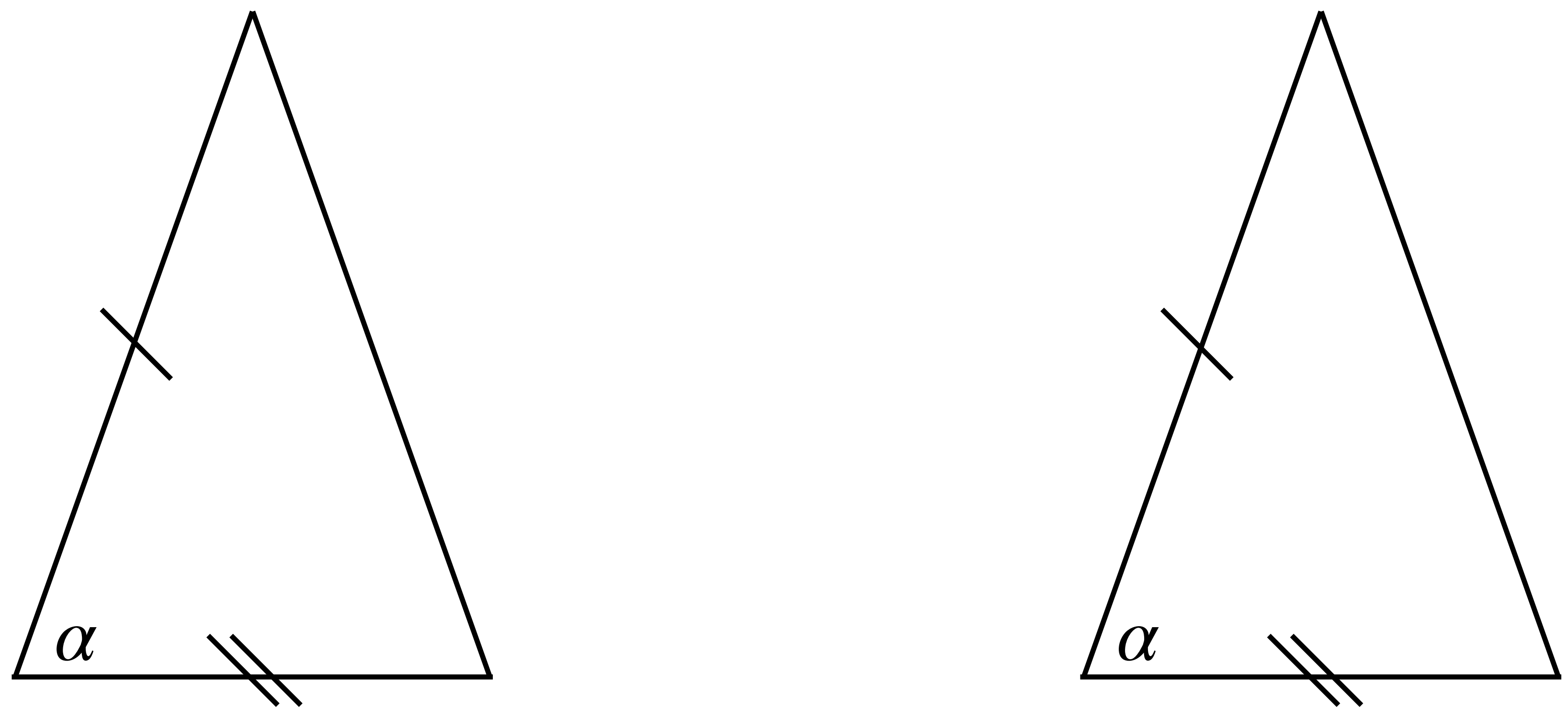

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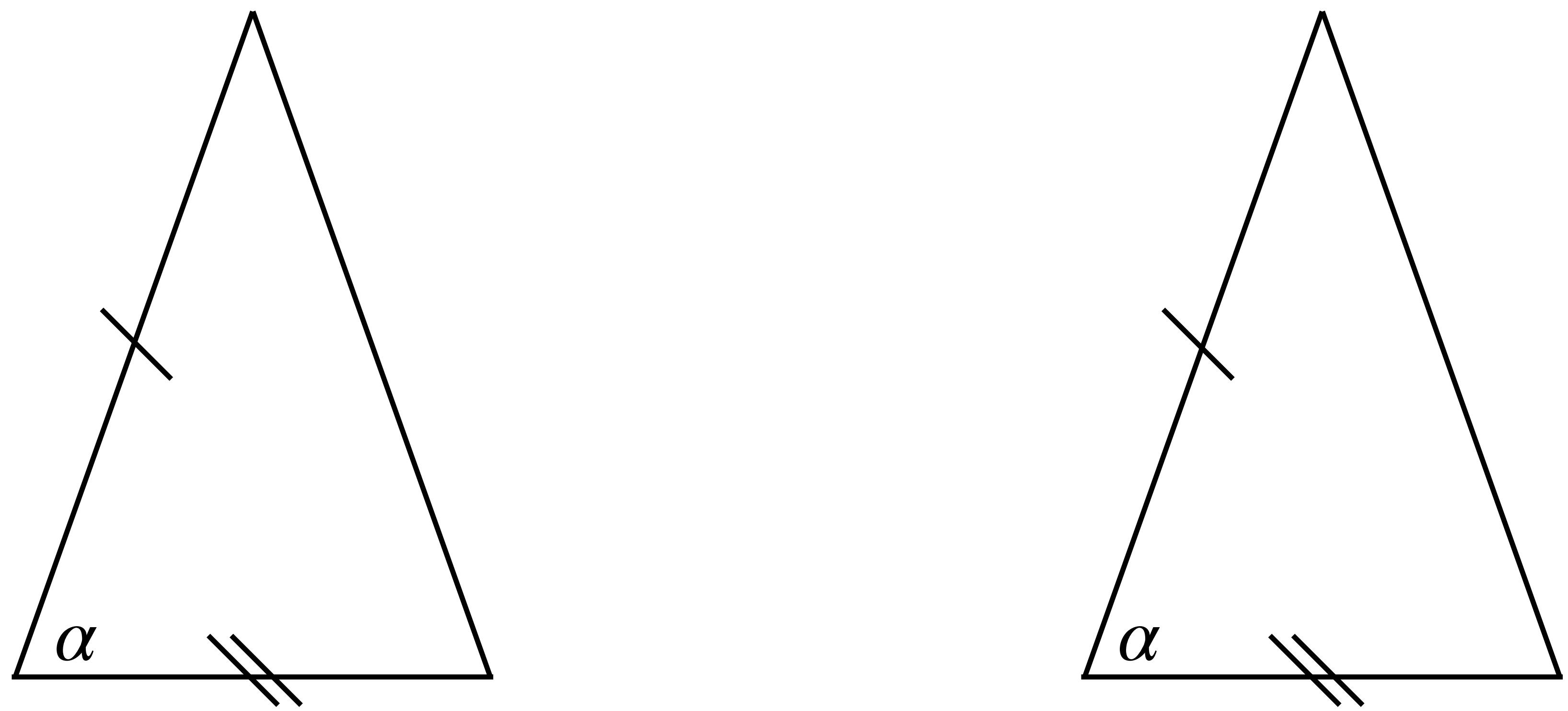
I.4: if two triangles have an angle the same and the angle-enclosing sides are the same, then the triangles are the same (SAS theorem)



Proof is by superposition

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I.5: in triangles, equal sides imply equal subtended angles.

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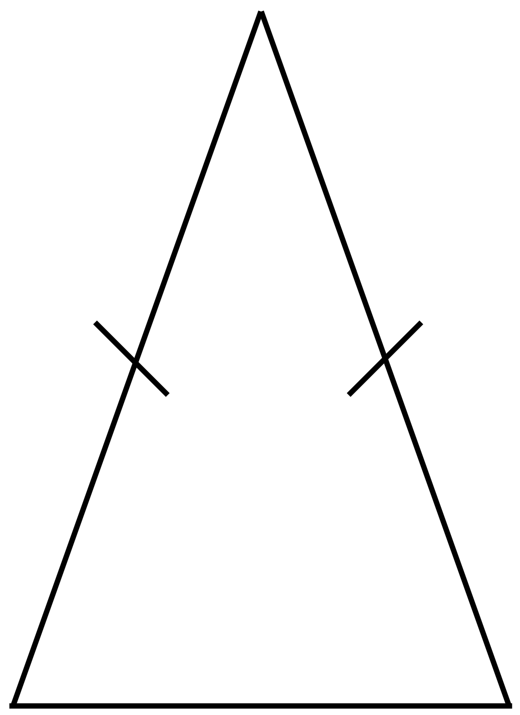

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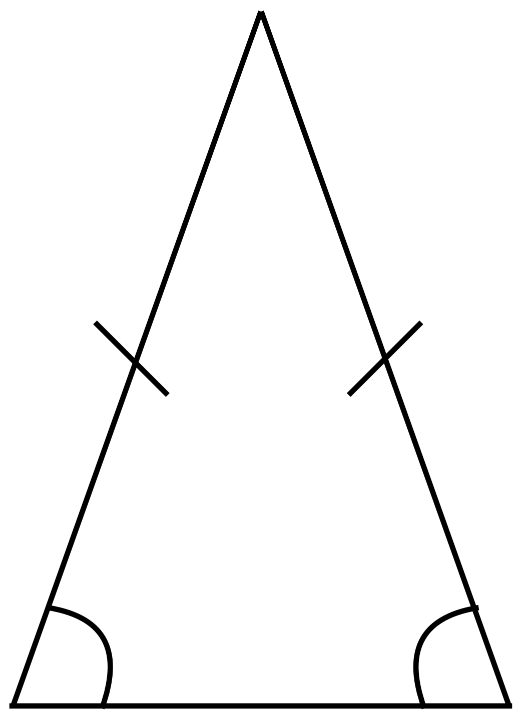

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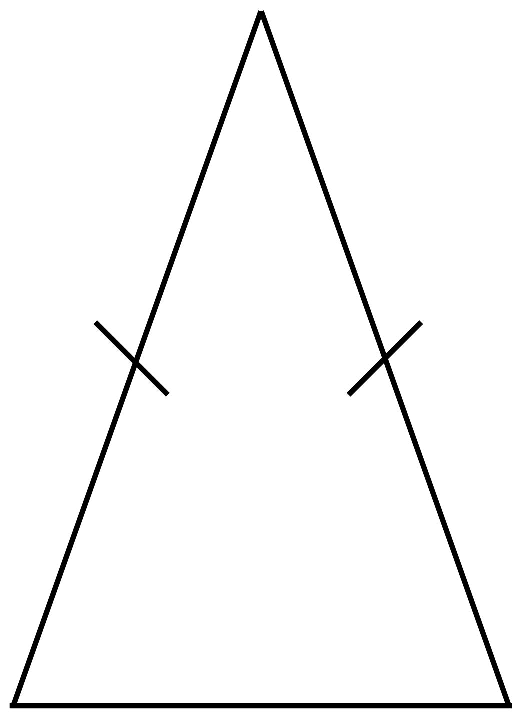

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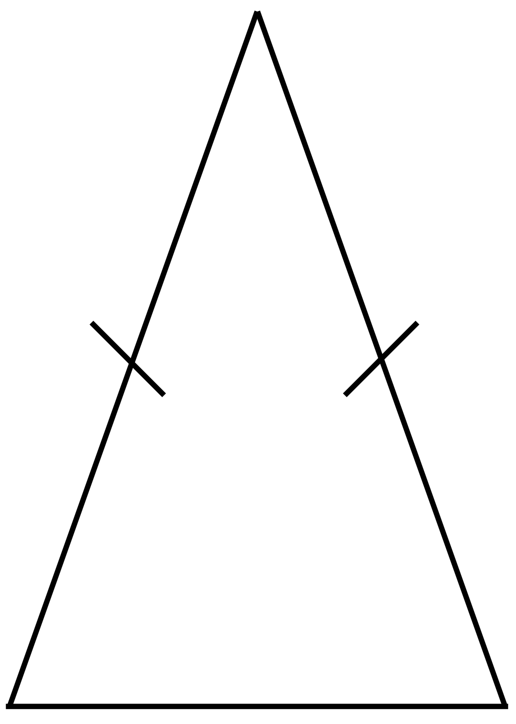

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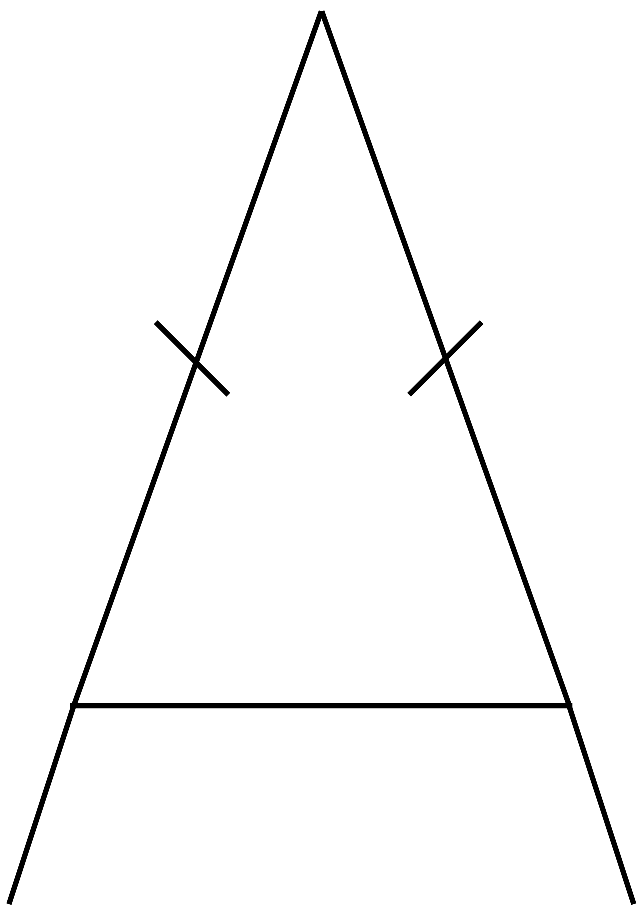

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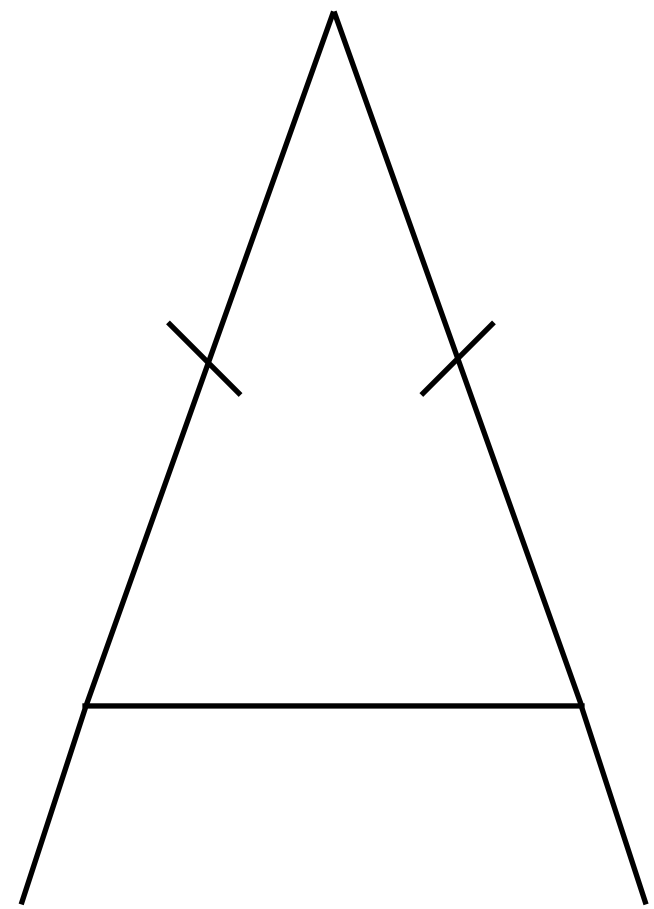

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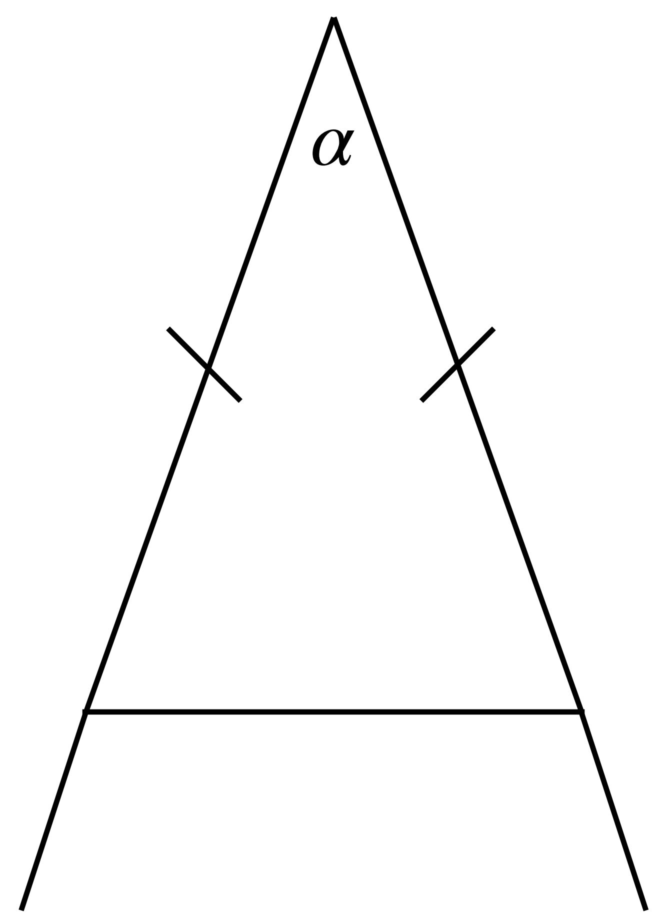

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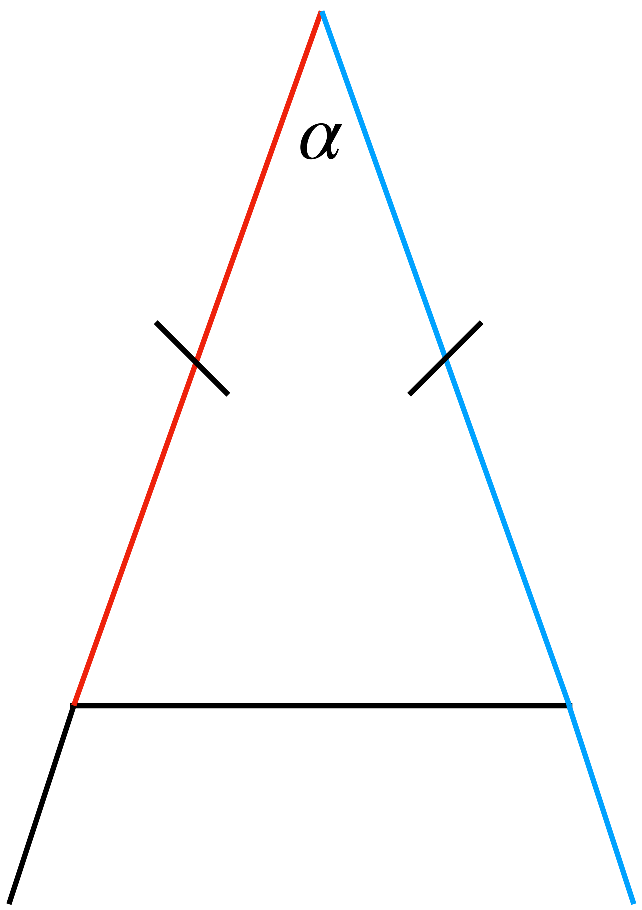

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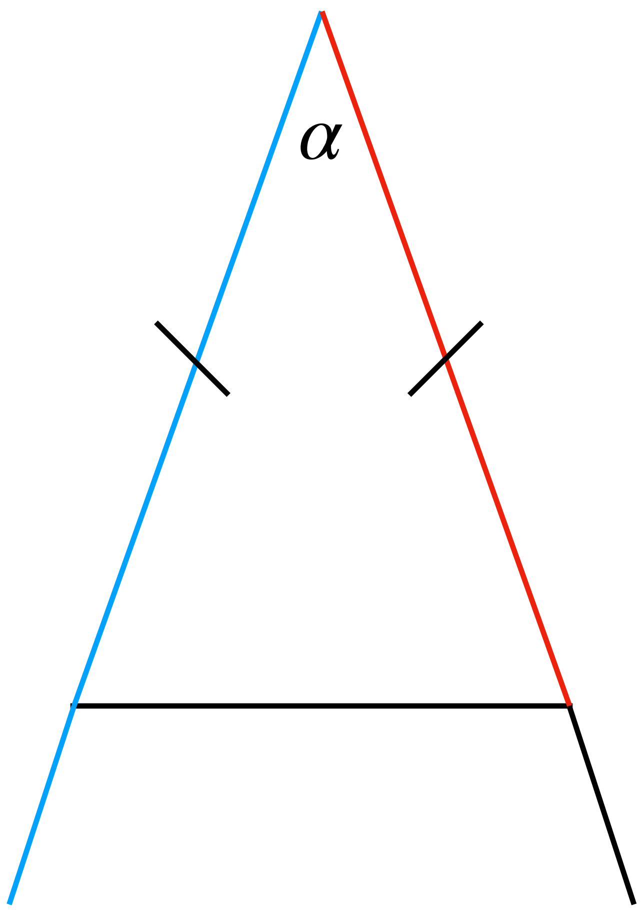

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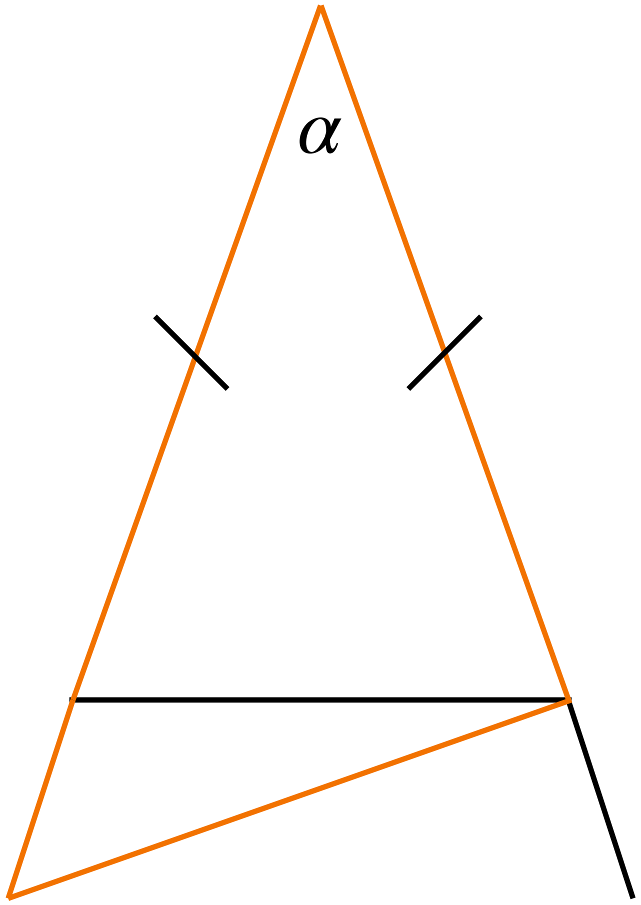

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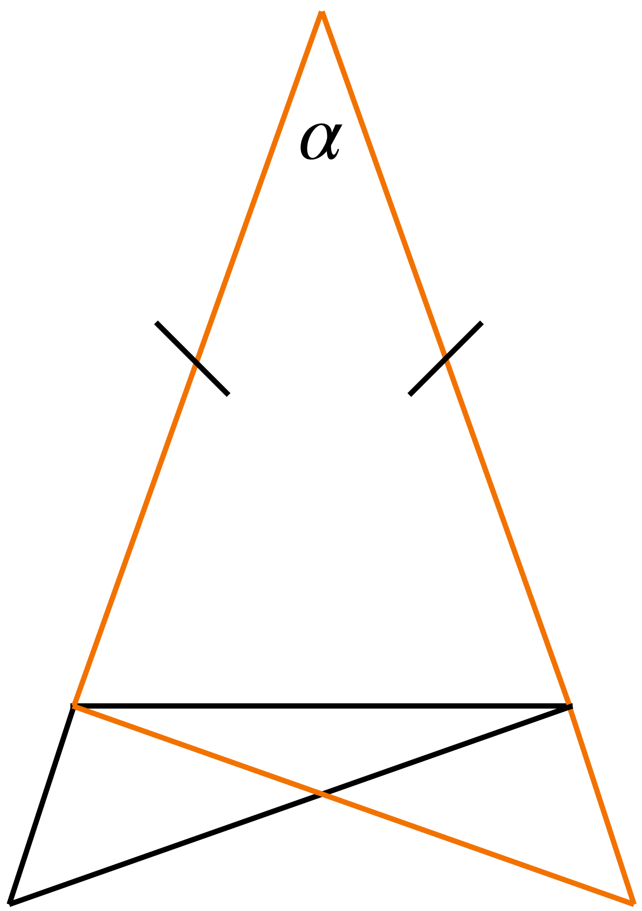

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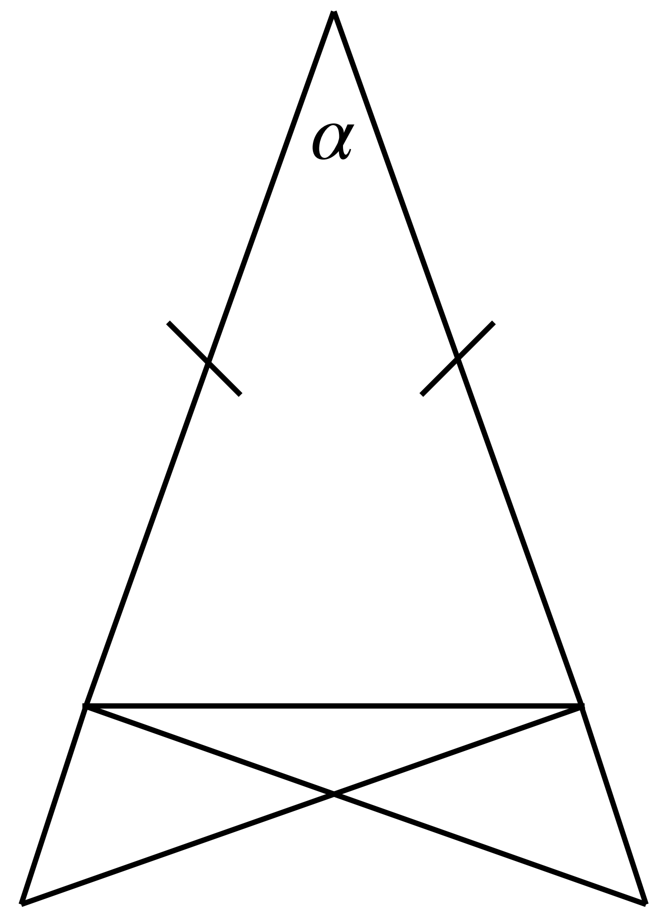

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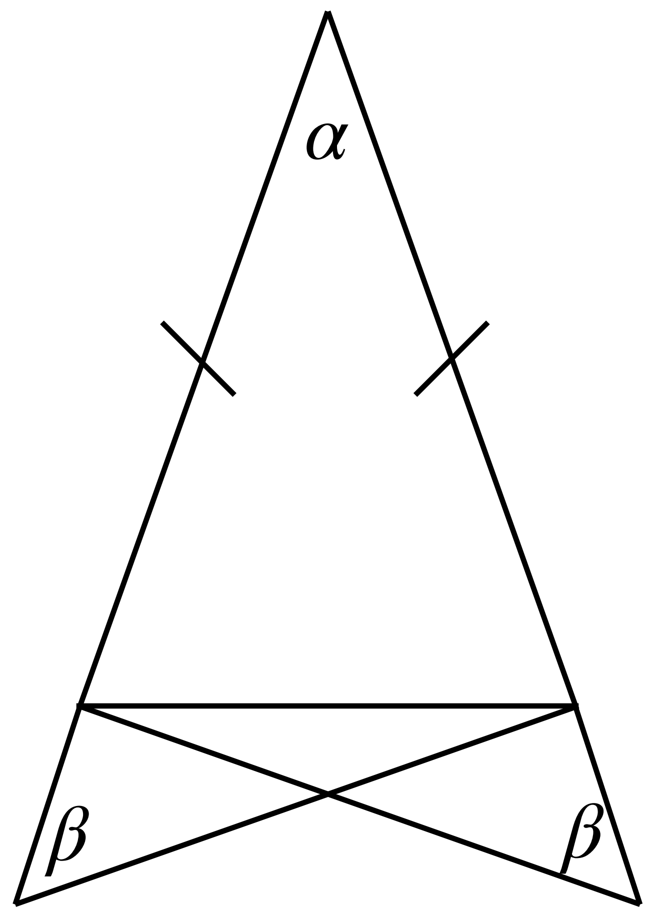

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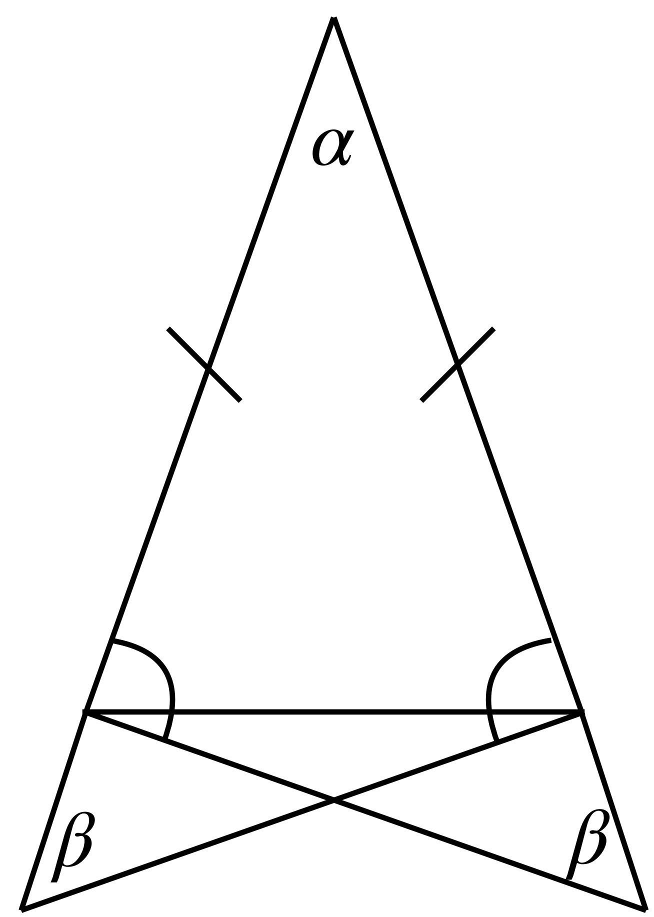

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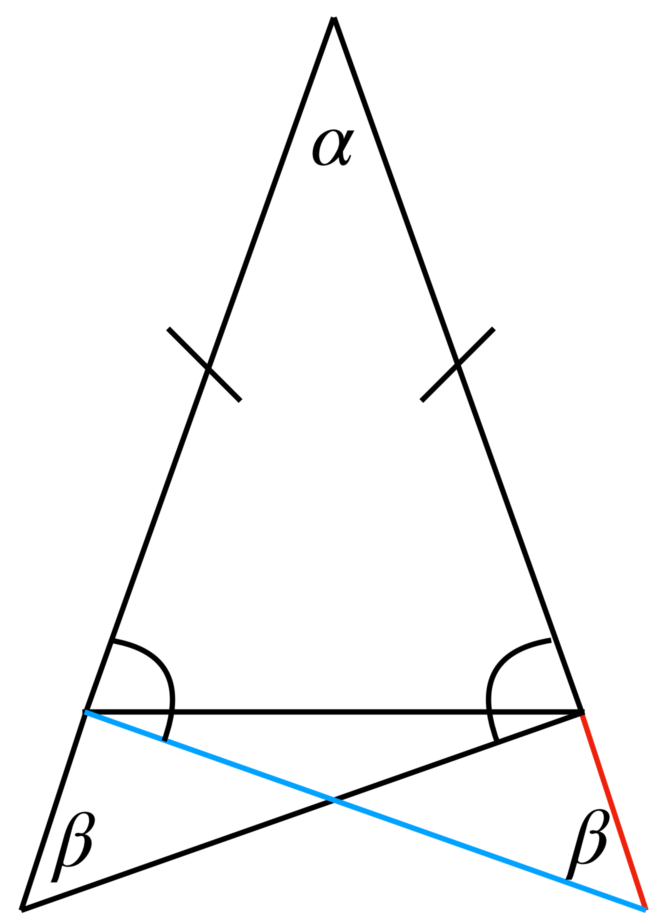

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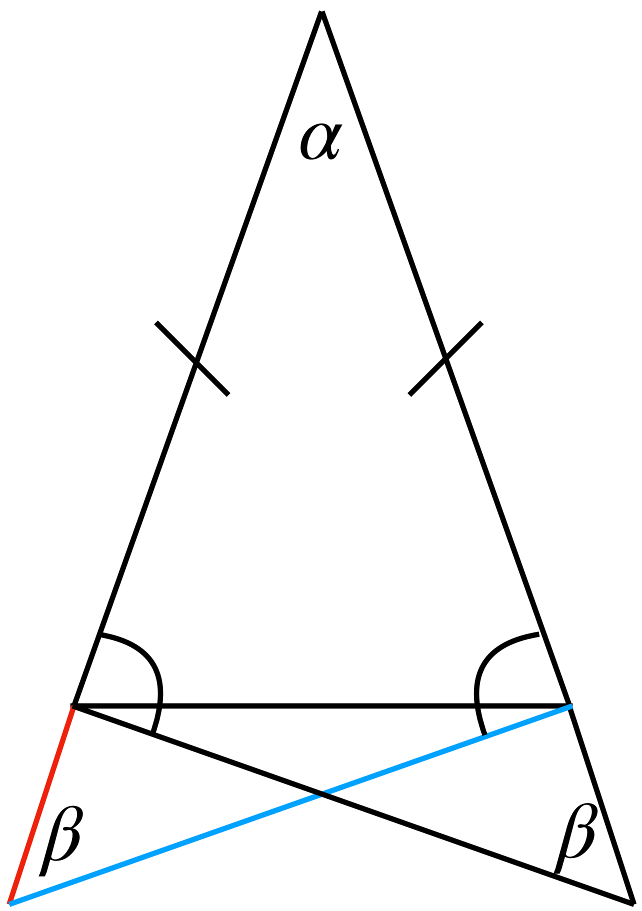

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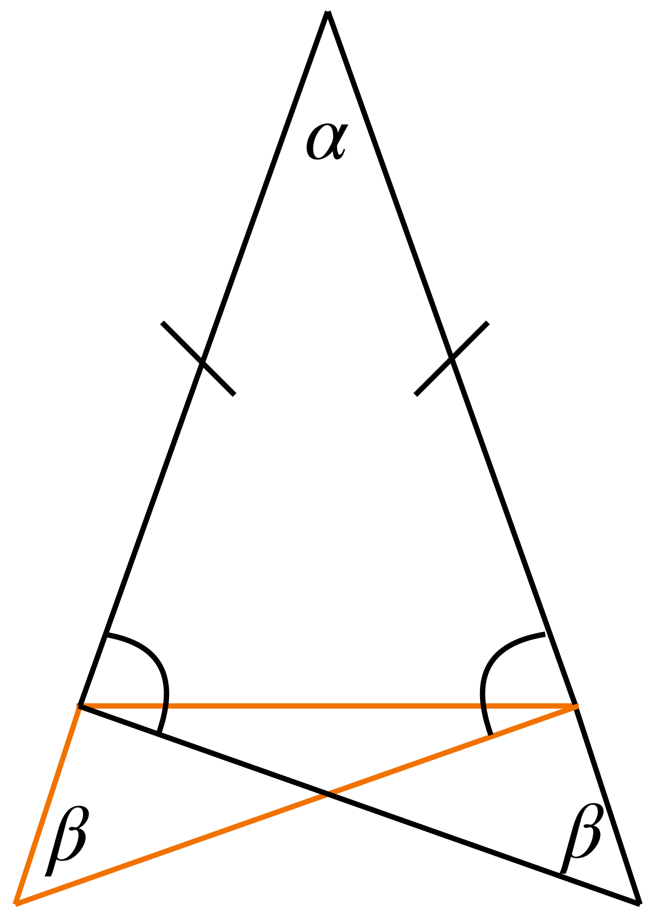

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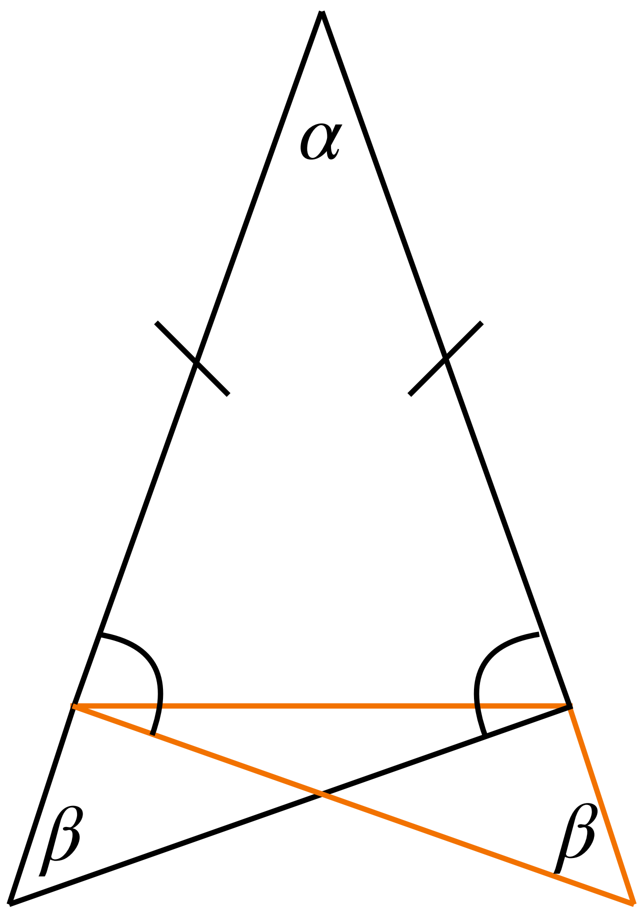

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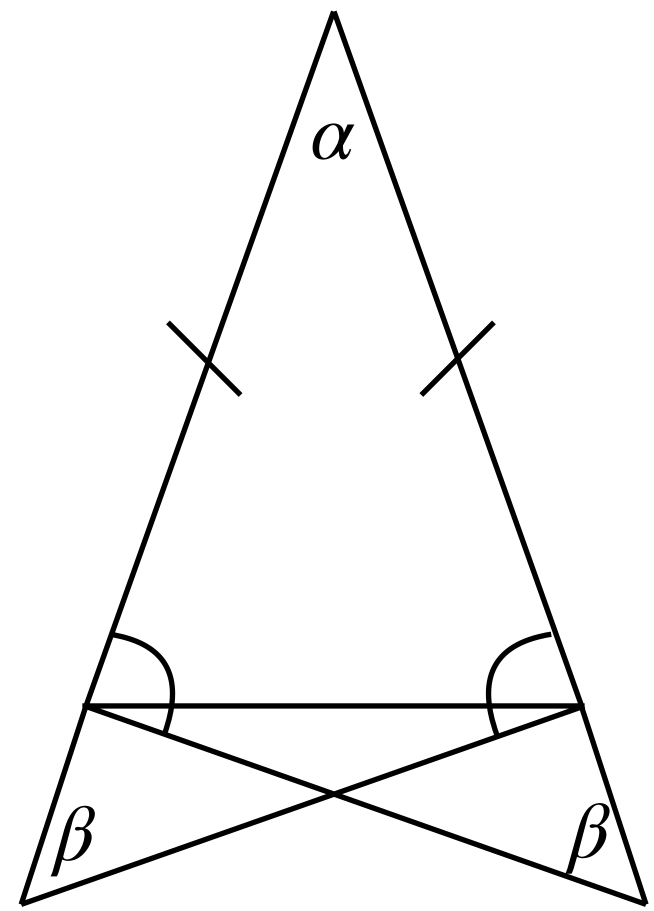

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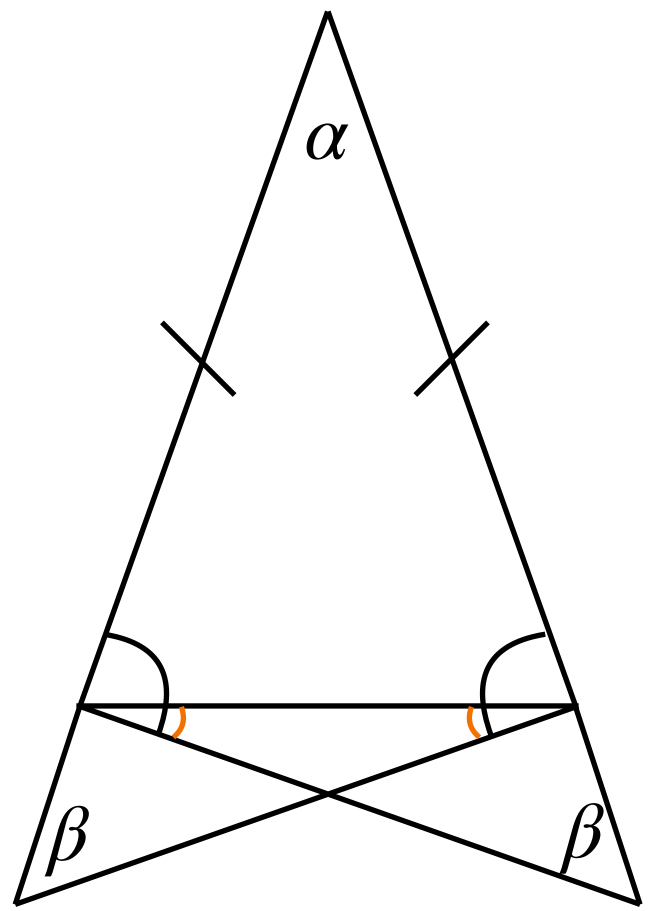

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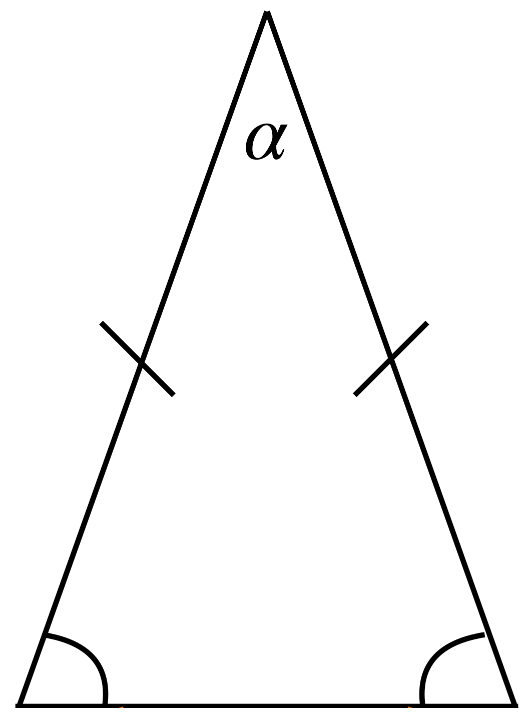

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I.6: in triangles, equal angles imply equal subtended sides.

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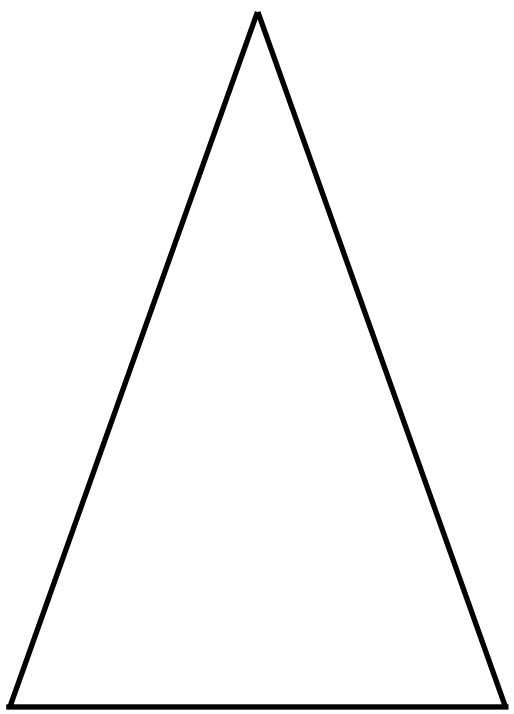

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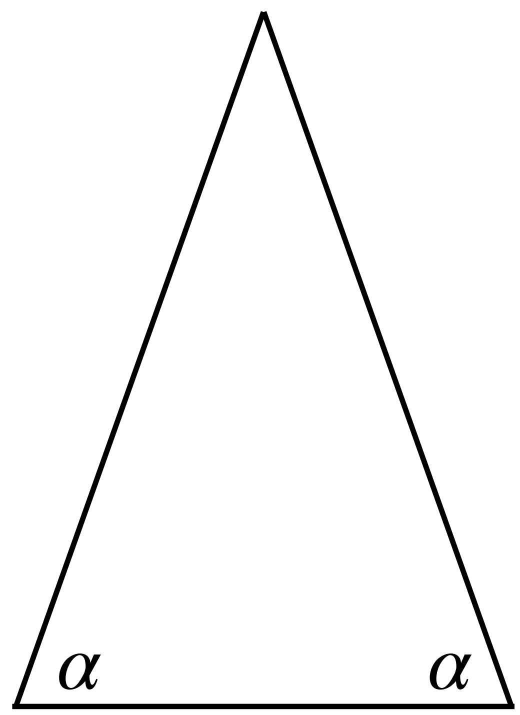

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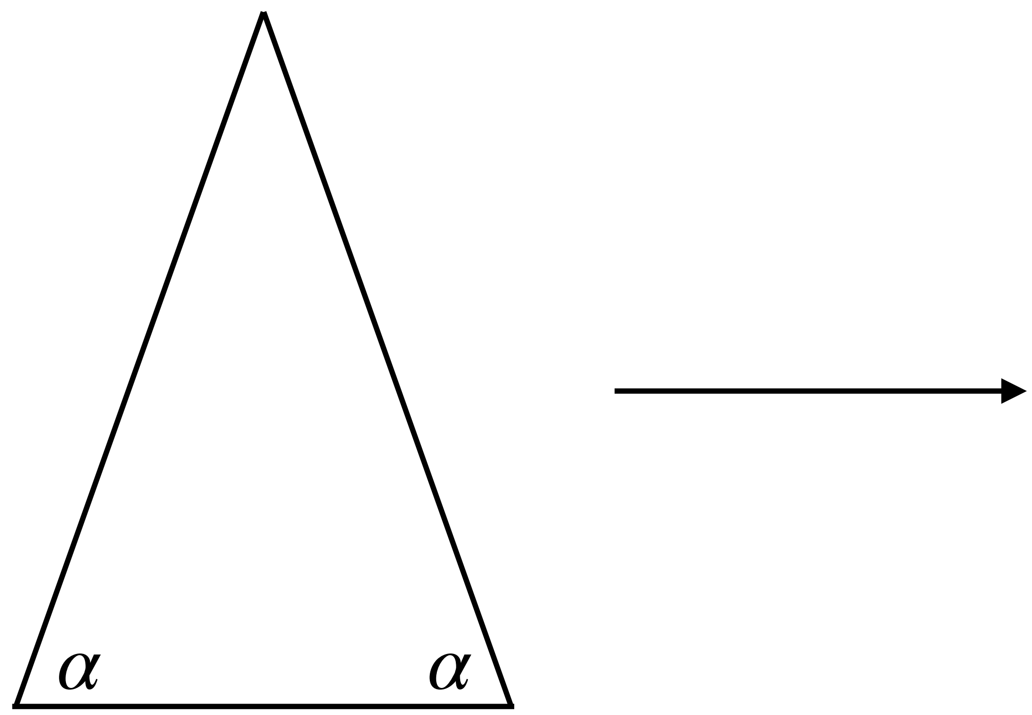

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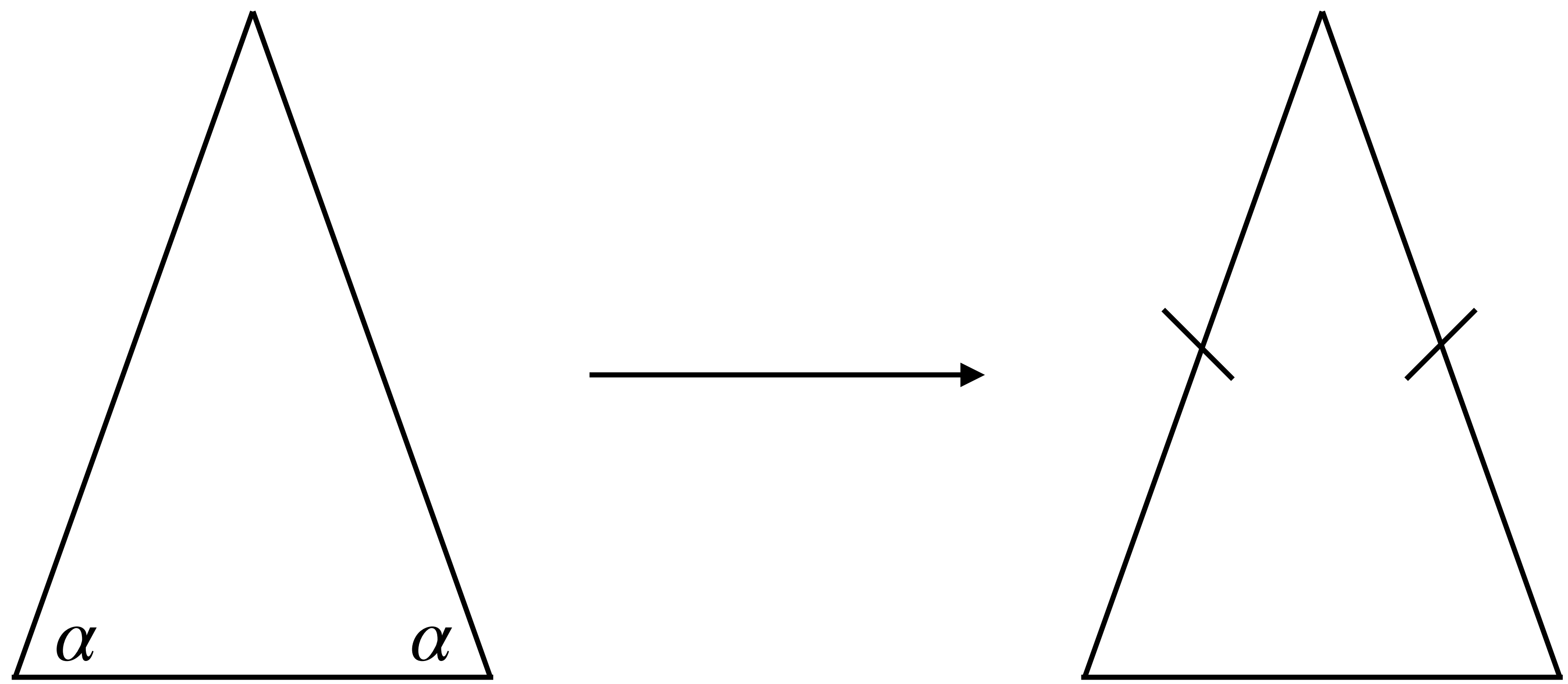

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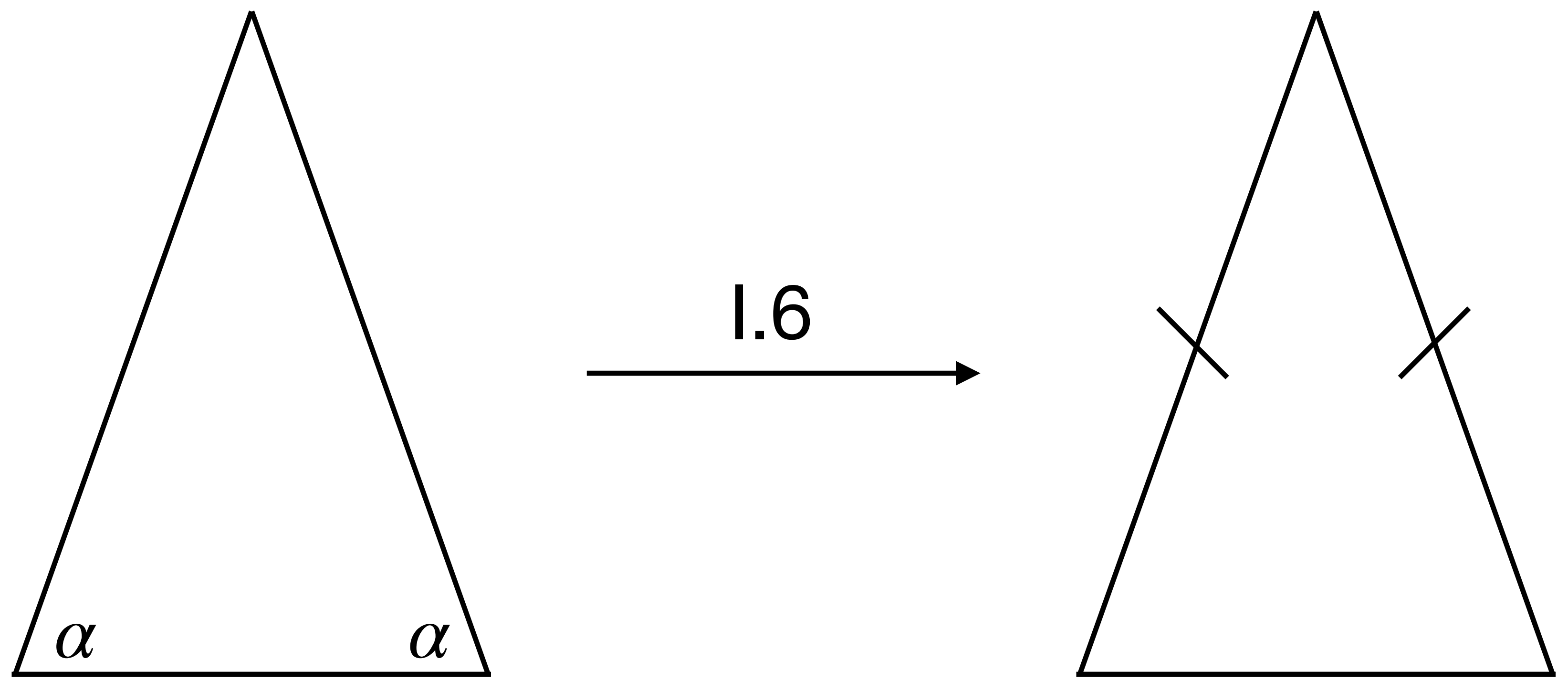

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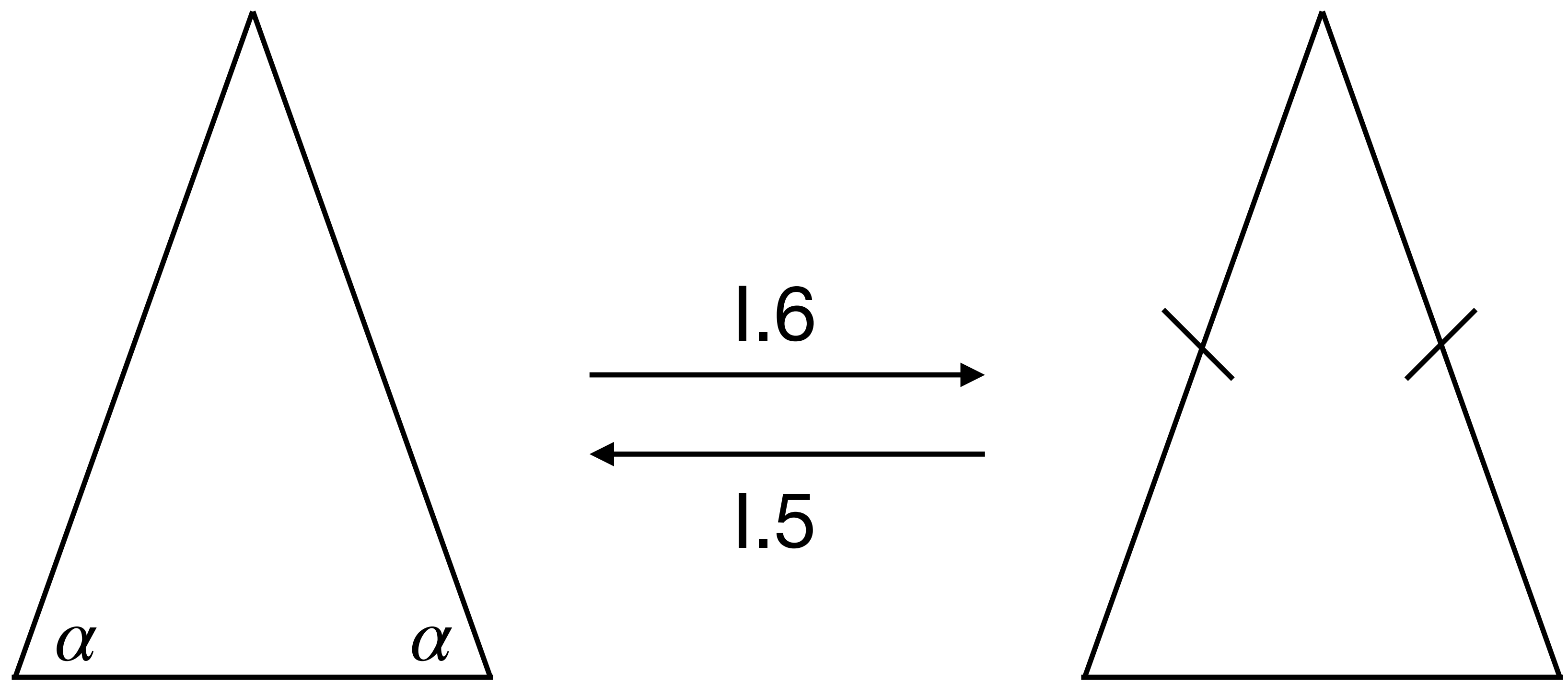

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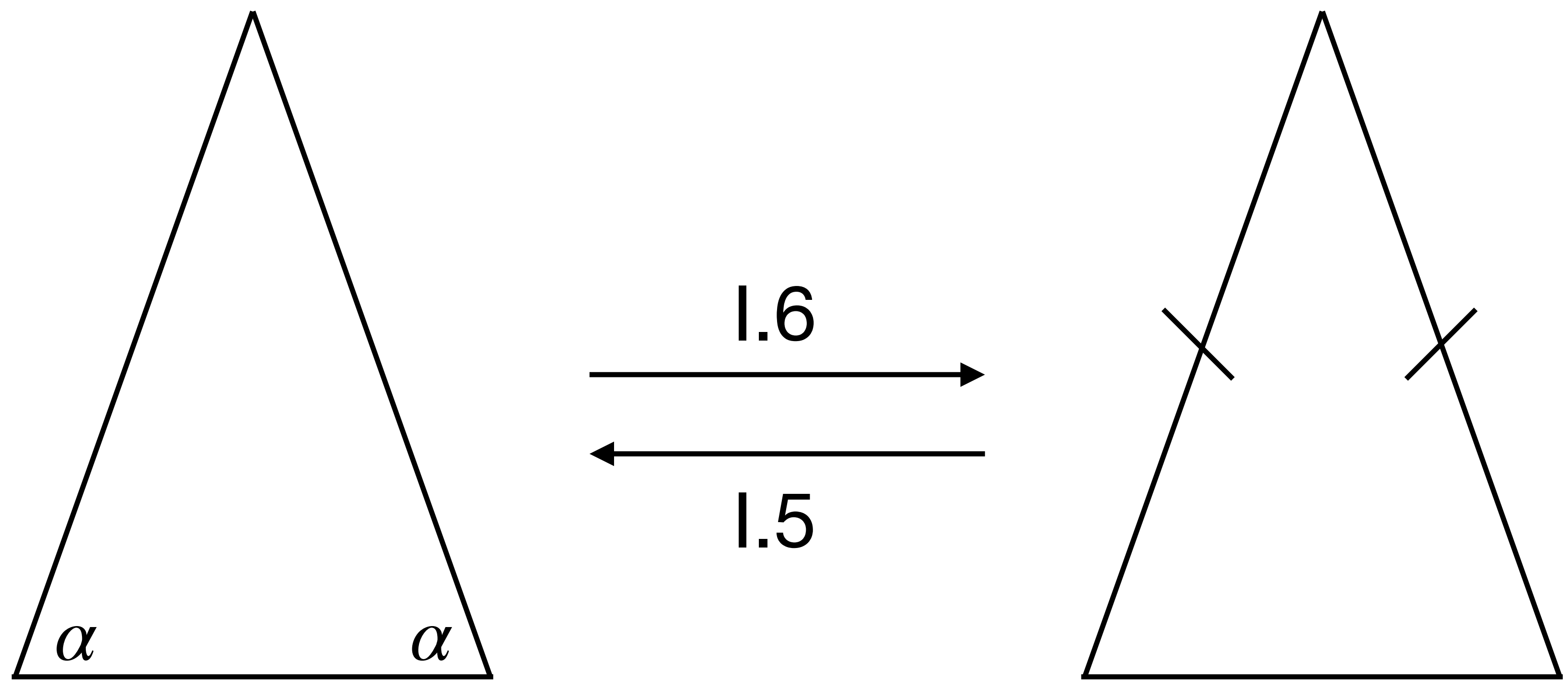

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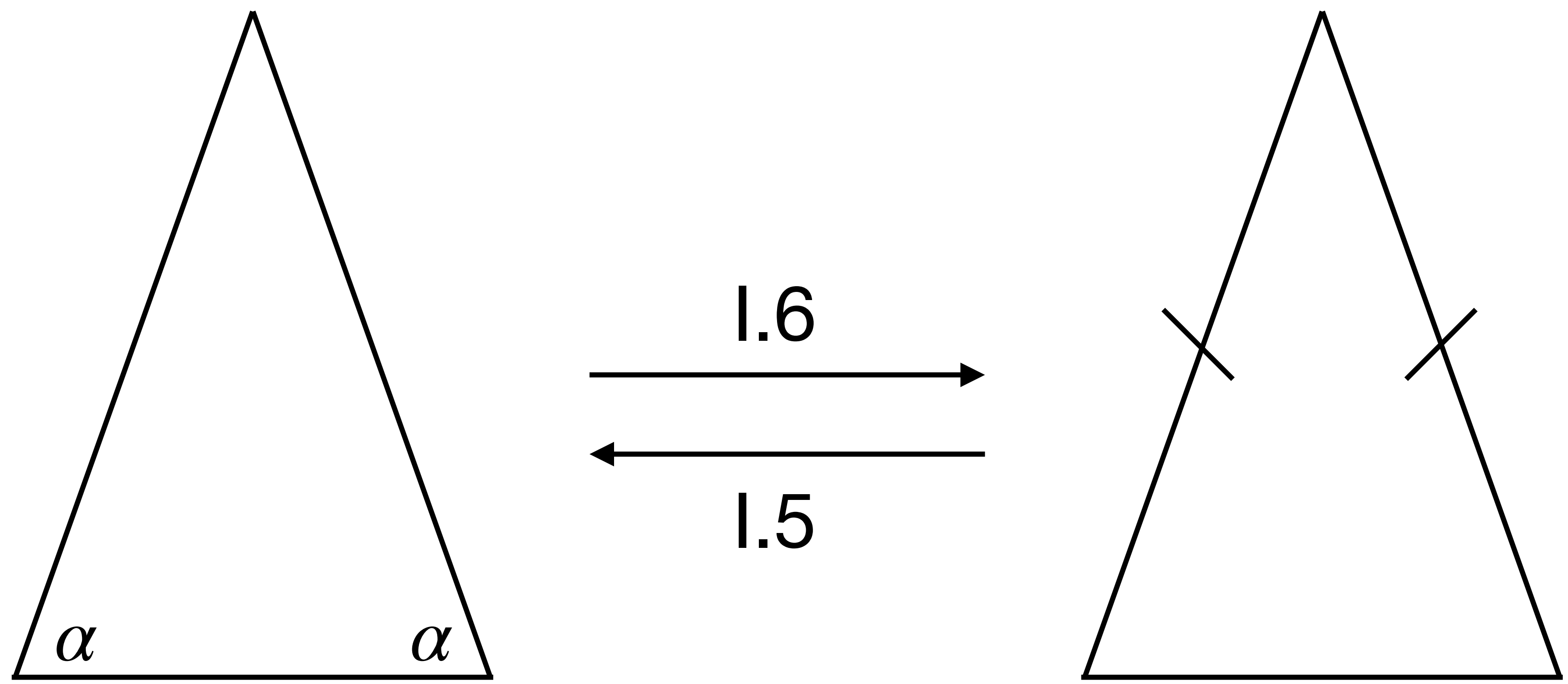

Note that I.1-I.5 were "constructive" proofs. We set up a situation, and then proved the consequence step by step. I.6 will be different.

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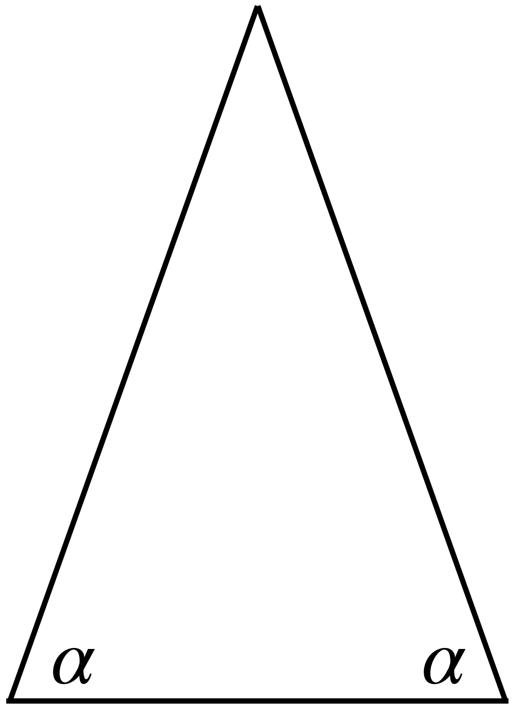

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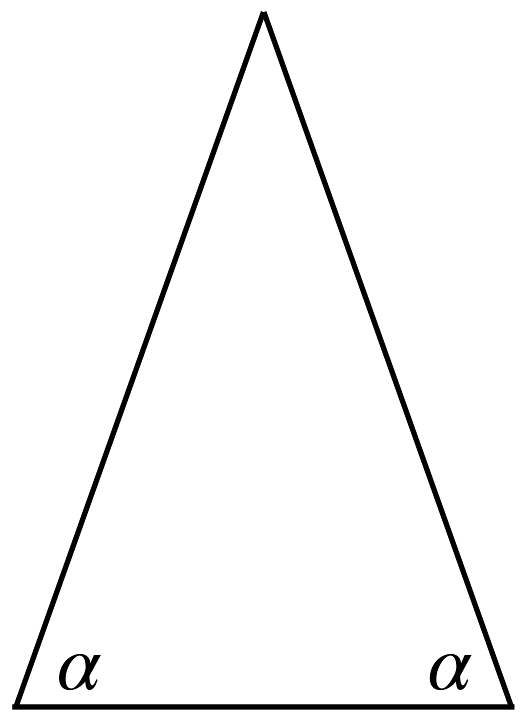

Step 1: suppose the subtended sides were NOT equal

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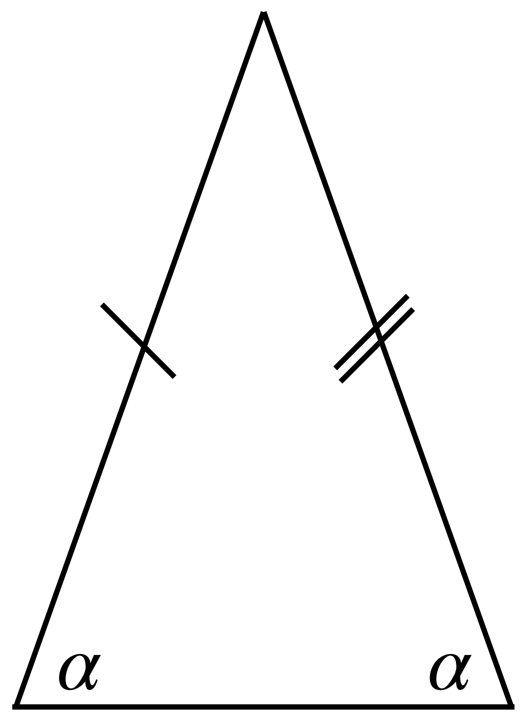

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Euclid's Elements

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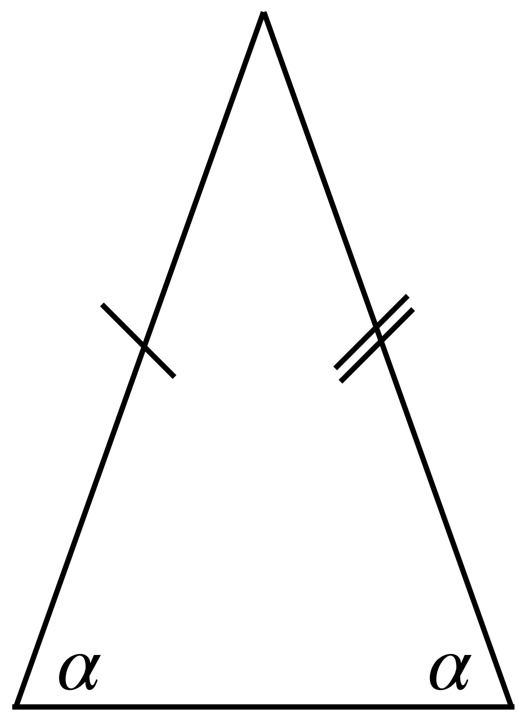

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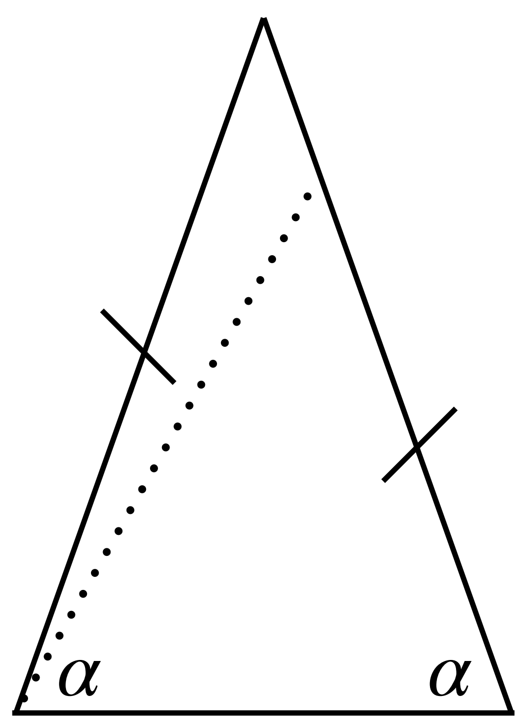

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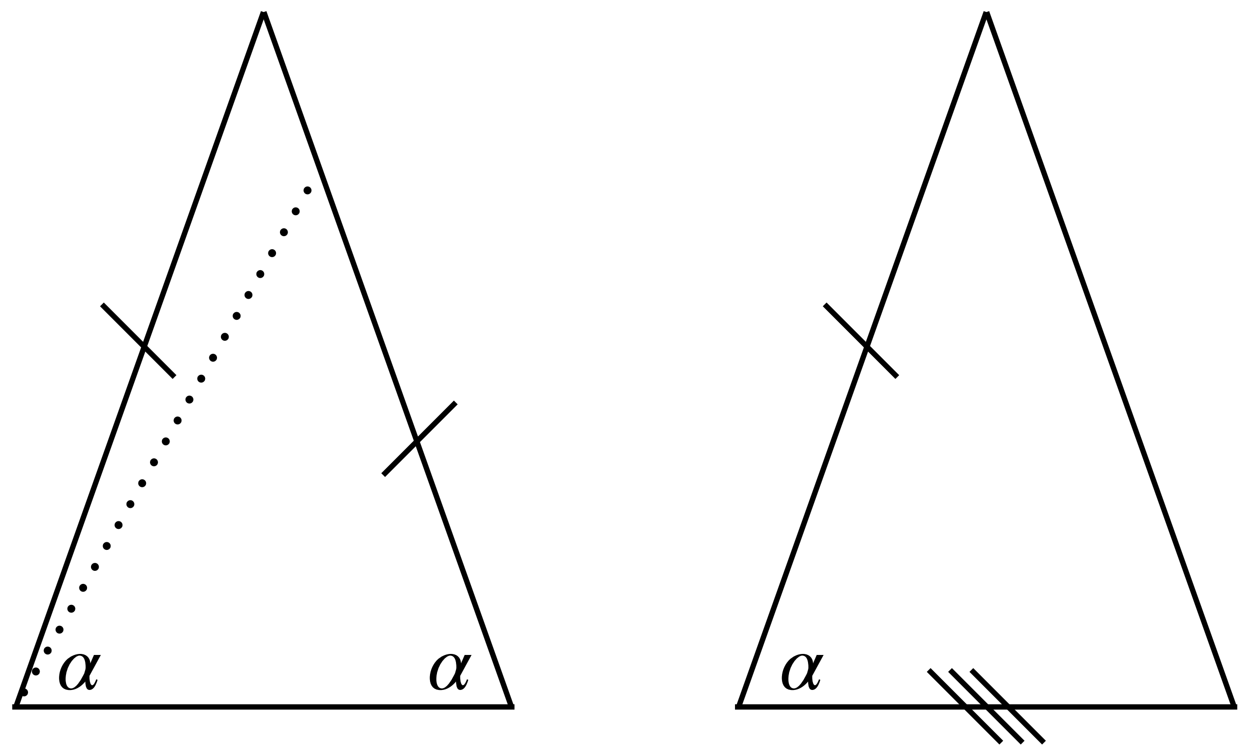

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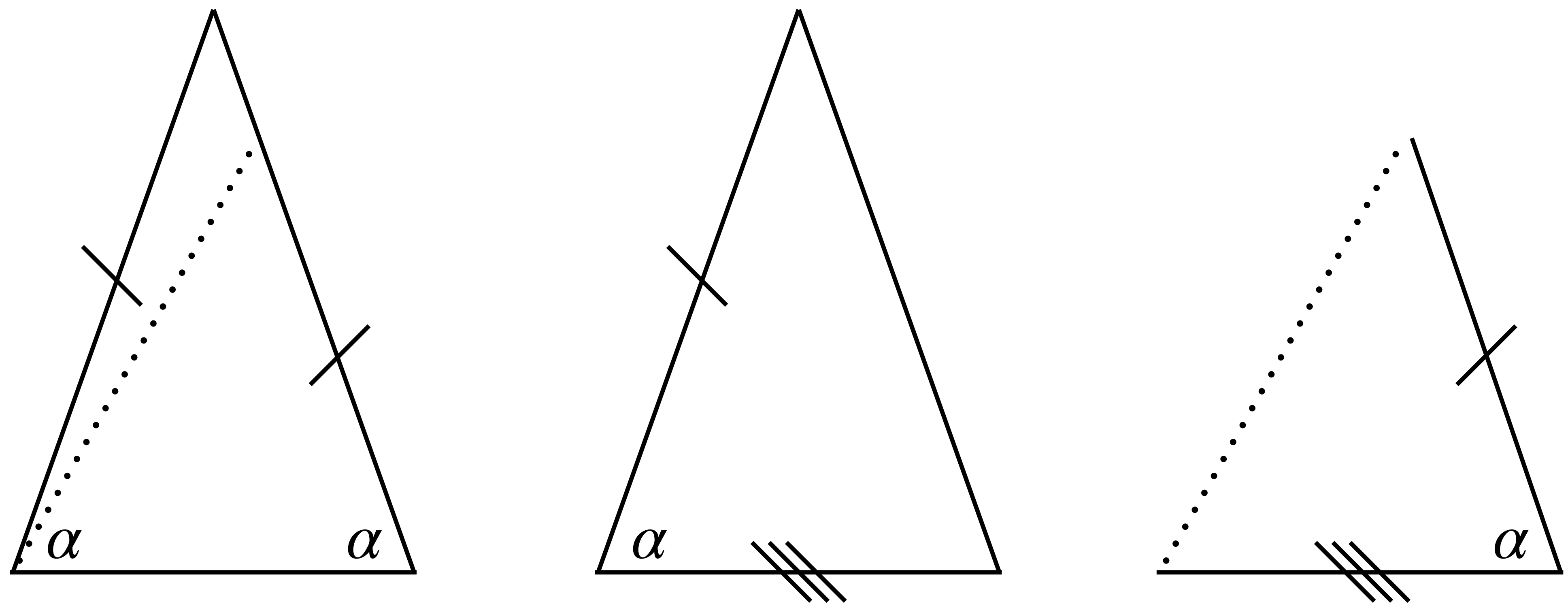

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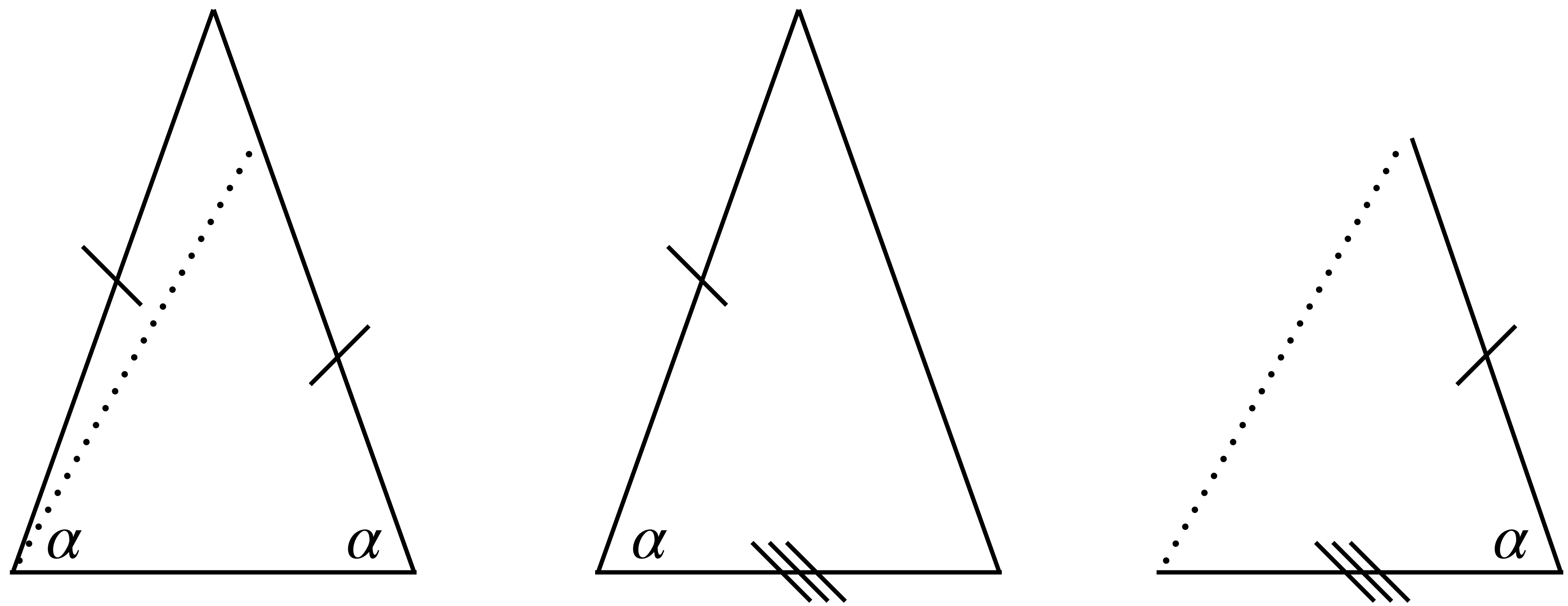

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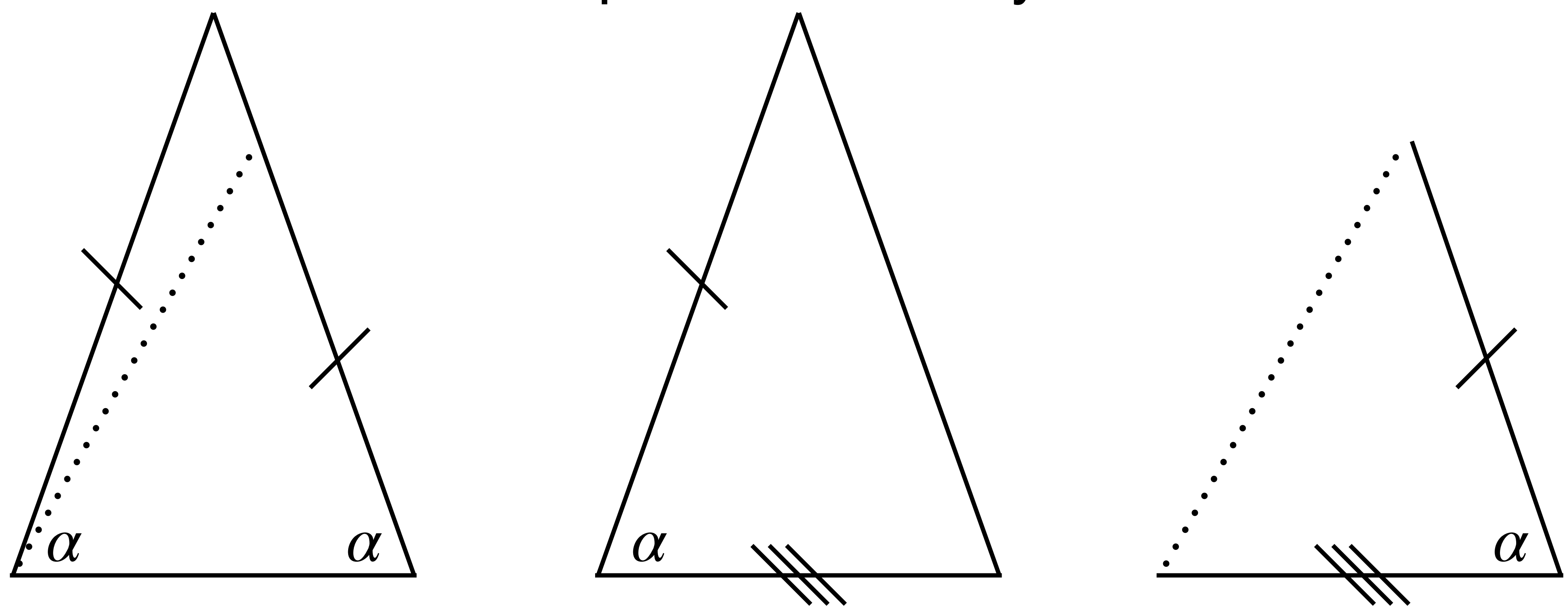

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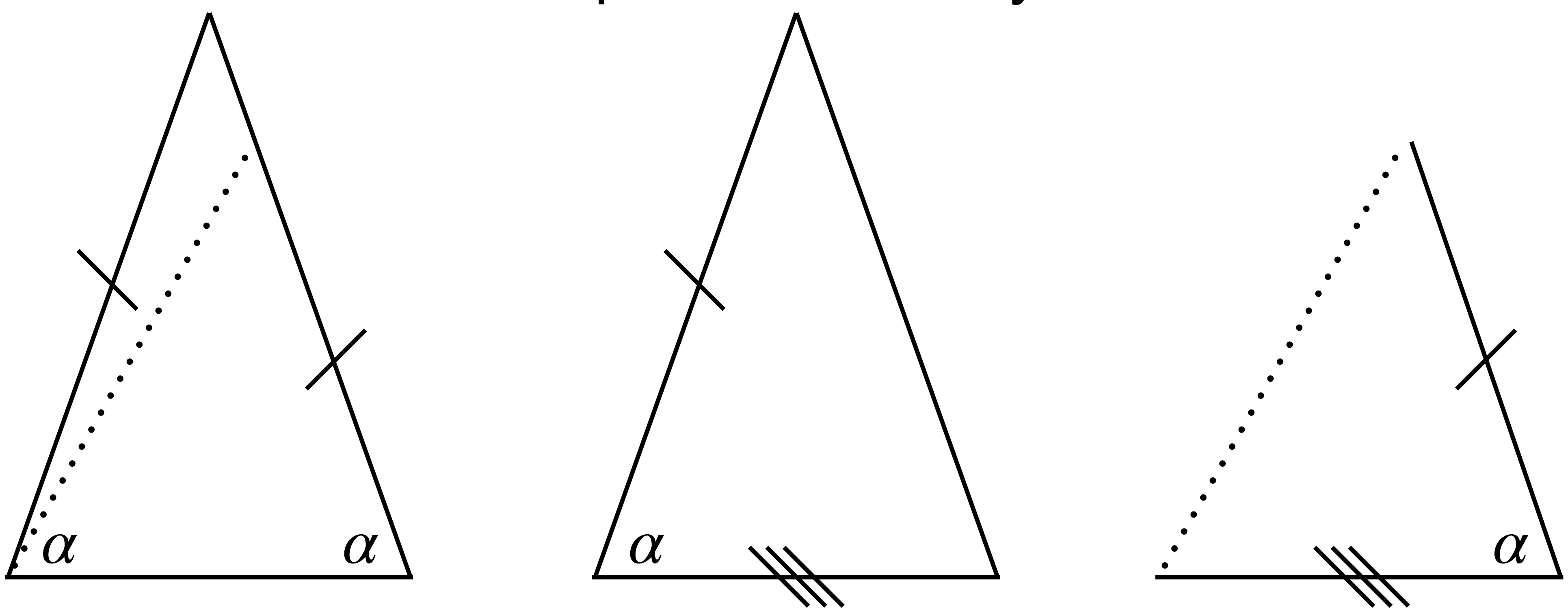

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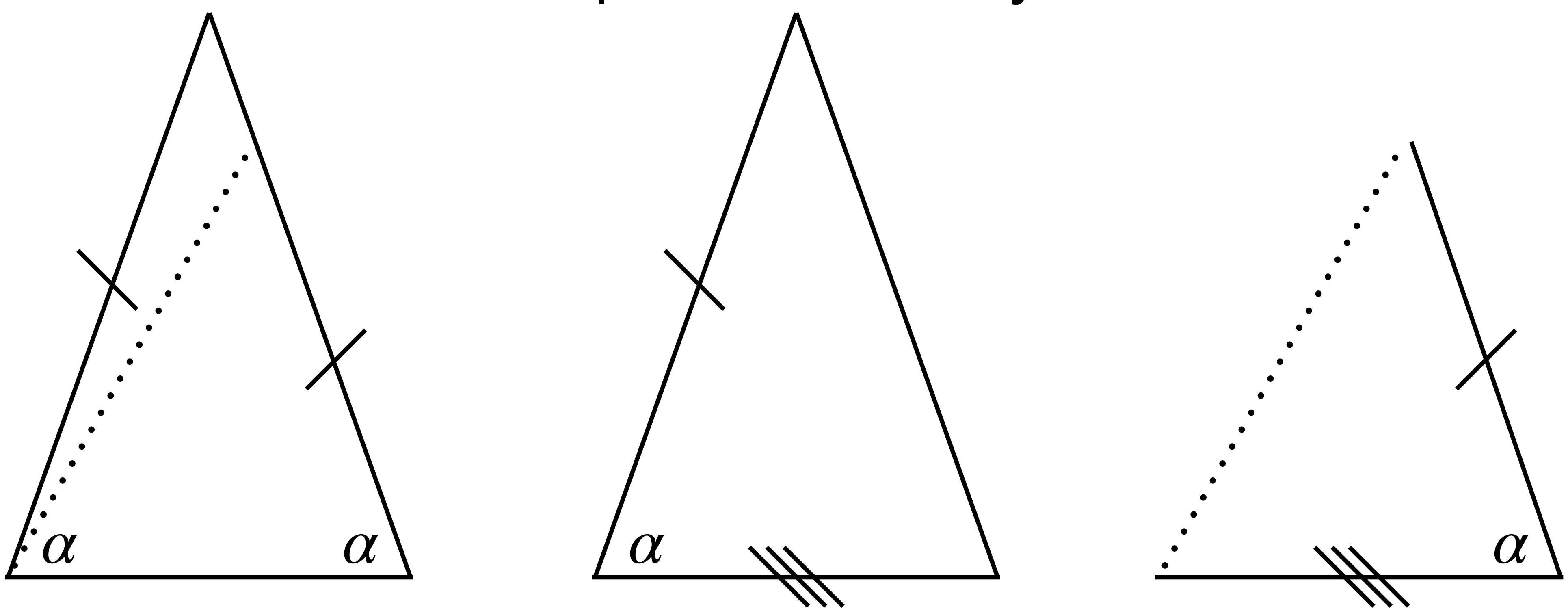

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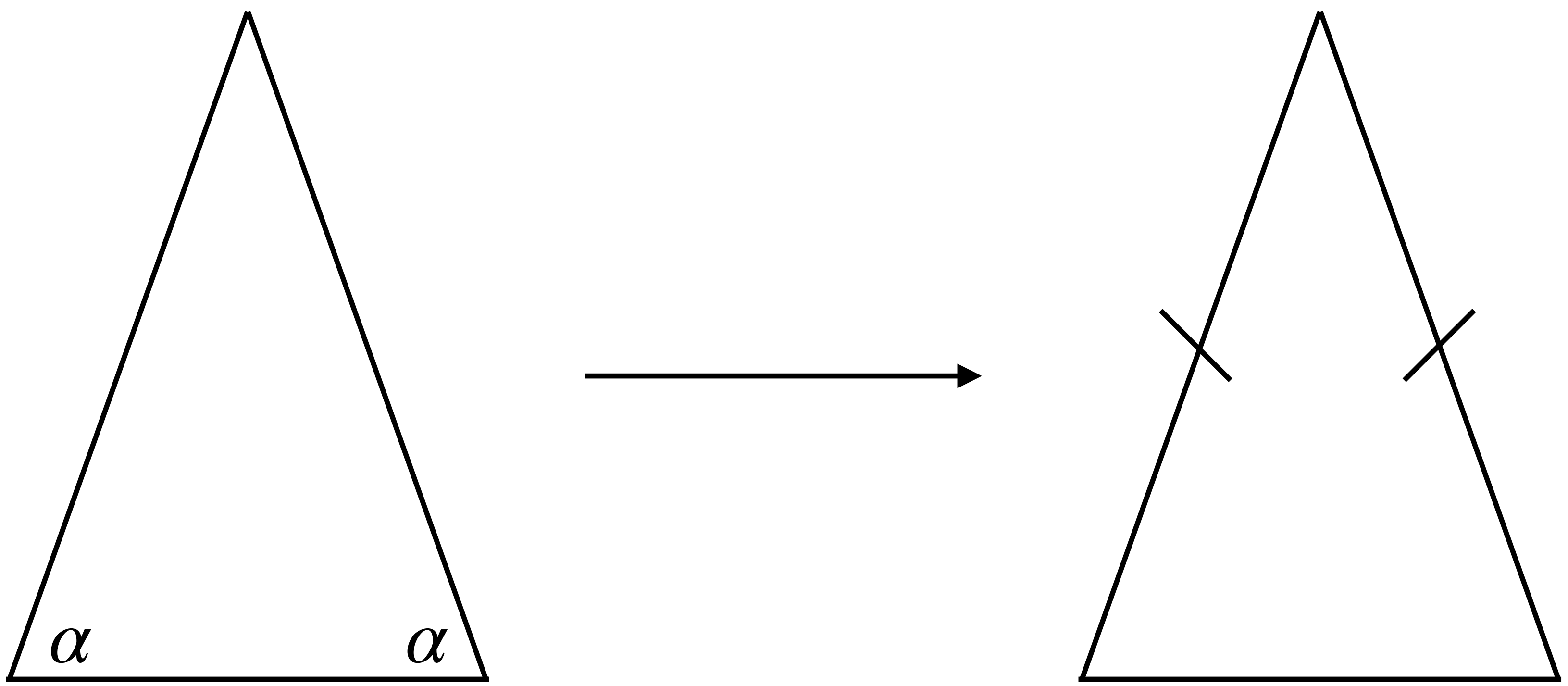

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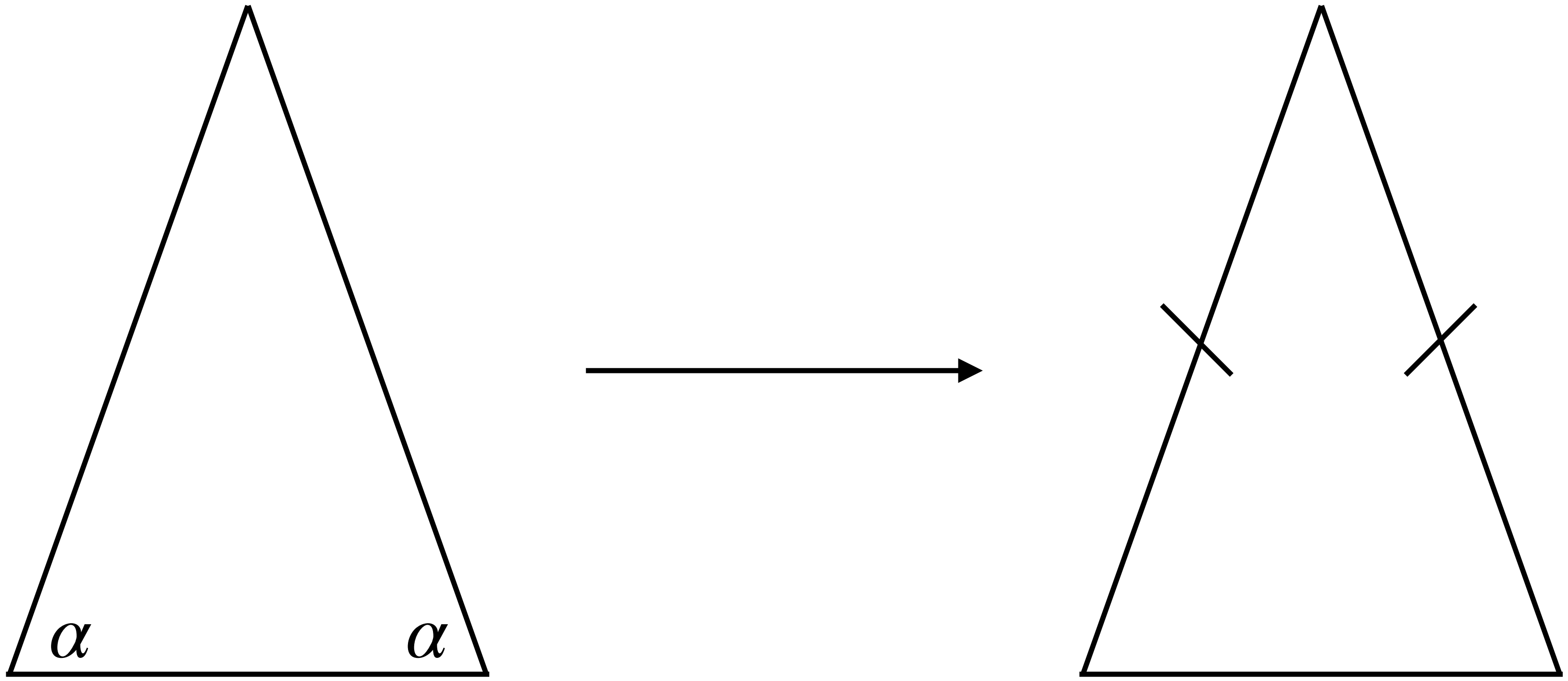

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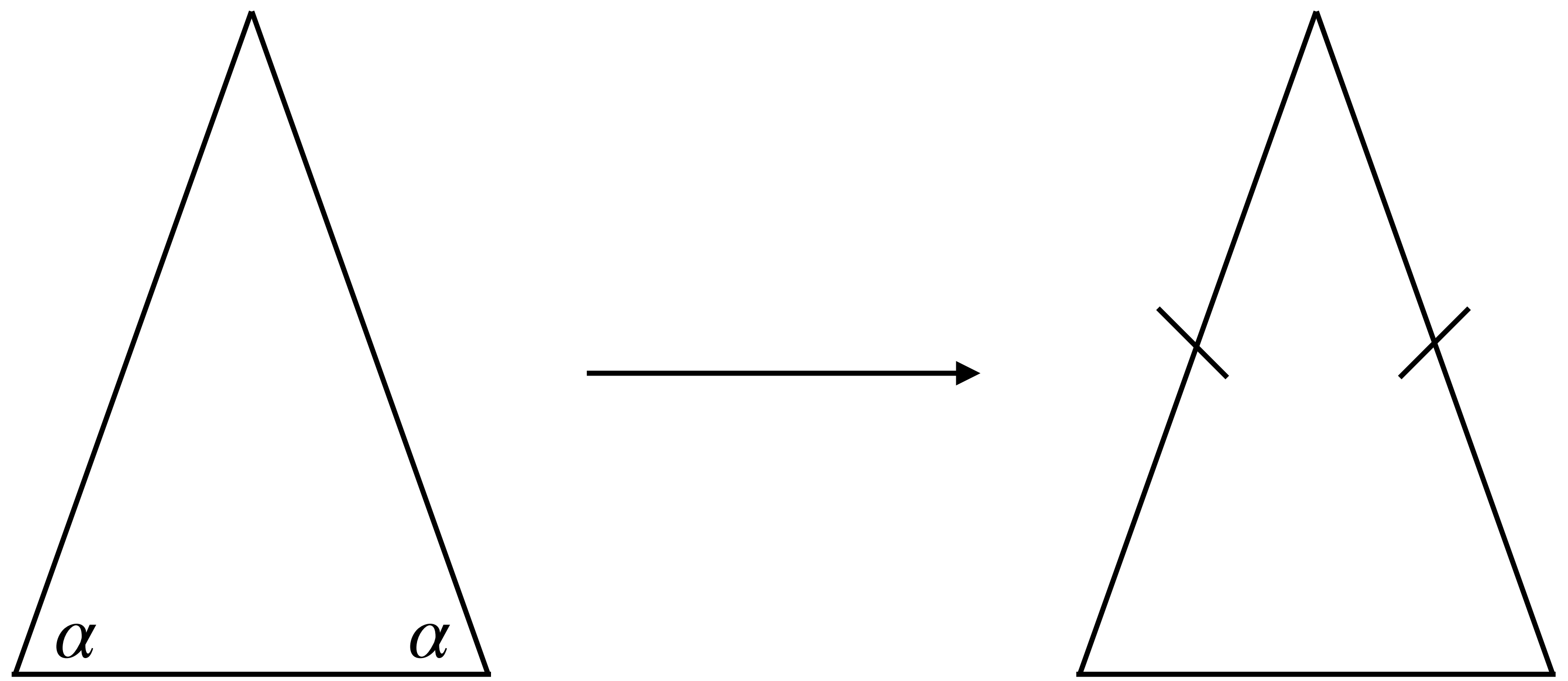

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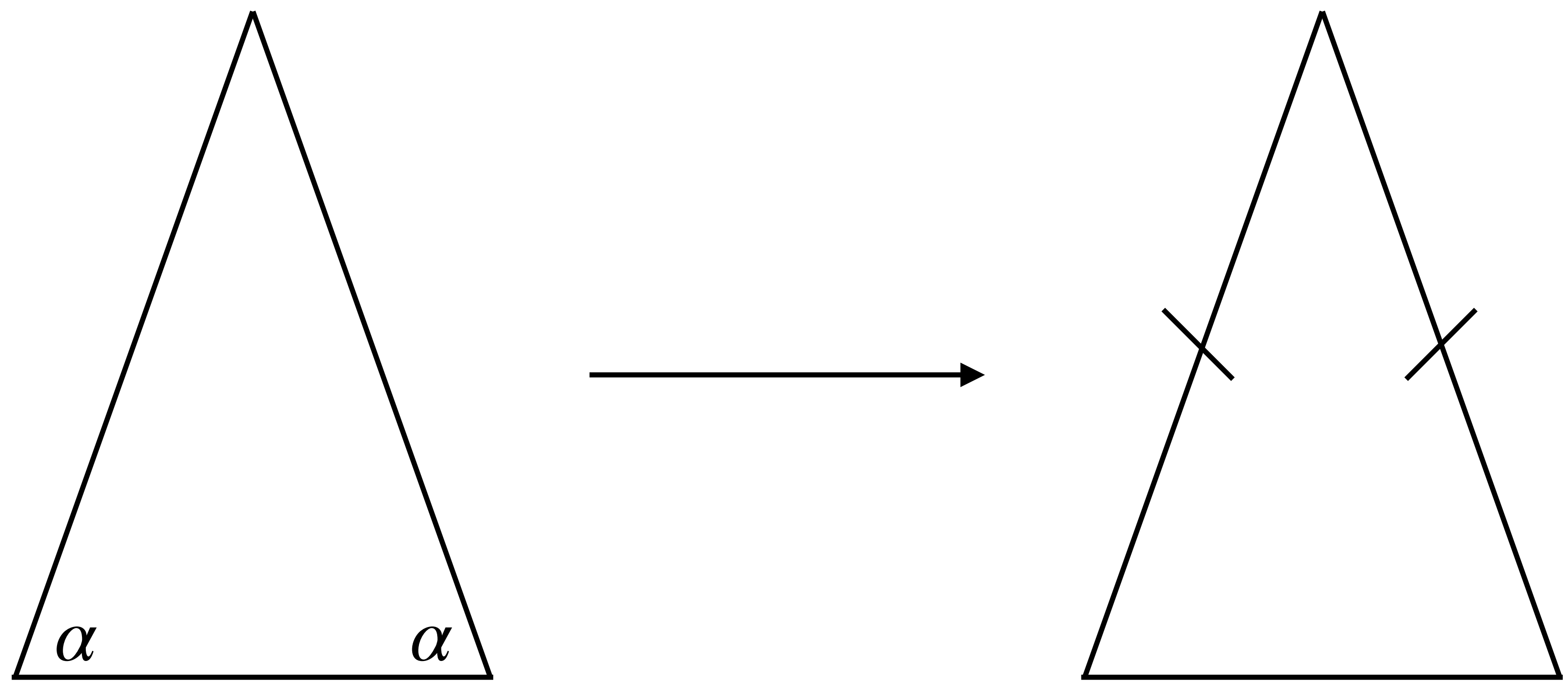

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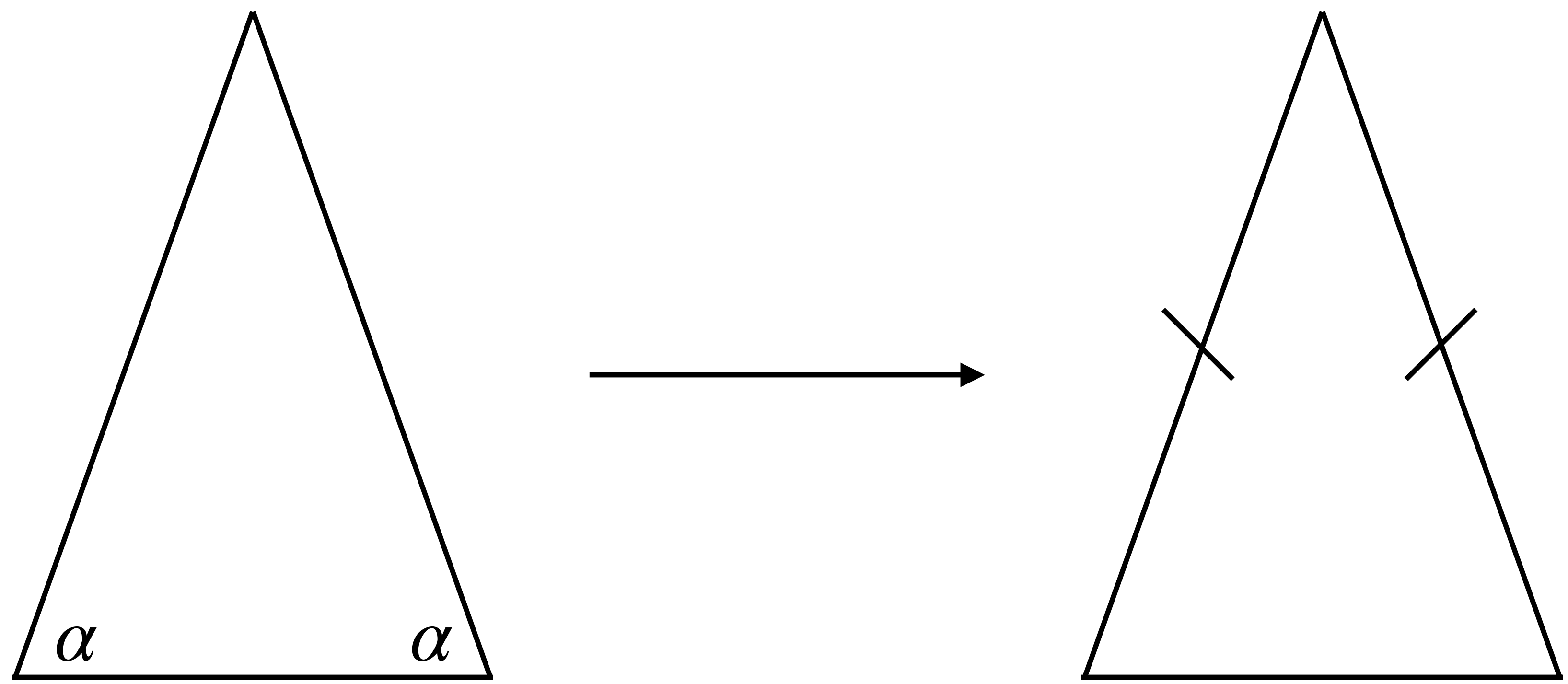

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So the false supposition must be abandoned - the sides are equal

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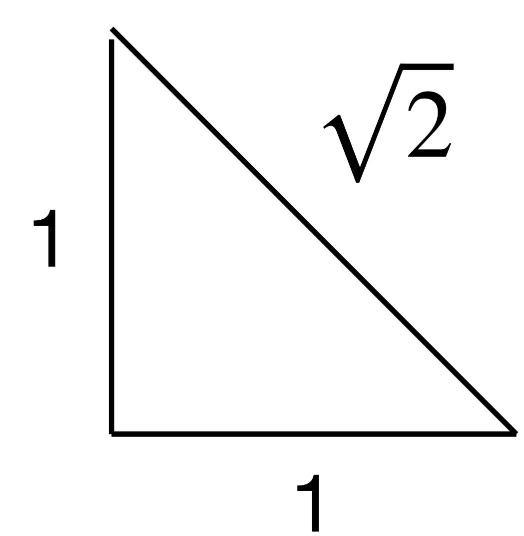

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What kind of number is sqrt(2)? Is it measurable?

Euclid's Elements (Book X)

Proof by contradiction #2: prove the existence of irrational numbers

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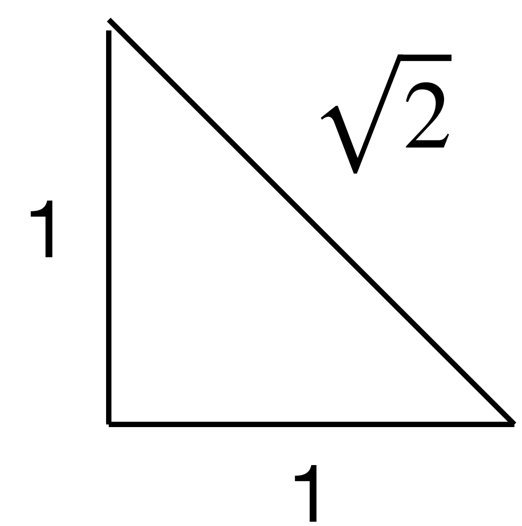

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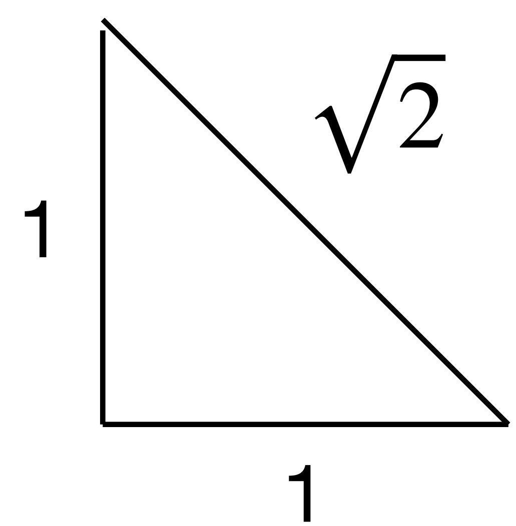
Proof by contradiction #2: prove the existence of irrational numbers



It would suffice to show that no integers p,q satisfy $\frac{p}{q} = \sqrt{2}$

Proof by contradiction #2: prove the existence of irrational numbers

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Even number

Proof by contradiction #2: prove the existence of irrational numbers

Even number $2s$

Proof by contradiction #2: prove the existence of irrational numbers

Even number $2s$

Odd number

Proof by contradiction #2: prove the existence of irrational numbers

Even number $2s$

Odd number $2s + 1$

Proof by contradiction #2: prove the existence of irrational numbers

Square

Even number	$2s$	
Odd number	$2s + 1$	

Proof by contradiction #2: prove the existence of irrational numbers

		Square
Even number	$2s$	$(2s)^2 = 4s^2$
Odd number	$2s + 1$	

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Thus, evens square to evens, odds to odds

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so p,q cannot exist satisfying $\frac{p}{q} = \sqrt{2}$

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Then found this led to an absurd situation

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Which could not be true

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And thus prove the existence of a non-rational kind of number

The null hypothesis

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This H0 construct, which reads awkwardly in language

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- is logically sound

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So even though the H0 construct isn't how we typically think,

and is certainly not how we typically speak,

it is a sensible way to frame statistical problems.

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In real-world data, we don't get the clarity of Euclid

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We CAN generate a good null hypothesis

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But we CANNOT say H0 is for sure false

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Instead, we say H0 is unlikely, and how unlikely

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Like so:

Example 1

Height in boys and girls

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I suspect they differ

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I suspect they differ

H0: height does not differ by sex

Example 1

Height in boys and girls

I suspect they differ, and I make measurements

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I suspect they differ, and I make measurements

H0: measurements grouped by sex do not differ

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(from the measurements grouped randomly)

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How can I test H0? See if it is true?

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Height in boys and girls

Sex	Height
1	45
1	35
1	64
1	75
0	54
0	42
0	67
0	43

$$\delta_{real} = \mu_1 - \mu_0$$

Example 1

Height in boys and girls

Sex	Height
1	45
0	35
0	64
1	75
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1	42
1	67
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$$\delta_{rand0001} = \mu_1 - \mu_0$$

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Sex	Height
0	45
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0	67
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$$\delta_{rand0002} = \mu_1 - \mu_0$$

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$$\delta_{rand0003} = \mu_1 - \mu_0$$

Example 1

Height in boys and girls

$$\delta_{real} = \mu_1 - \mu_0$$

$$\delta_{rand0001} = \mu_1 - \mu_0$$
$$\delta_{rand0002} = \mu_1 - \mu_0$$
$$\delta_{rand0003} = \mu_1 - \mu_0$$
$$\cdots$$
$$\delta_{rand0100} = \mu_1 - \mu_0$$

Example 1

$$\delta_{rand0001} = \mu_1 - \mu_0$$
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Example 1

Largest

Smallest

$$\delta_{rand0001} = \mu_1 - \mu_0$$
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Largest

$\delta_{rand0001}$

Smallest

$$\delta_{rand0001} = \mu_1 - \mu_0$$
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Largest

$\delta_{rand0001}$

$\delta_{rand0002}$

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Example 1

Largest

$\delta_{rand0003}$

$\delta_{rand0001}$

$\delta_{rand0002}$

Smallest

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$$\delta_{rand0002} = \mu_1 - \mu_0$$
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Example 1

Largest

$\delta_{rand0004}$

$\delta_{rand0003}$

$\delta_{rand0001}$

$\delta_{rand0002}$

Smallest

$$\delta_{rand0001} = \mu_1 - \mu_0$$
$$\delta_{rand0002} = \mu_1 - \mu_0$$
$$\delta_{rand0003} = \mu_1 - \mu_0$$
$$\cdots$$
$$\delta_{rand0100} = \mu_1 - \mu_0$$

Example 1

Largest

$\delta_{rand0004}$

$\delta_{rand0003}$

$\delta_{rand0001}$

$\delta_{rand0005}$

$\delta_{rand0002}$

Smallest

$$\delta_{rand0001} = \mu_1 - \mu_0$$
$$\delta_{rand0002} = \mu_1 - \mu_0$$
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Example 1

Largest

$\delta_{rand0006}$

$\delta_{rand0004}$

$\delta_{rand0003}$

$\delta_{rand0001}$

$\delta_{rand0005}$

$\delta_{rand0002}$

Smallest

$$\delta_{rand0001} = \mu_1 - \mu_0$$
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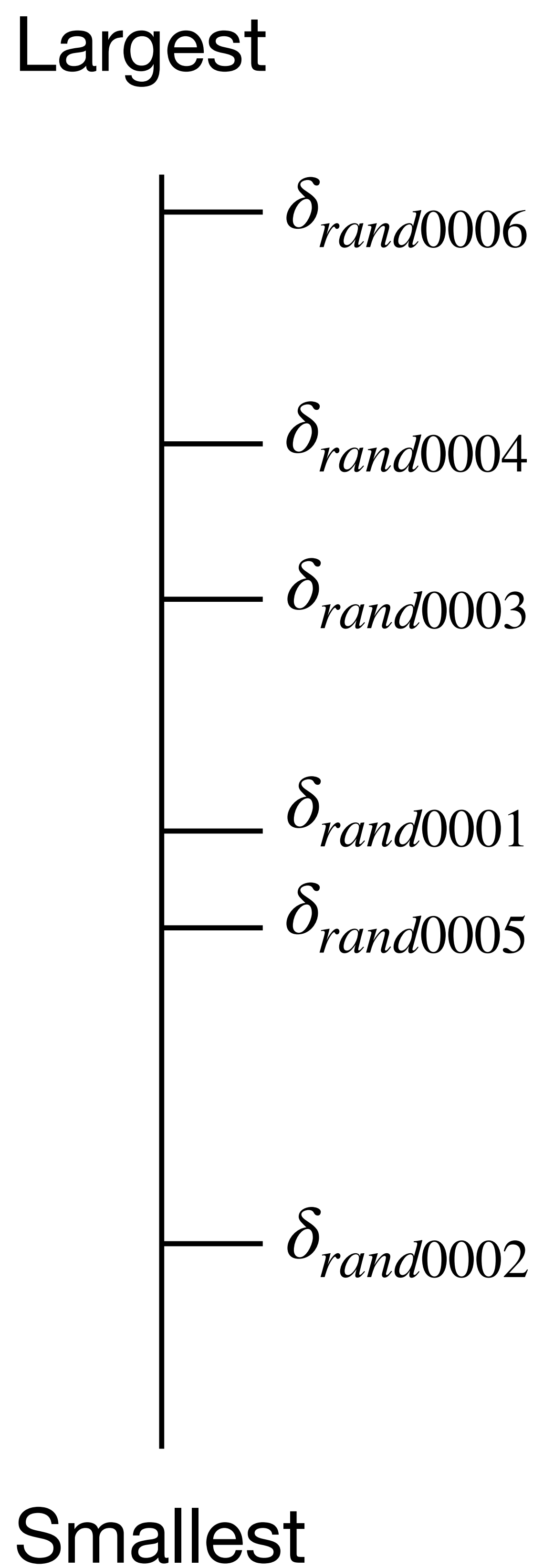

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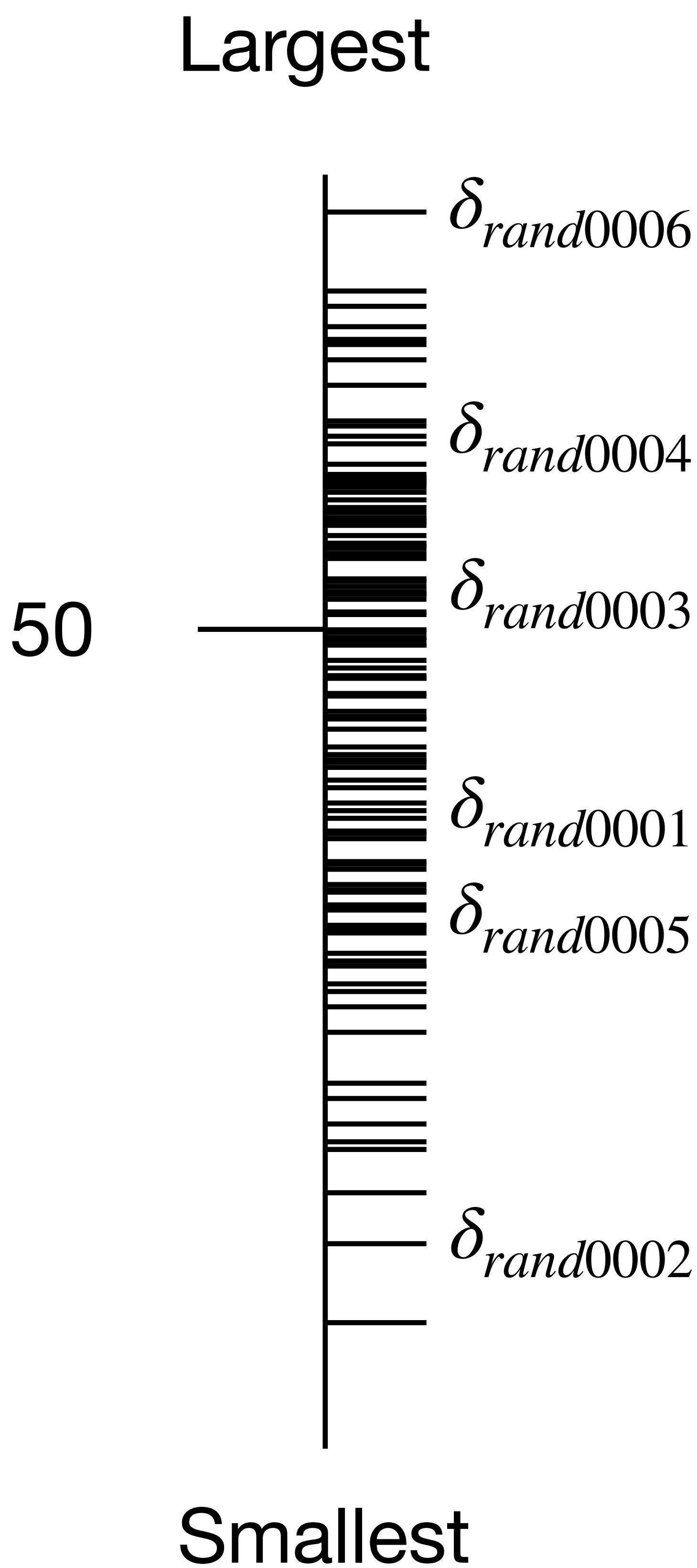

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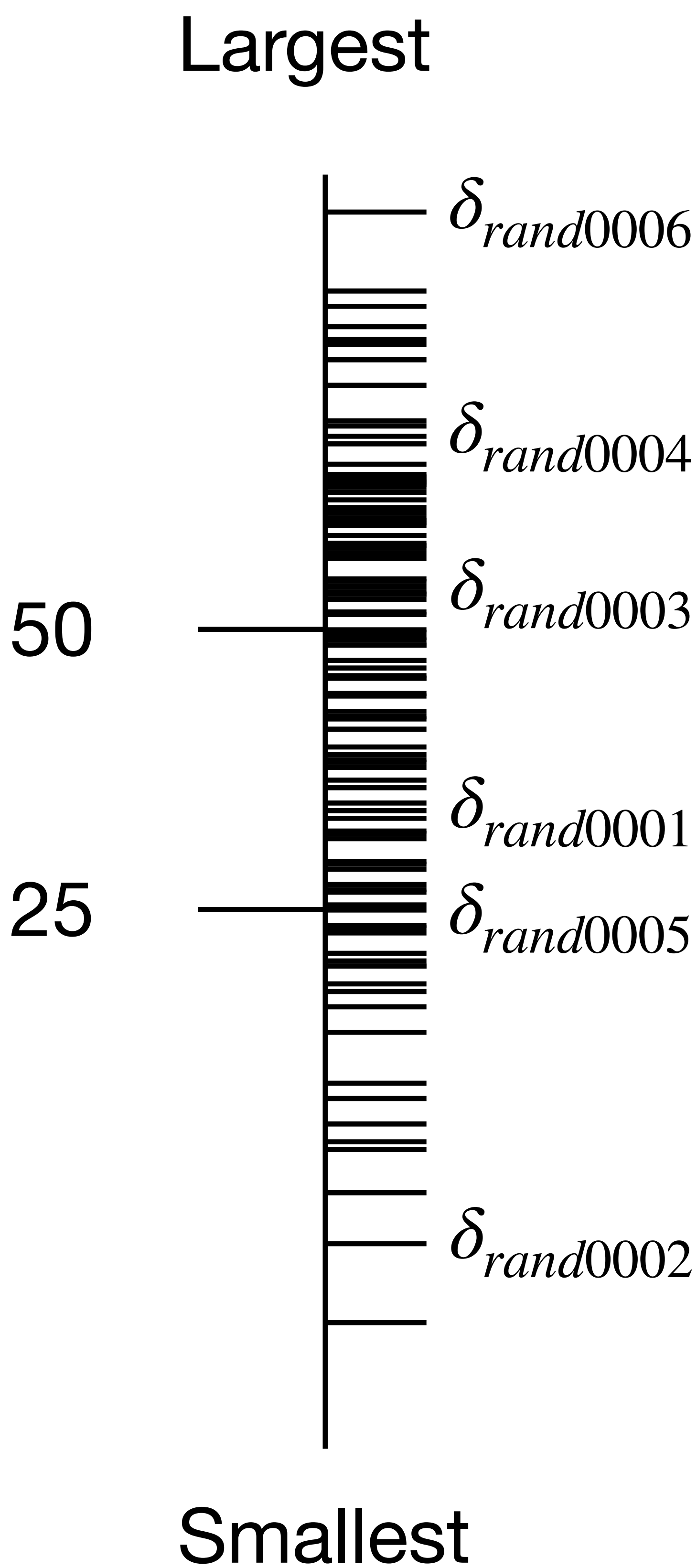

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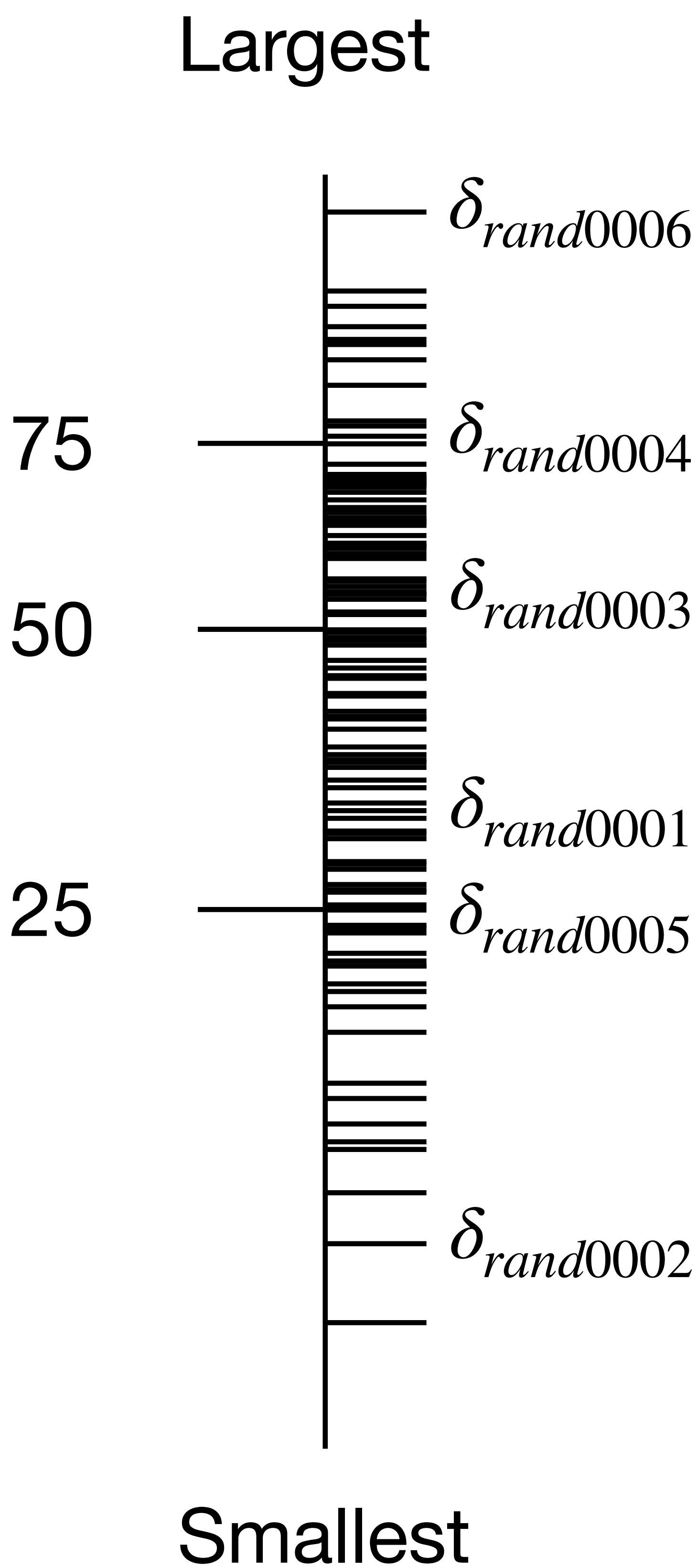

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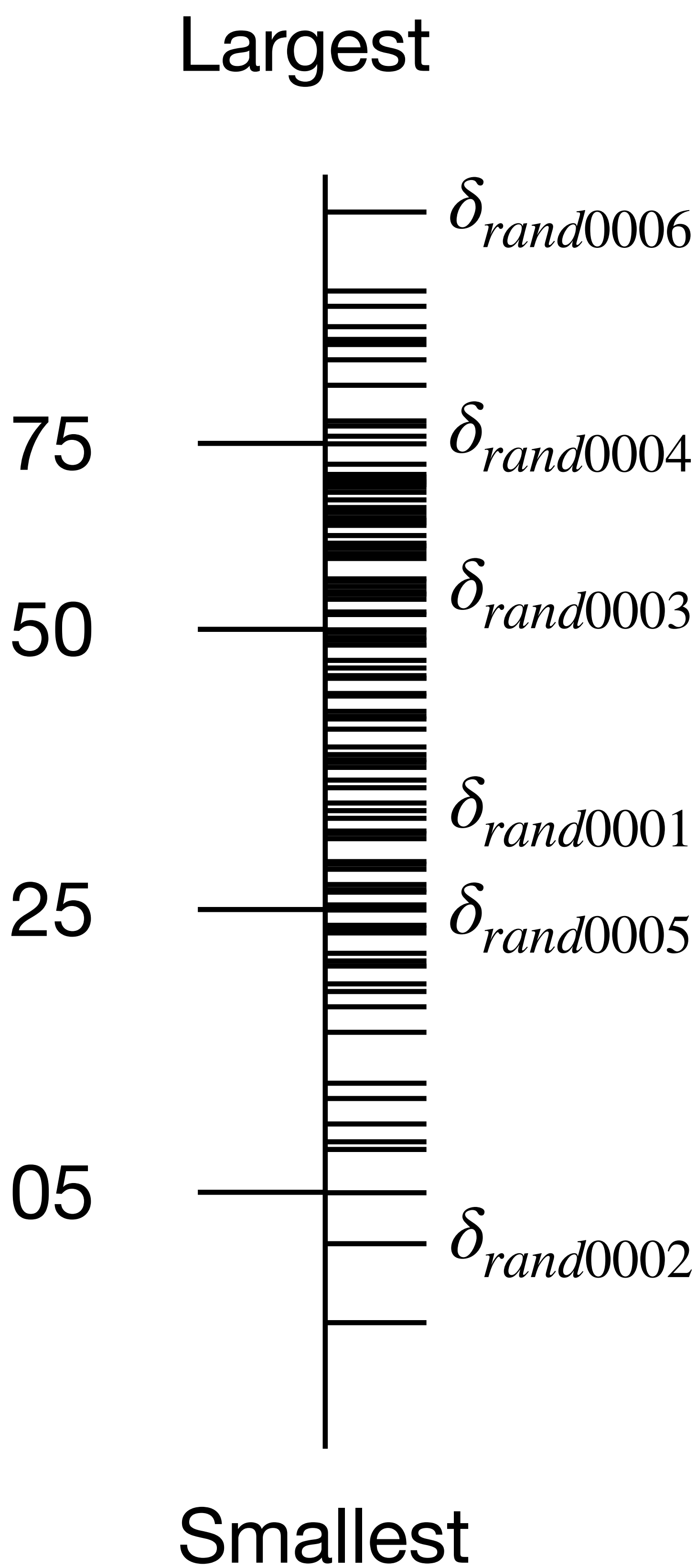

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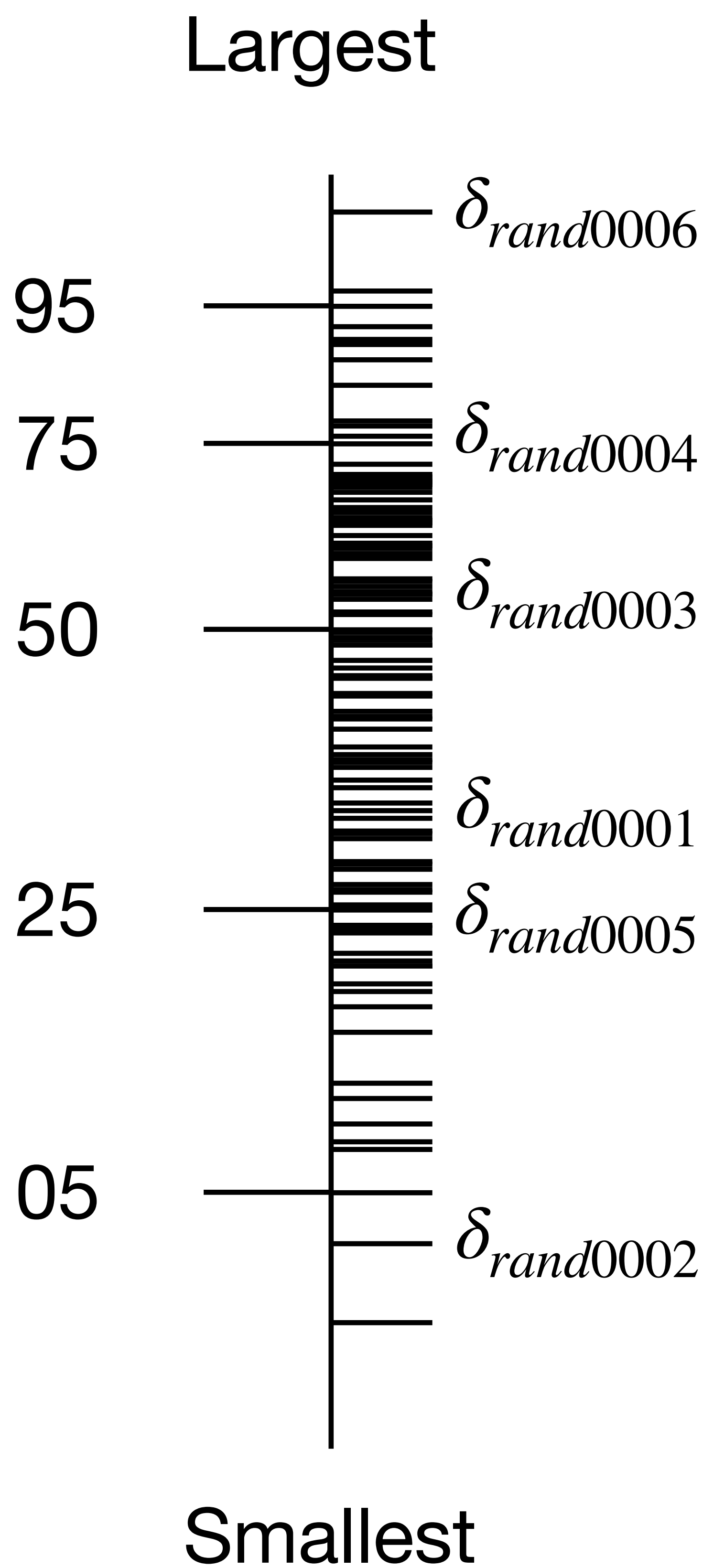

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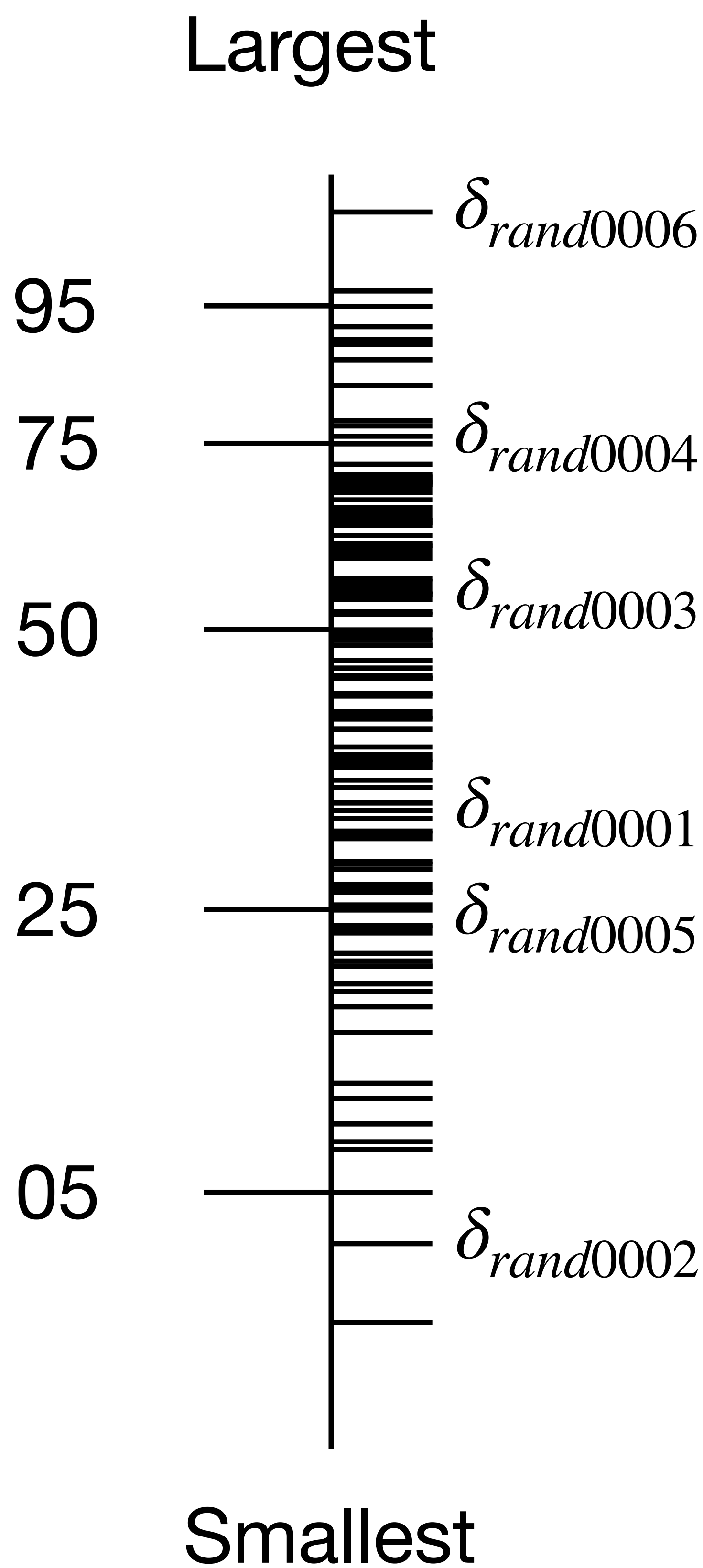

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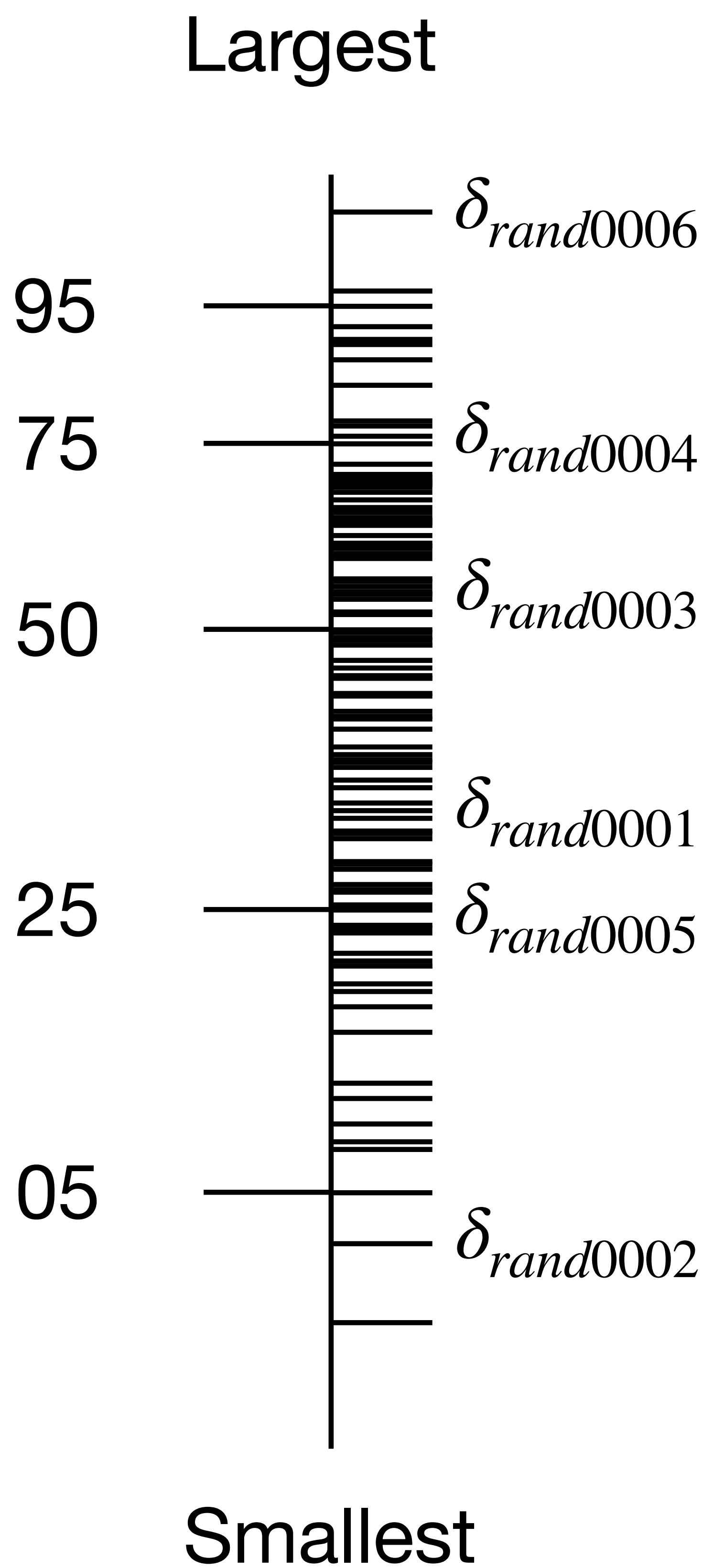

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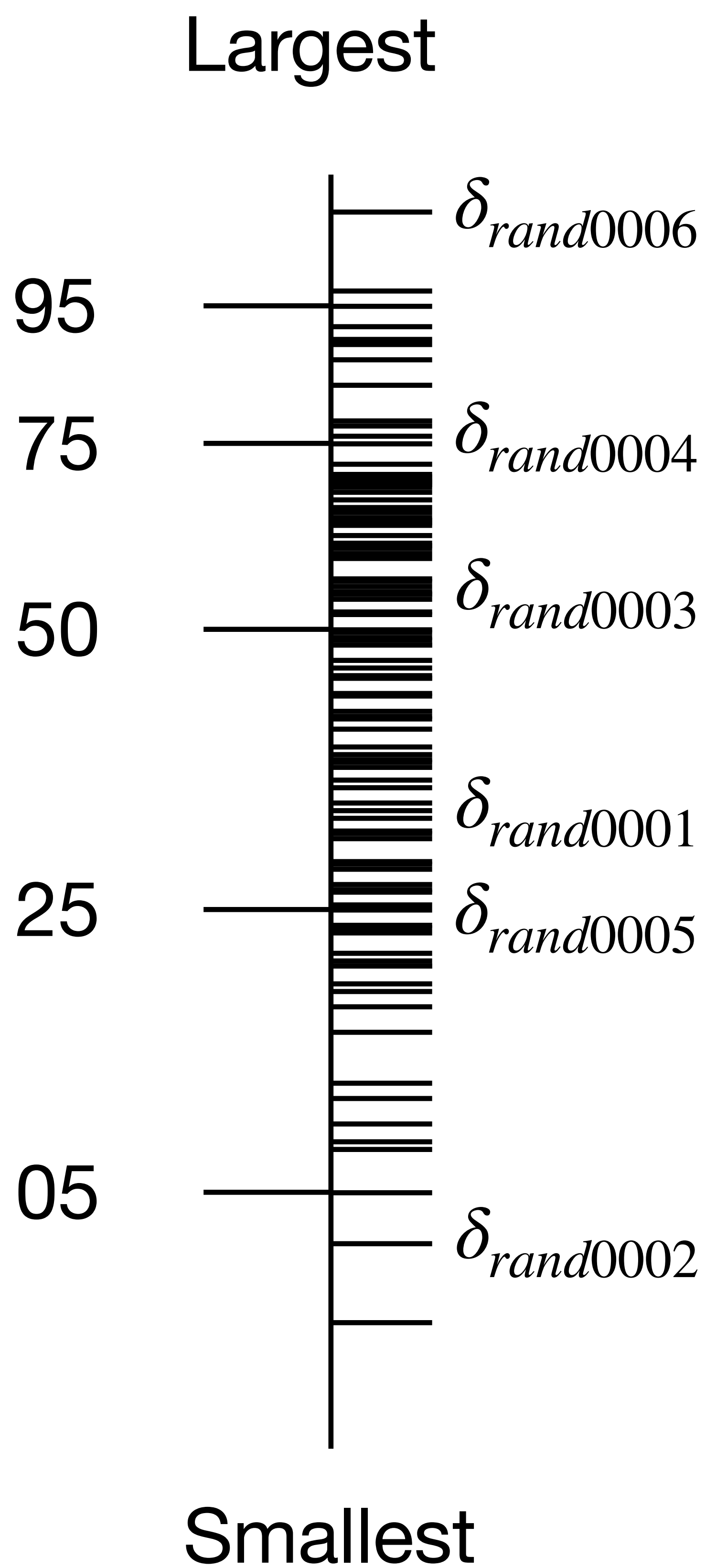

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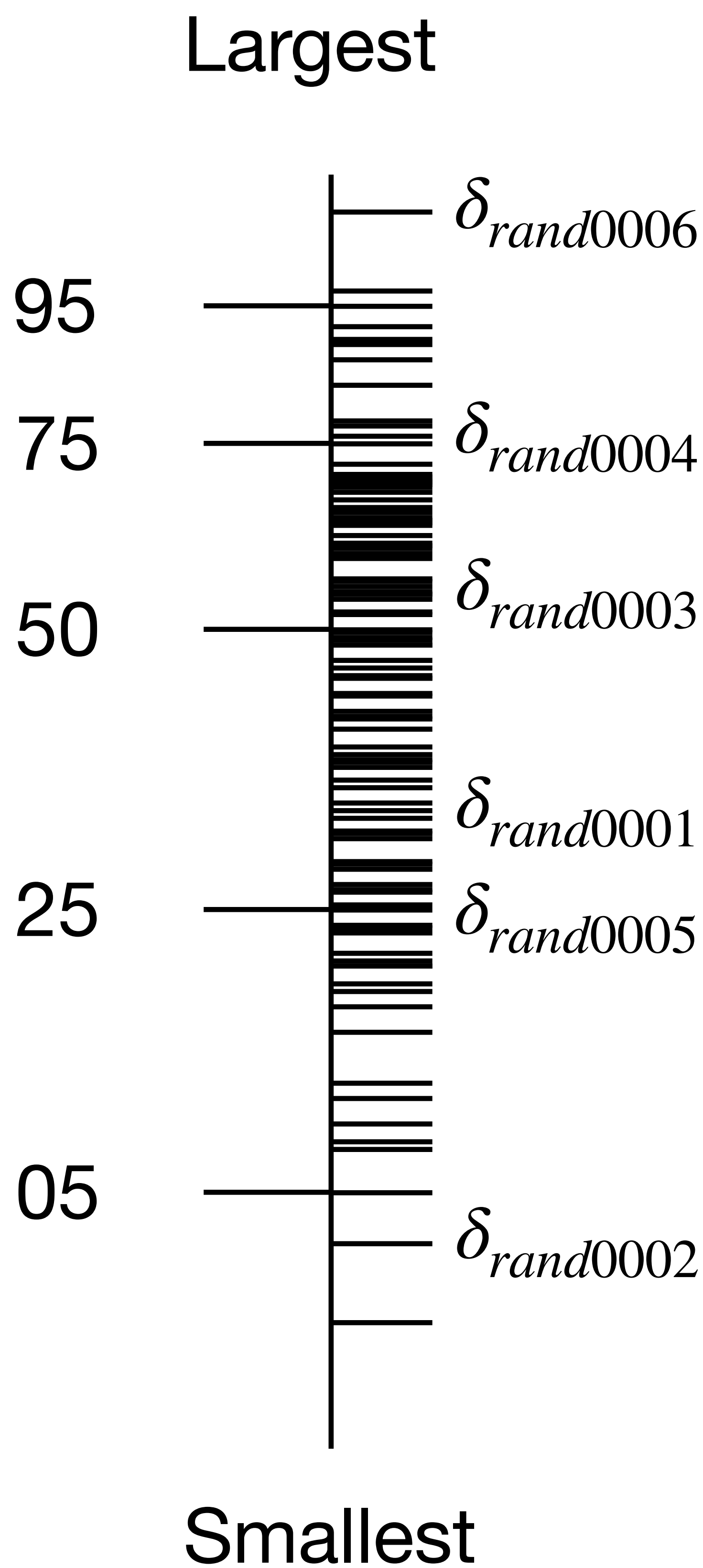

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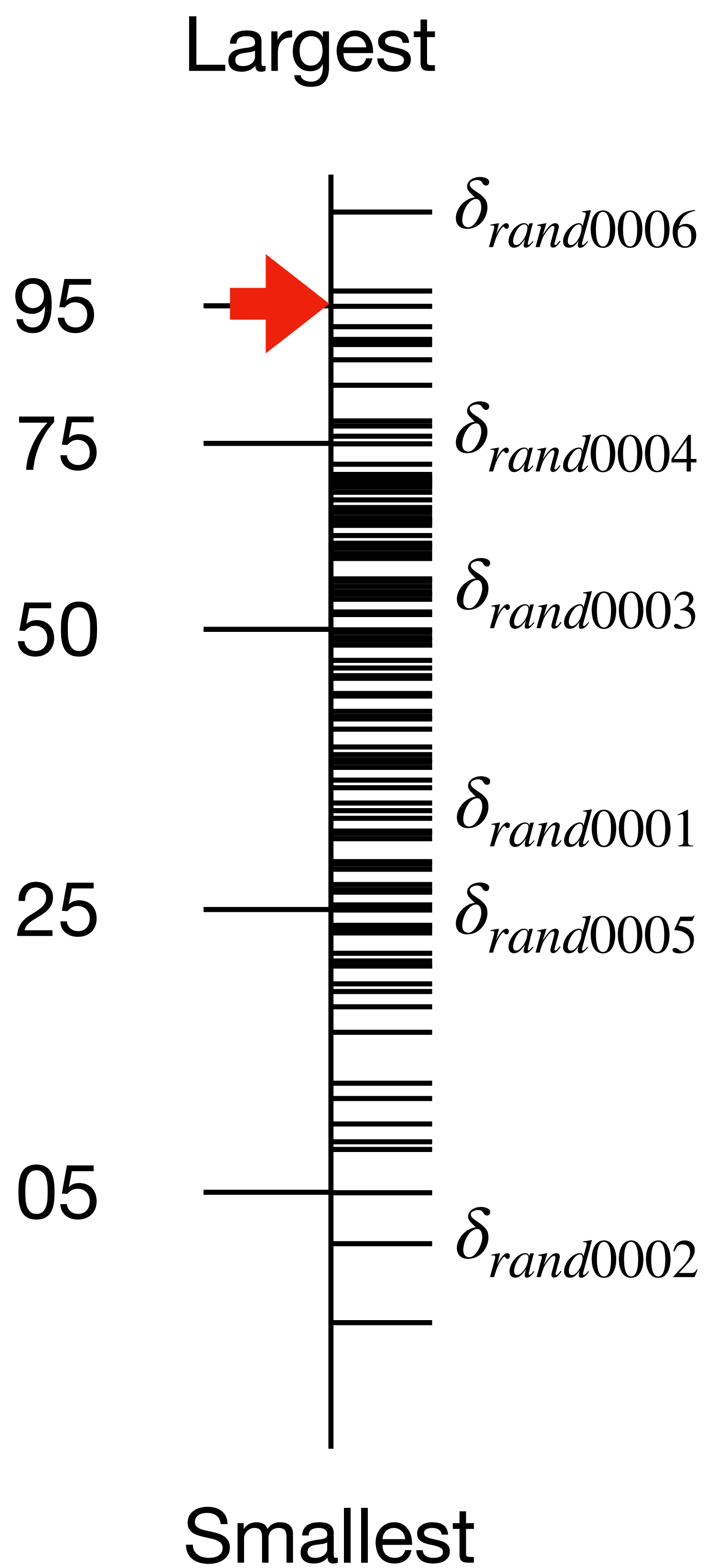

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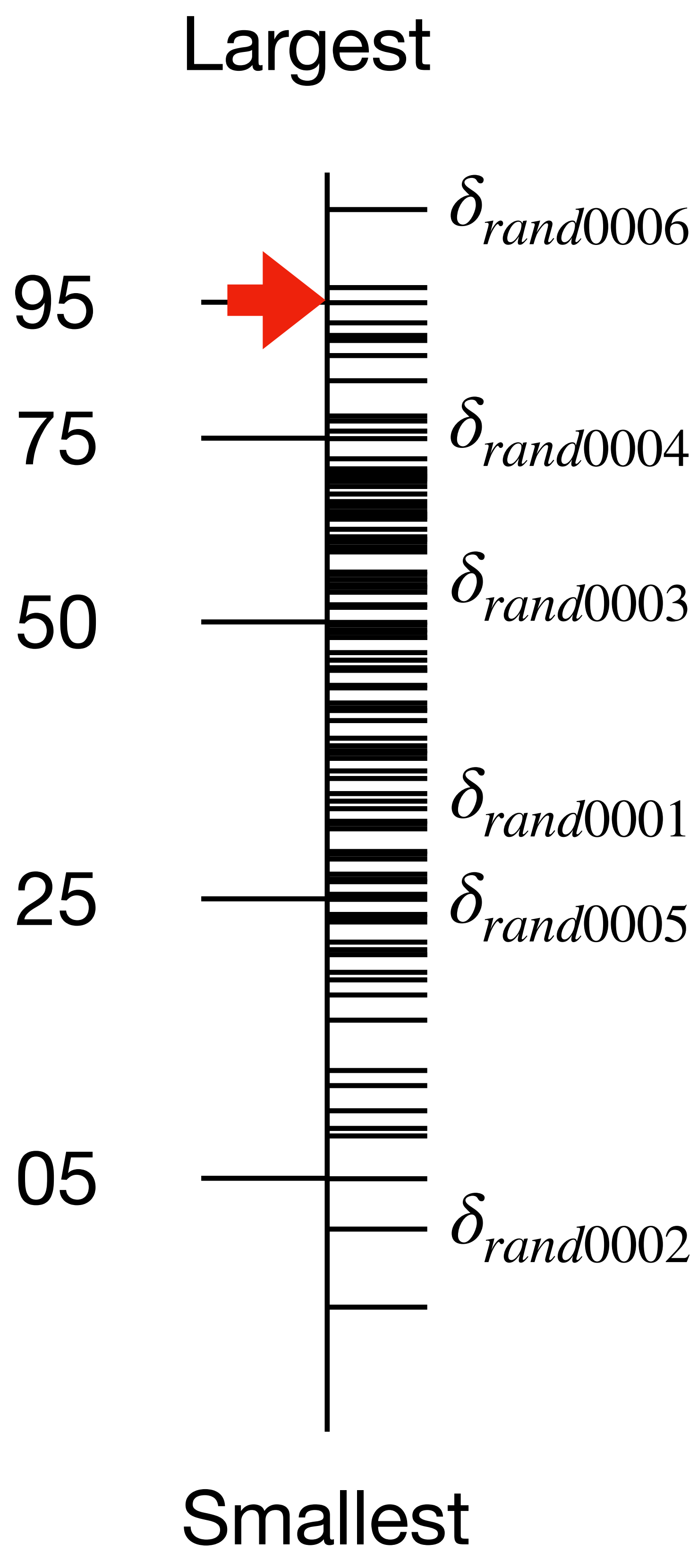

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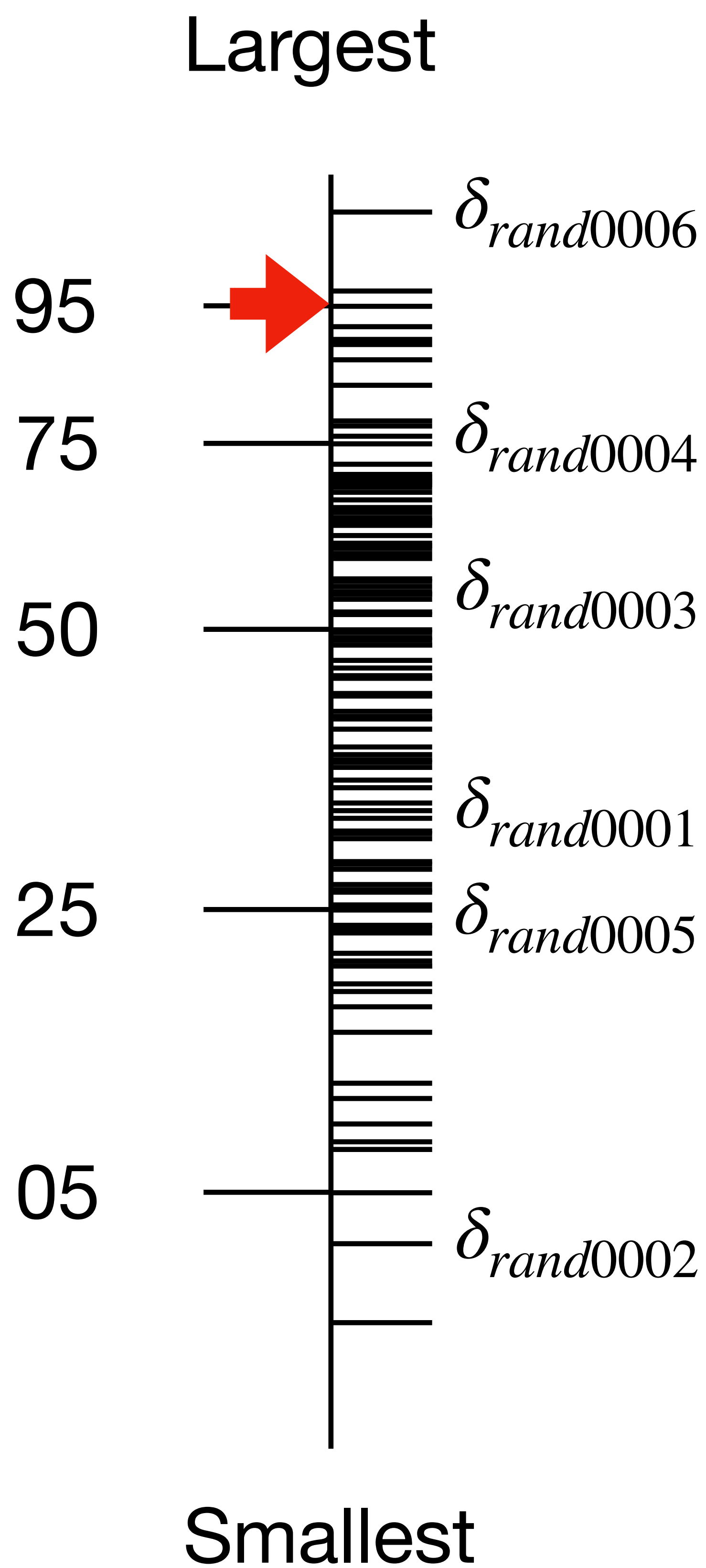

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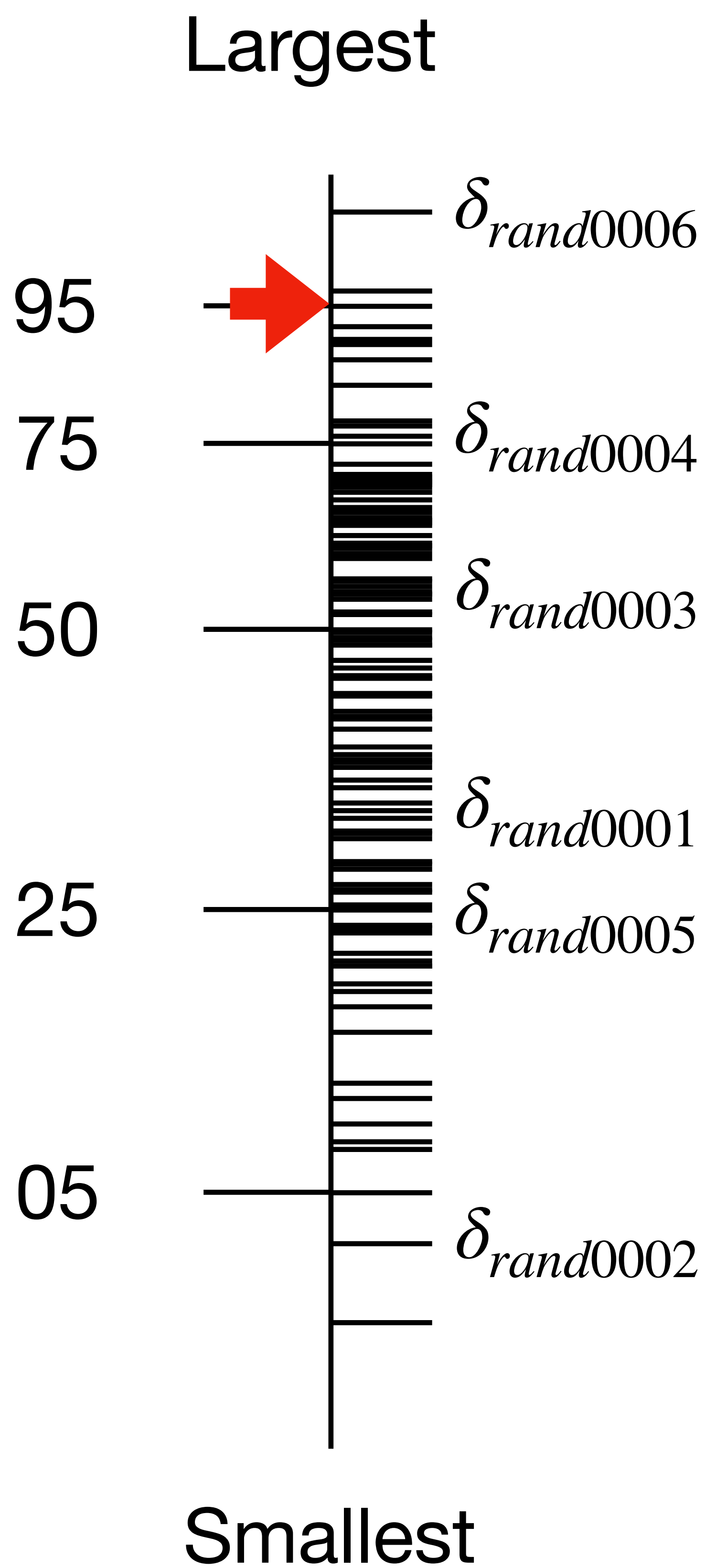

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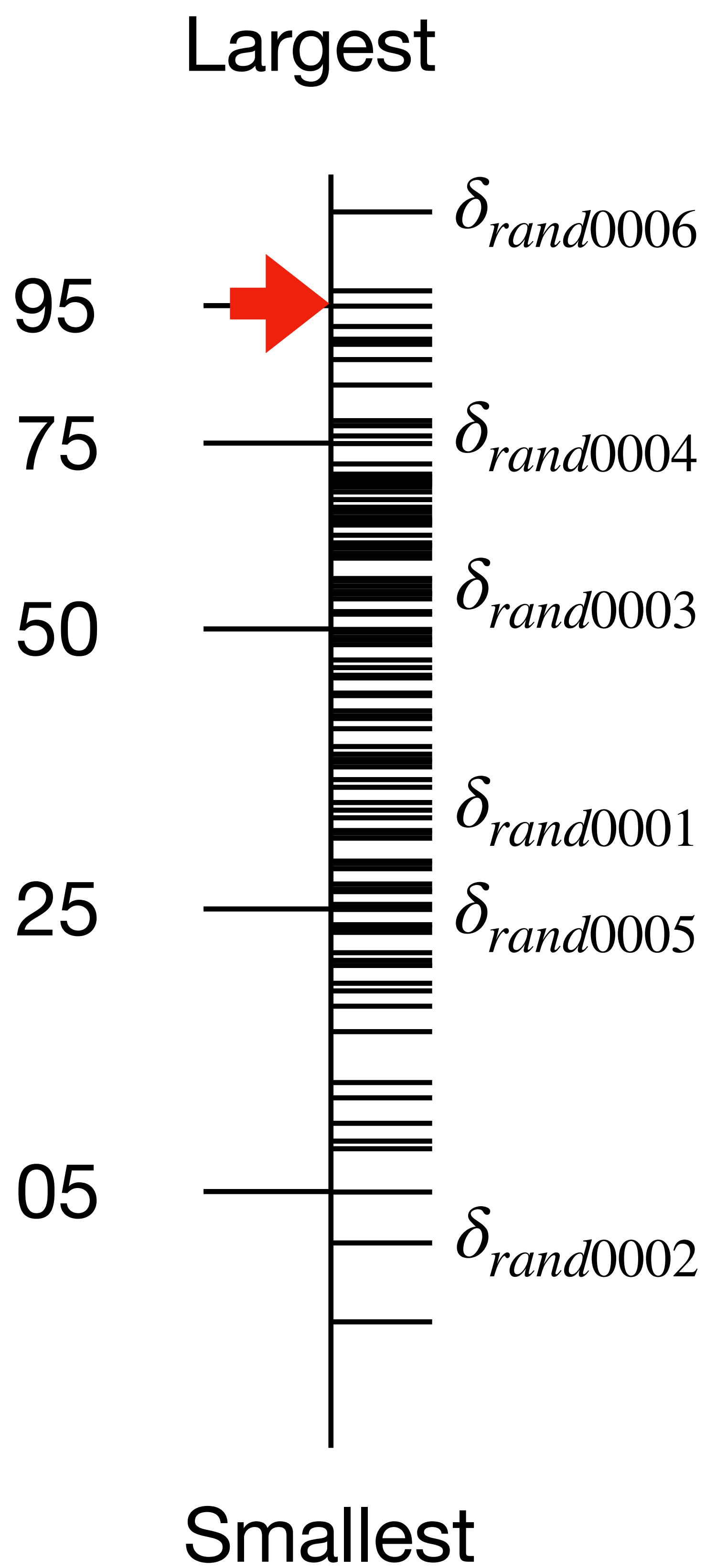

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Suppose I take sample 0004

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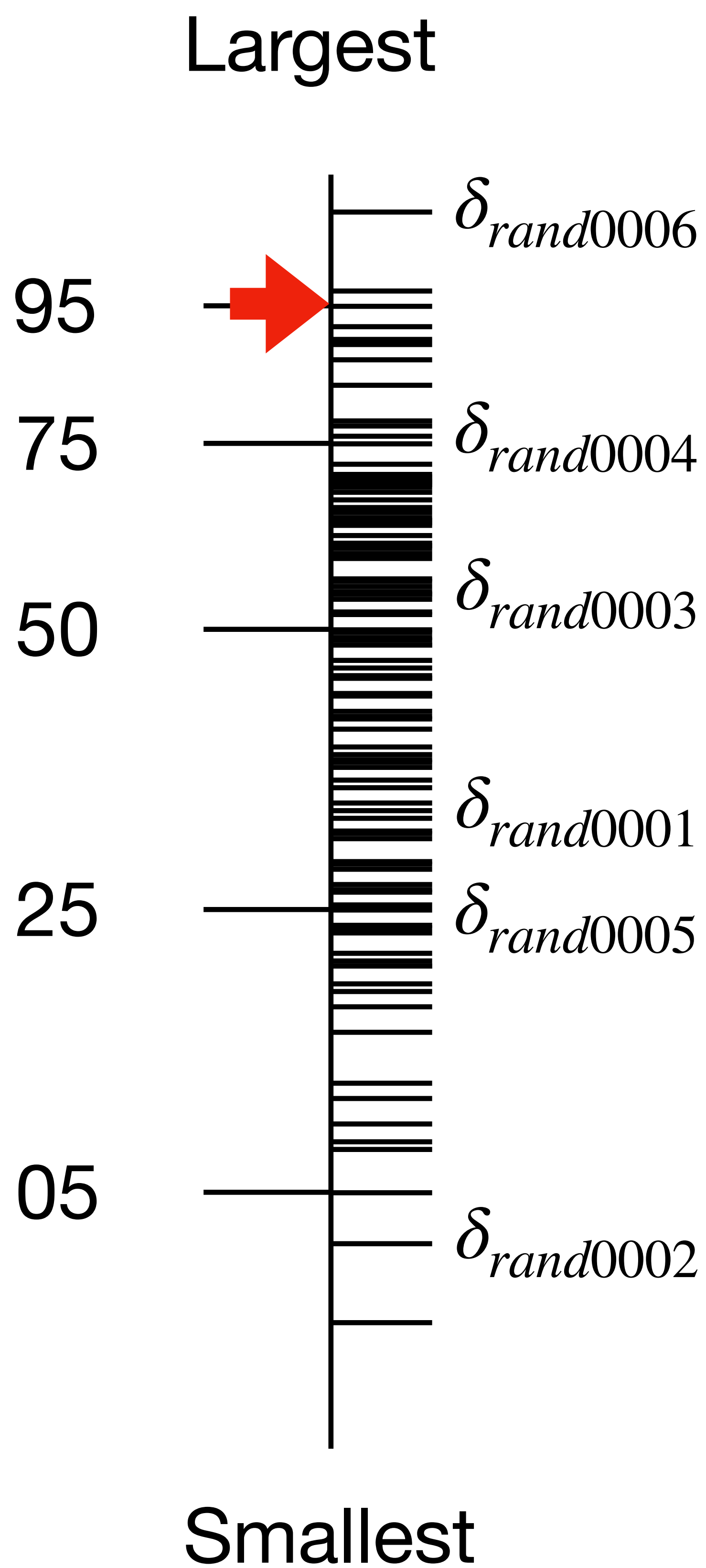

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Suppose I take sample 0004

I accept H0 that far up. And the groups do not in fact differ. Good!

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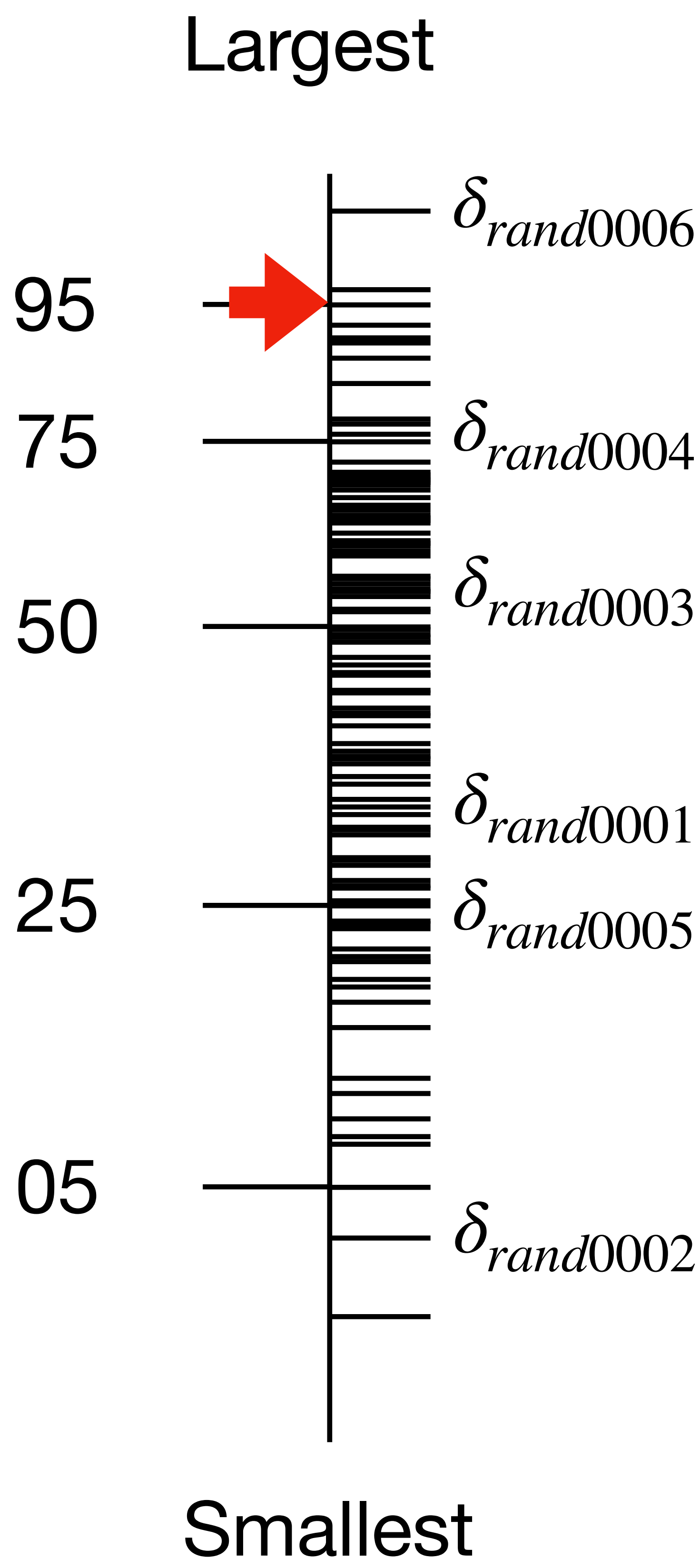

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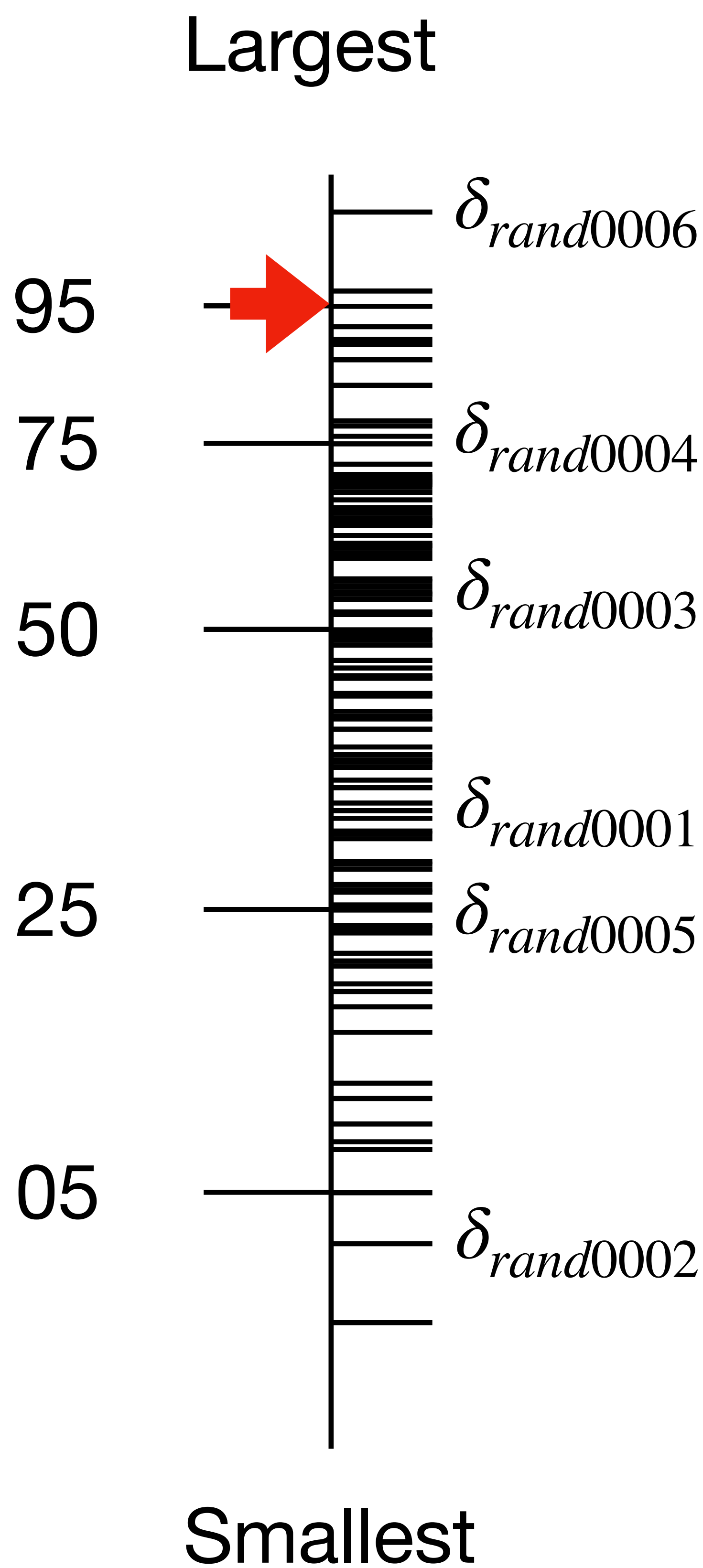

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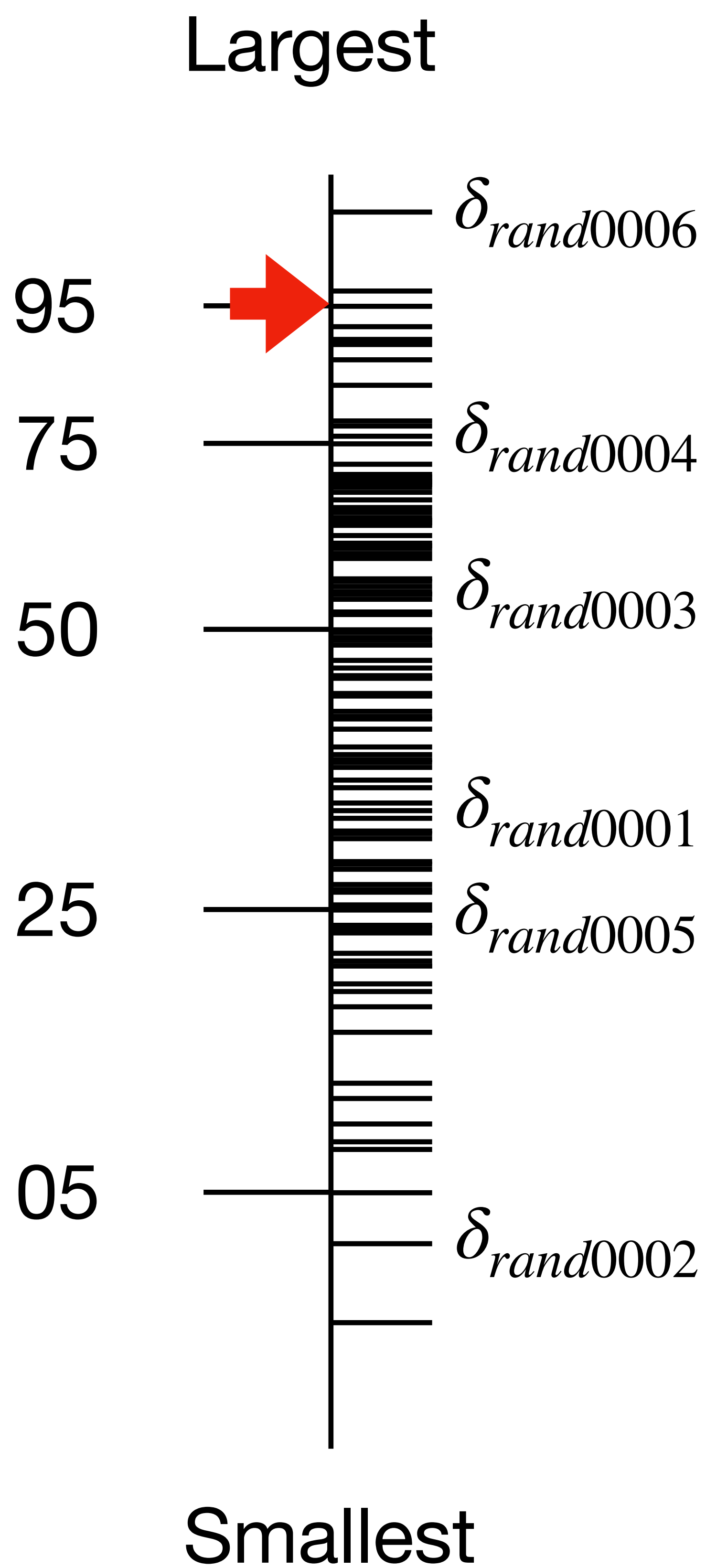

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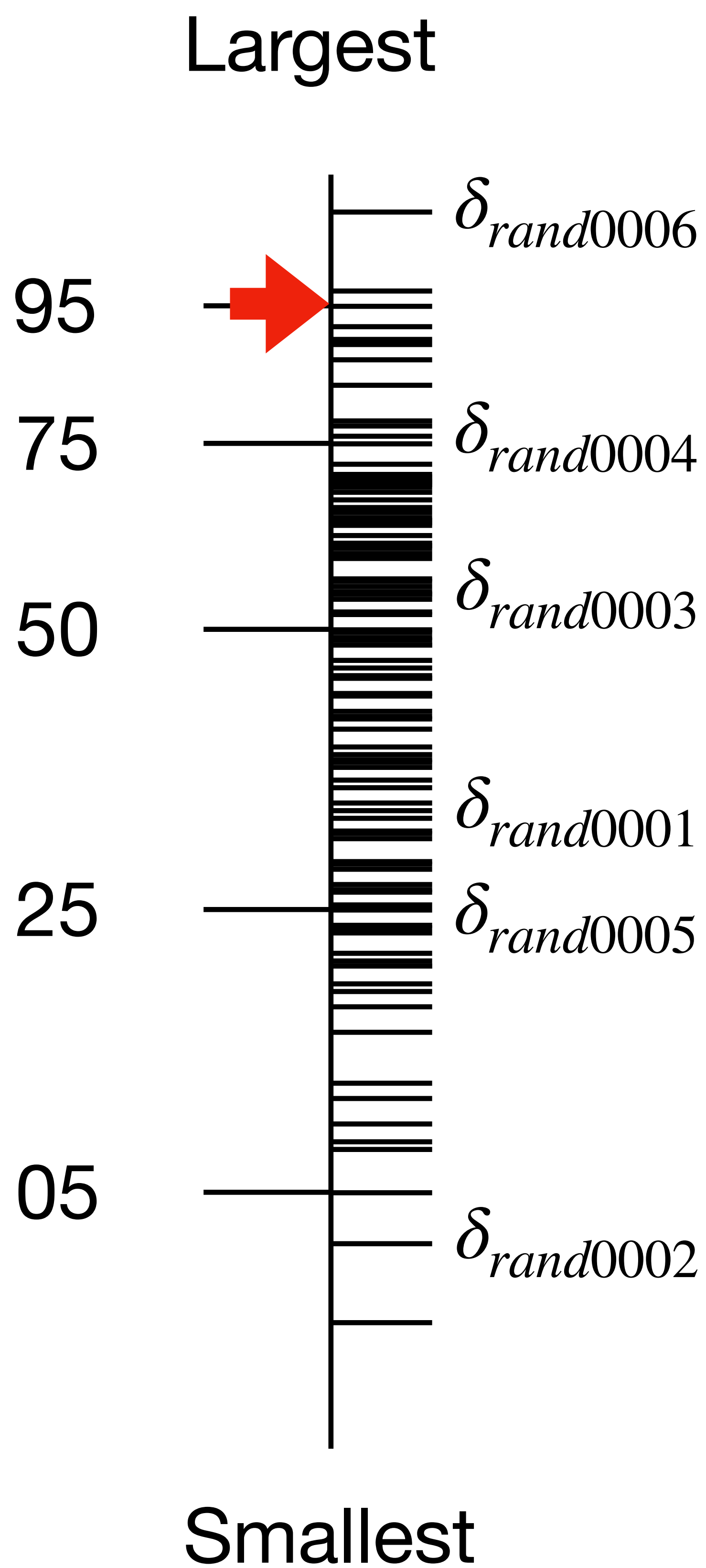

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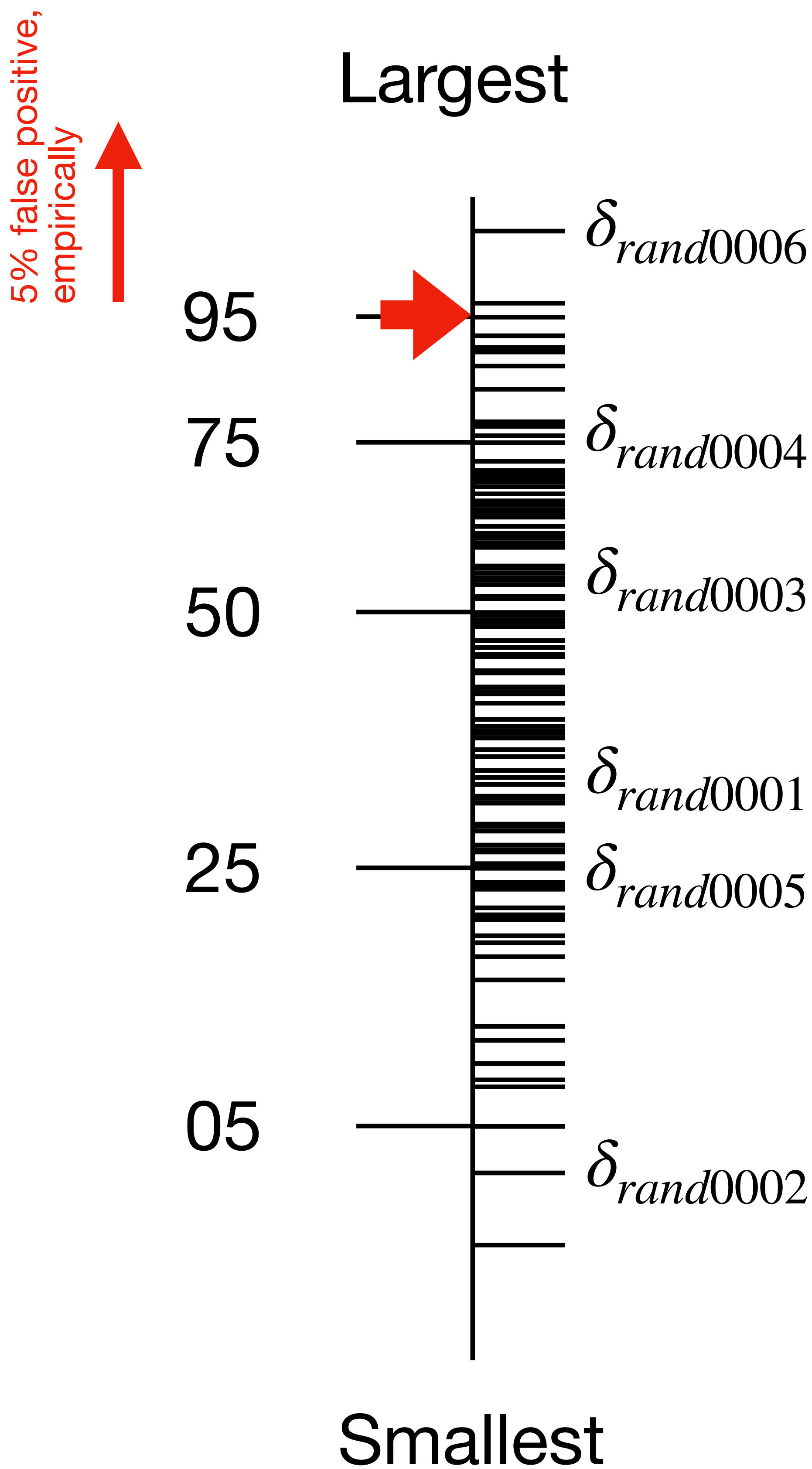
Suppose I take sample 0006

I reject H0 that far up. But by construction H0 is true! I say the groups differ but they do not! This is a false positive.

Example 1



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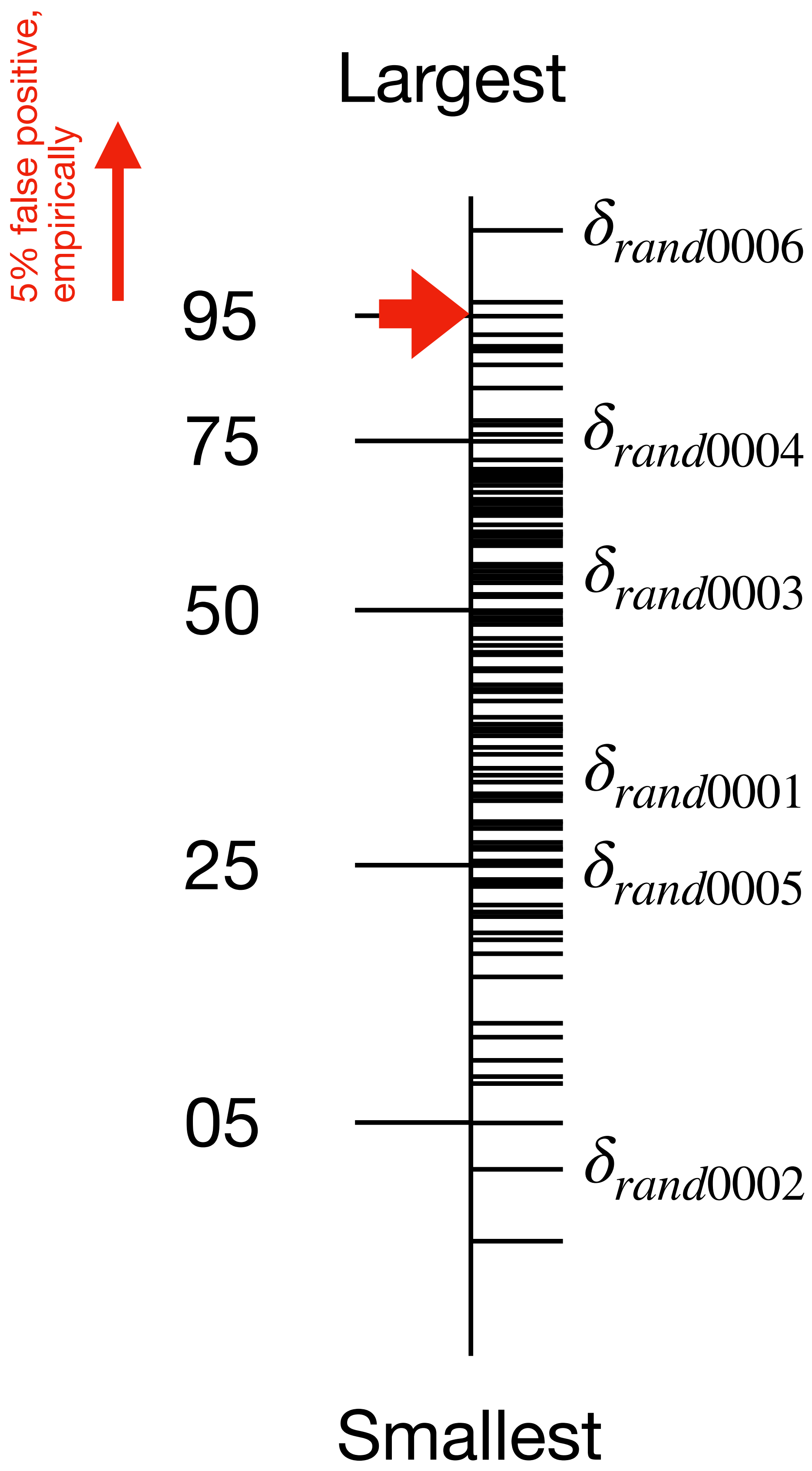

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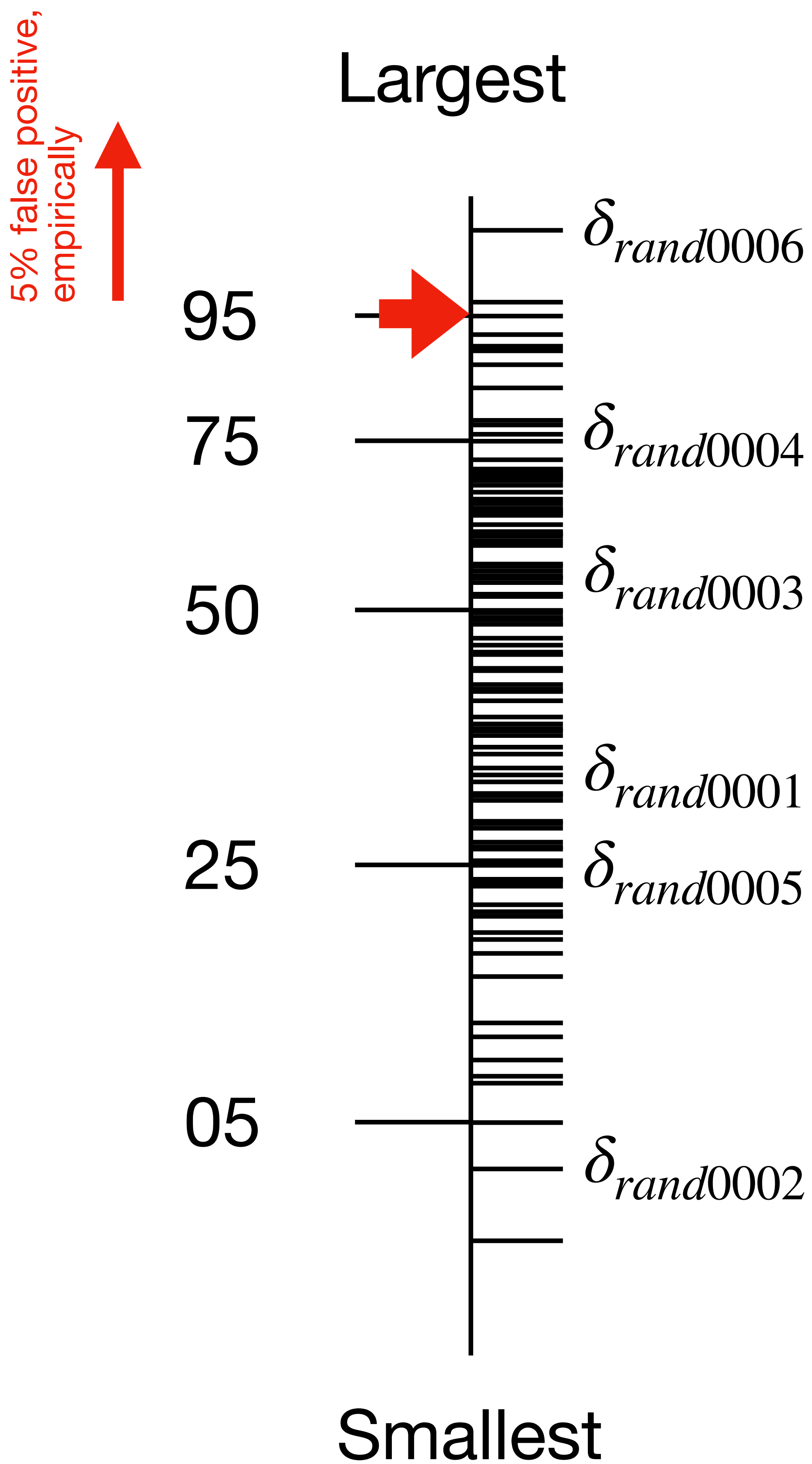

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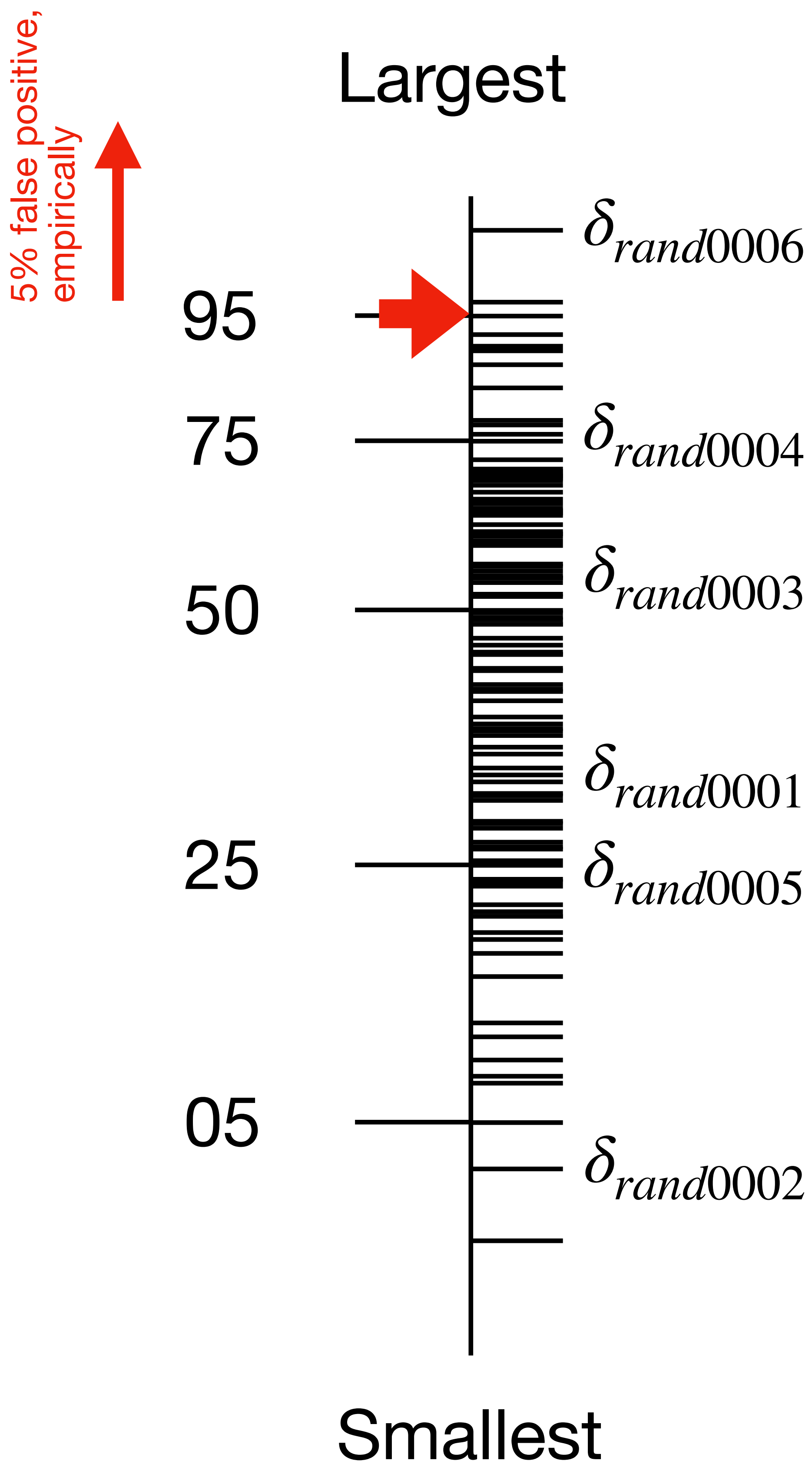

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We made H0 true!

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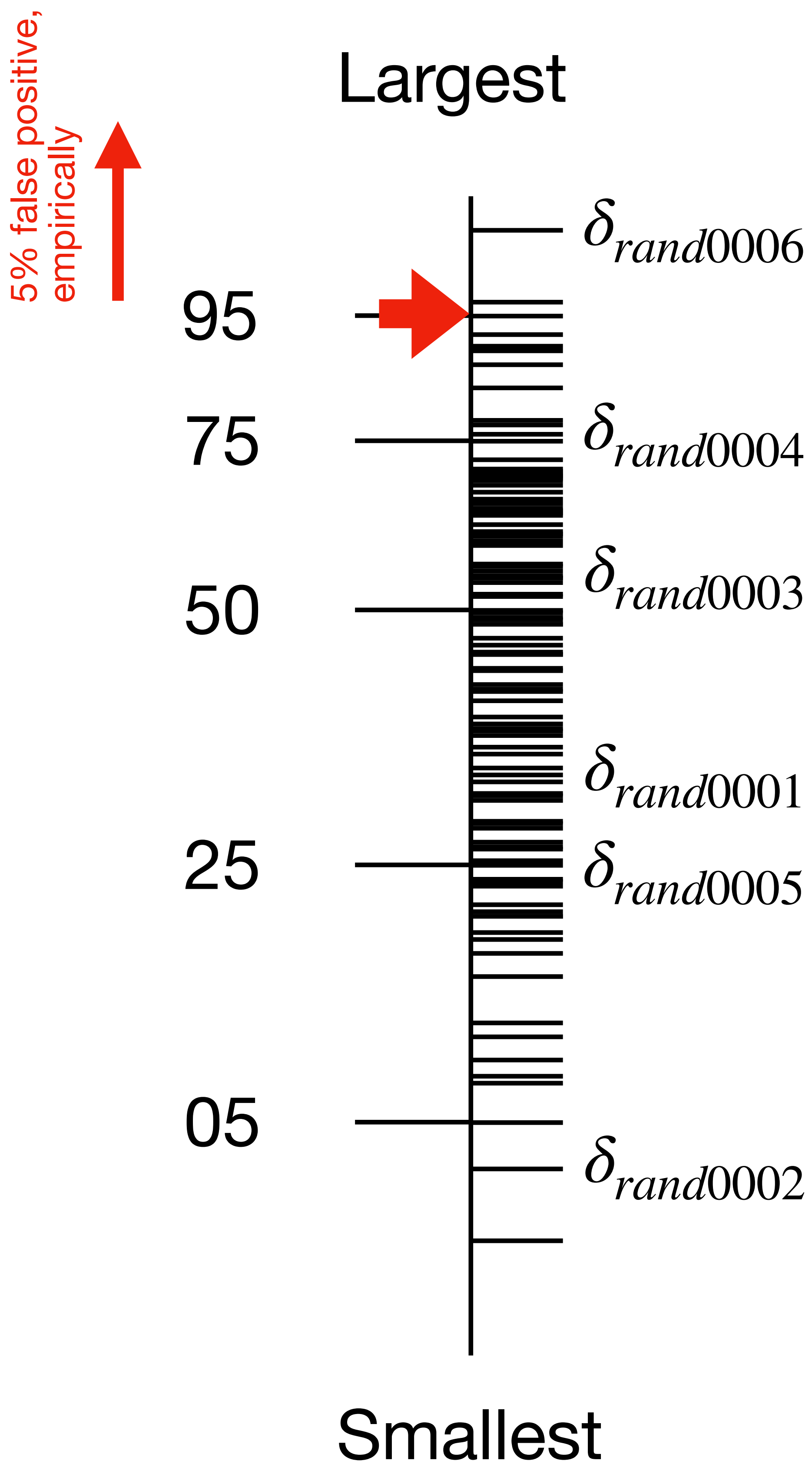

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What have we done here?

We made H0 true!

We did that by permuting labels

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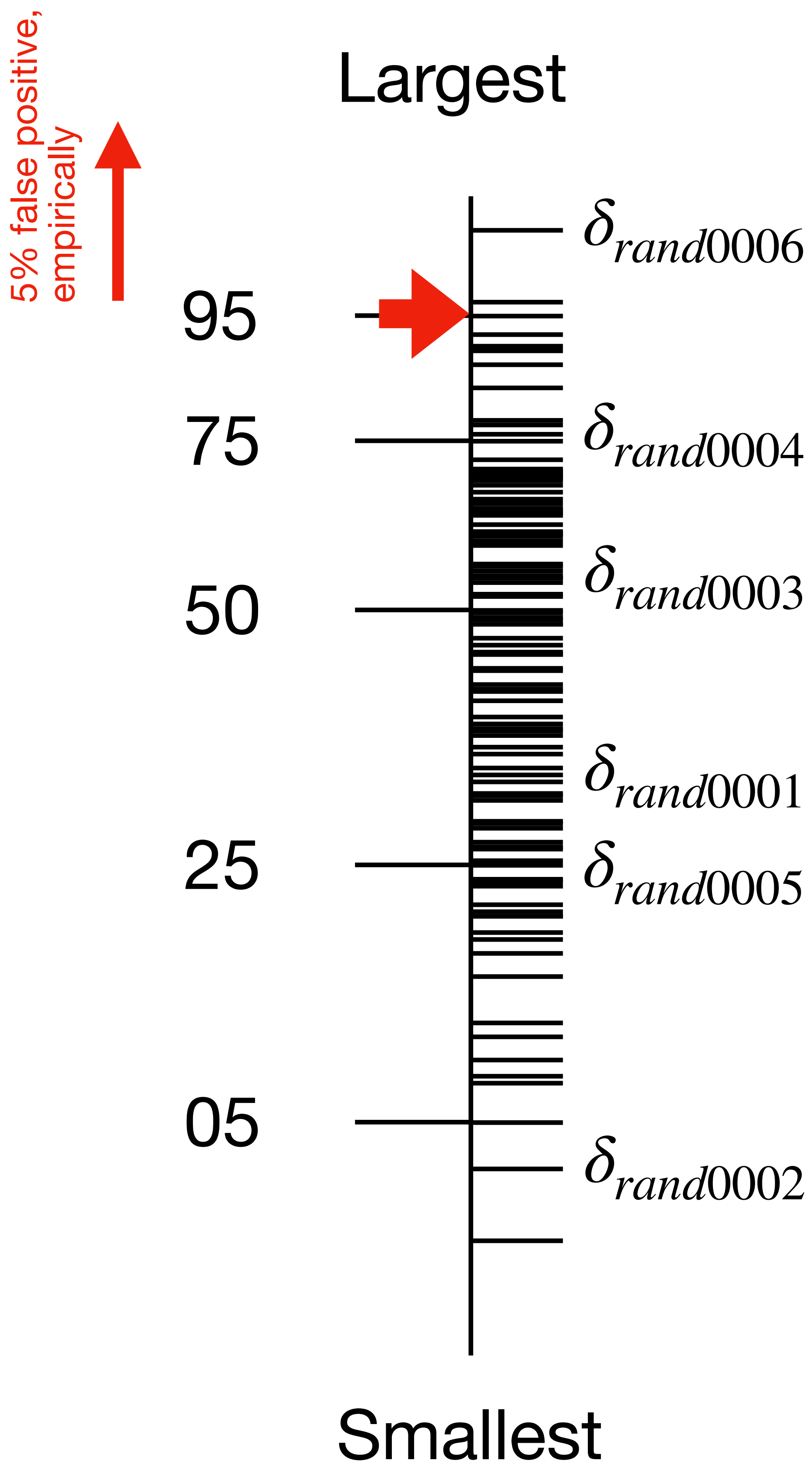

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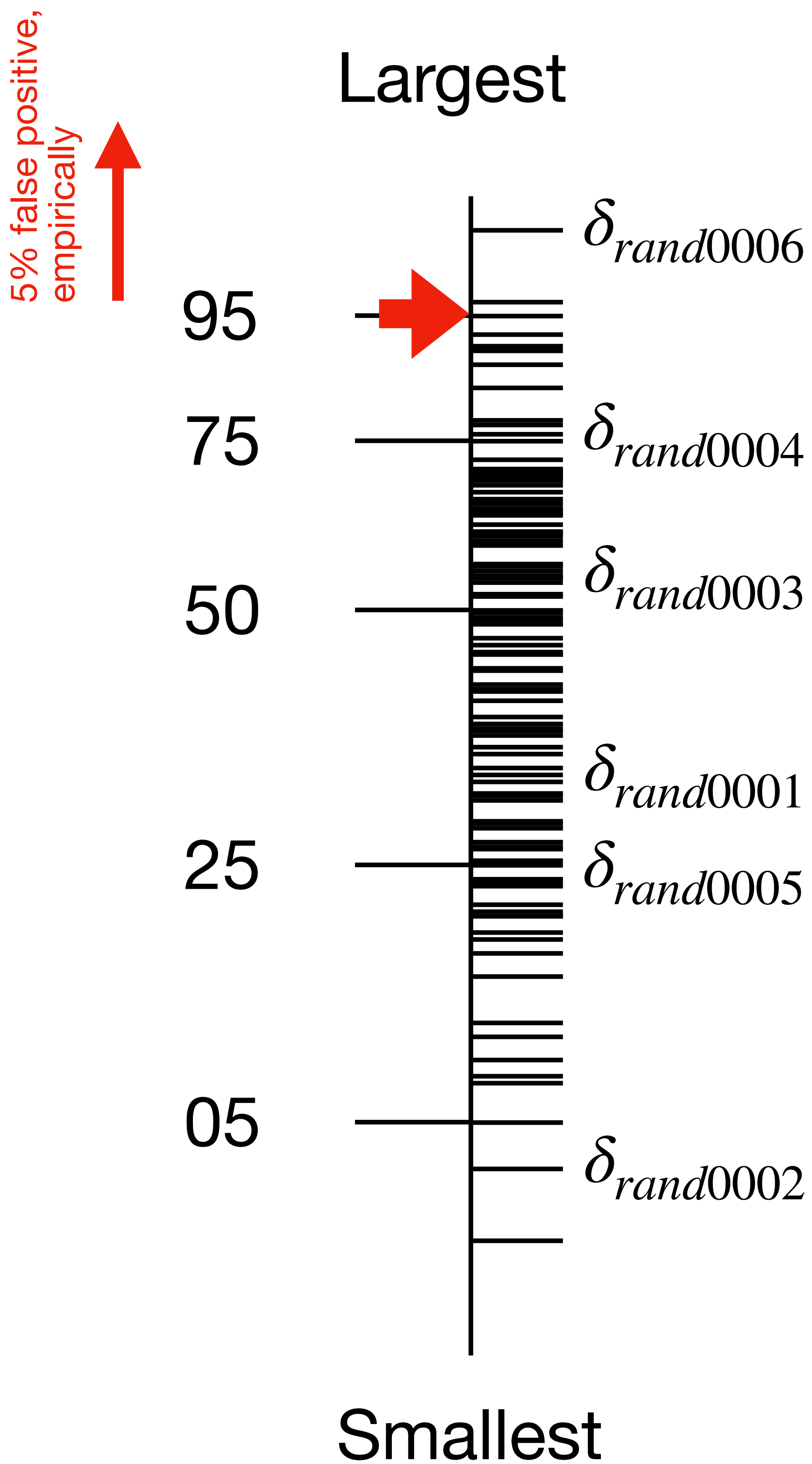

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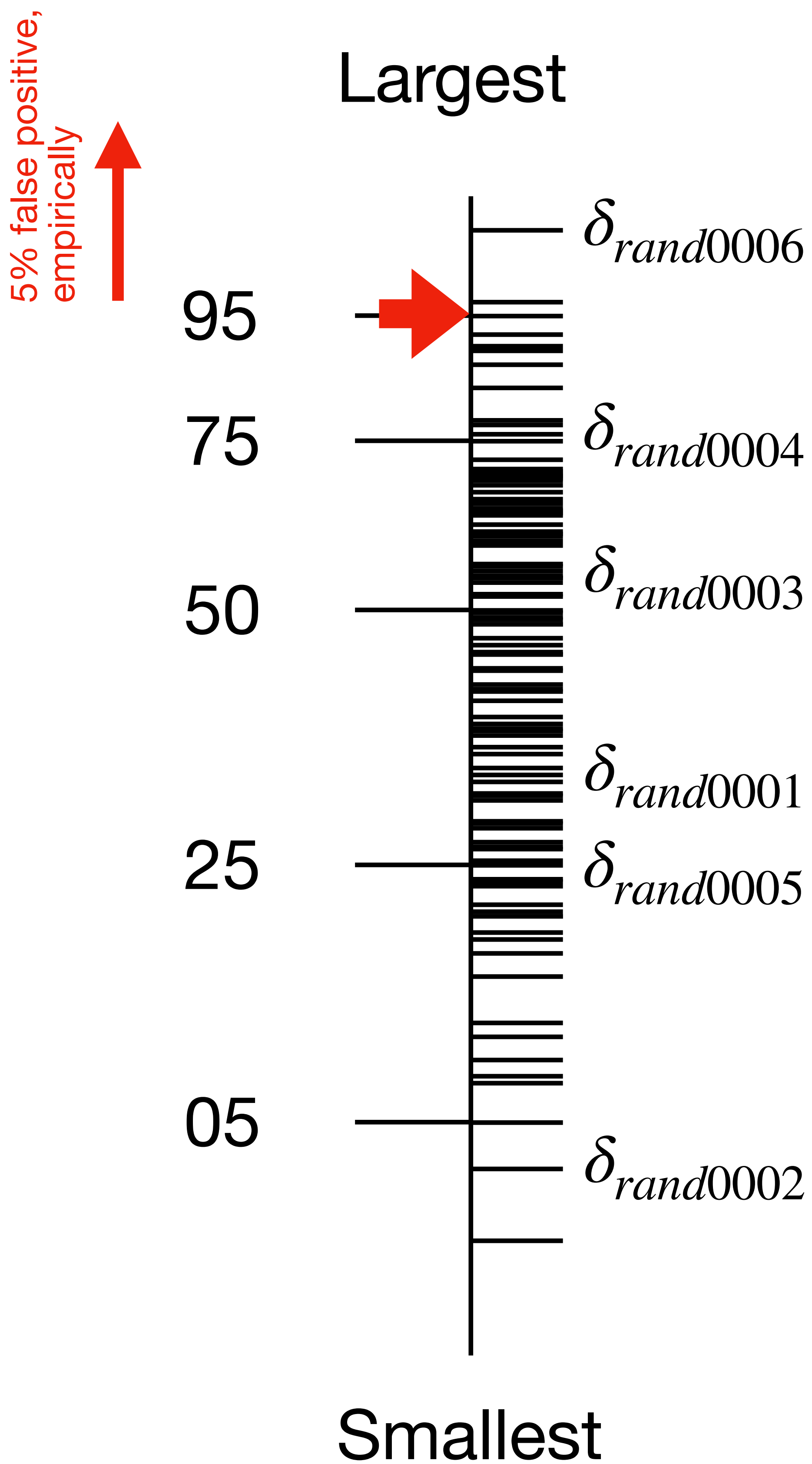

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So that we can usually reject true H0

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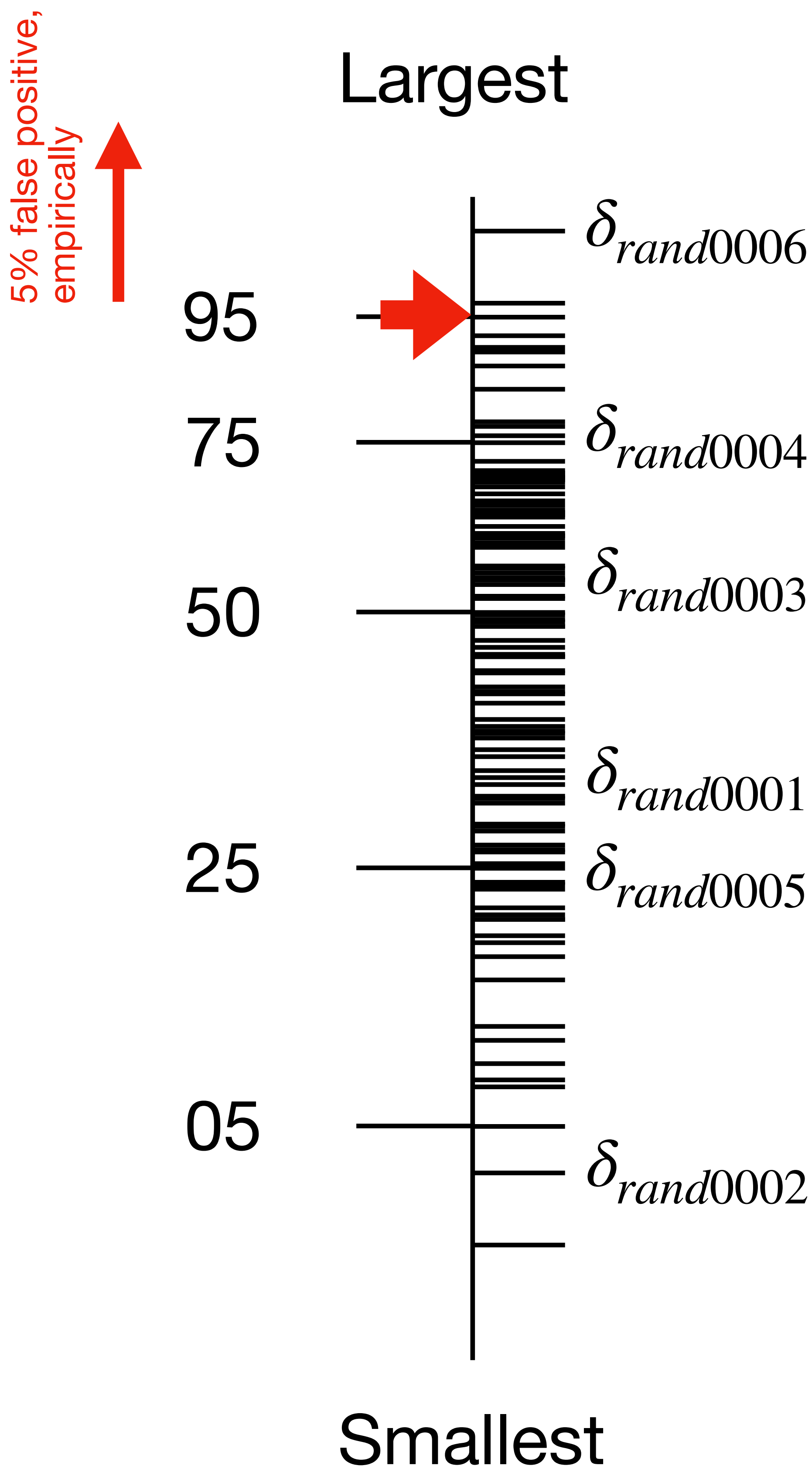

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Customarily at a 95% success rate

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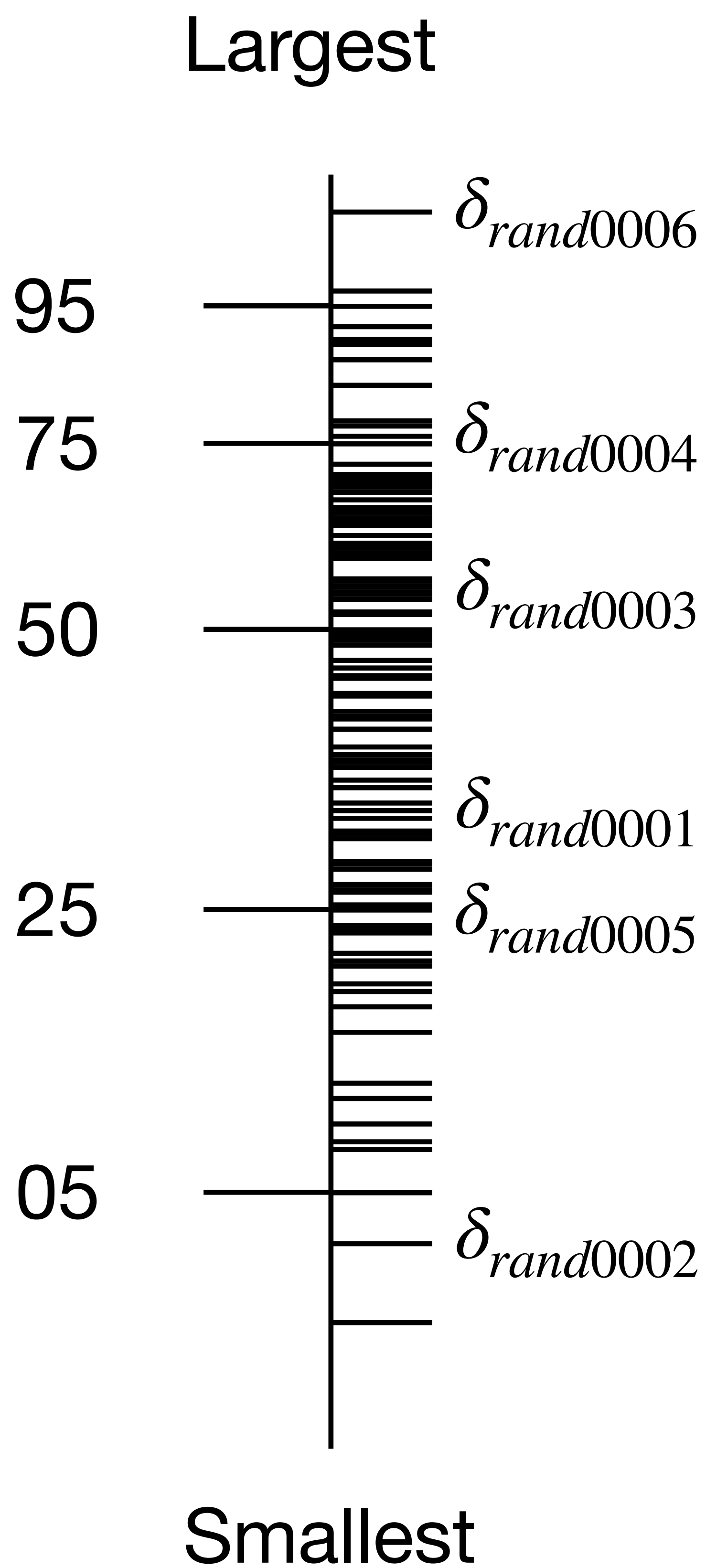

Largest
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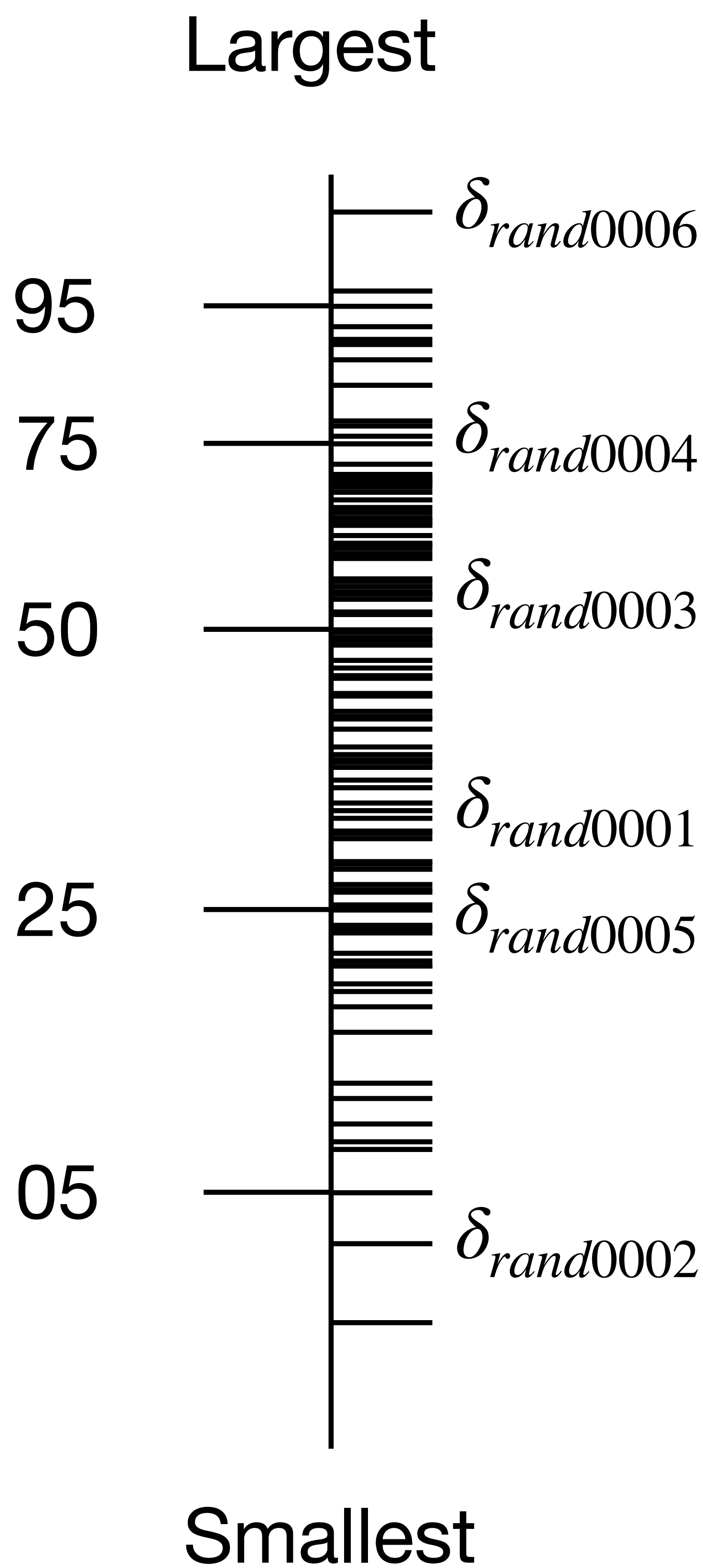

Now, where is the real delta?

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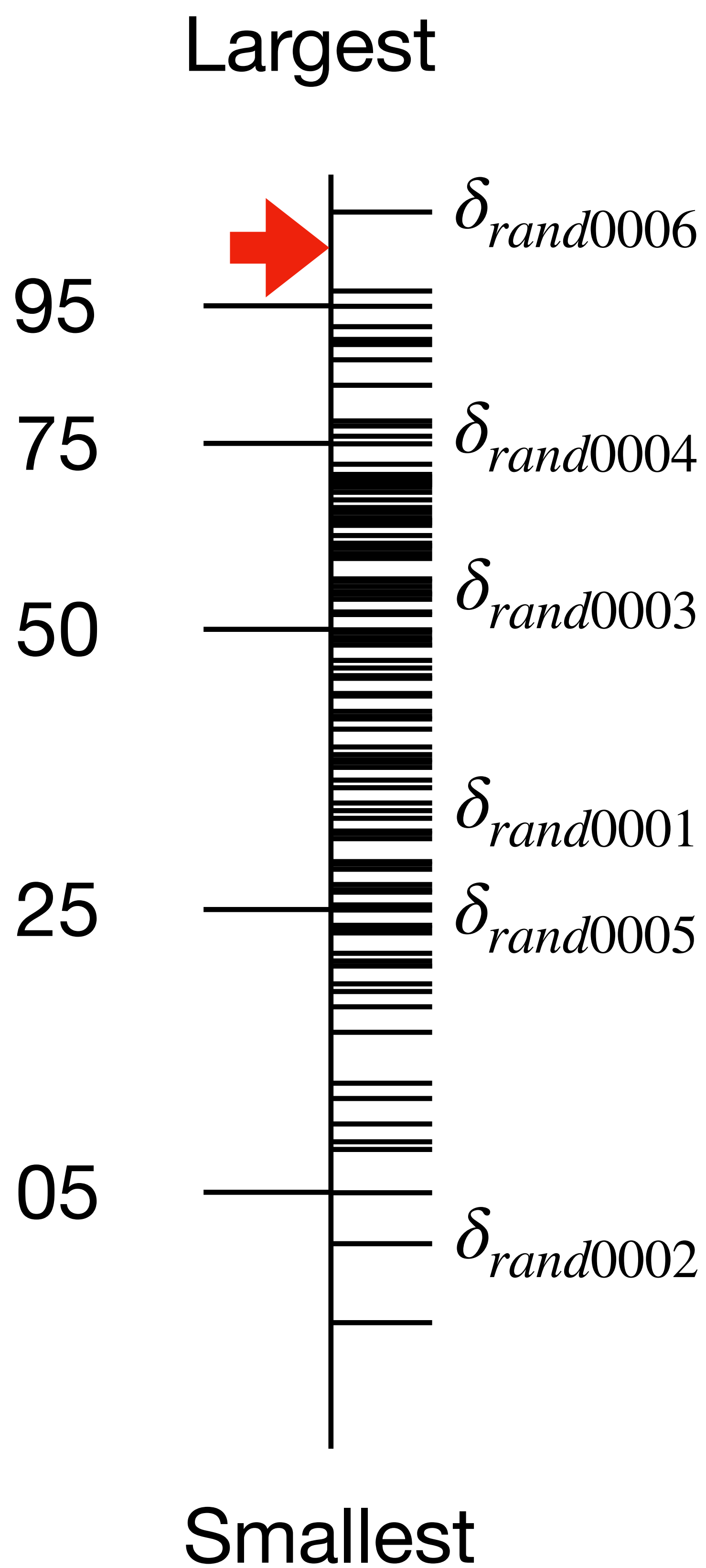

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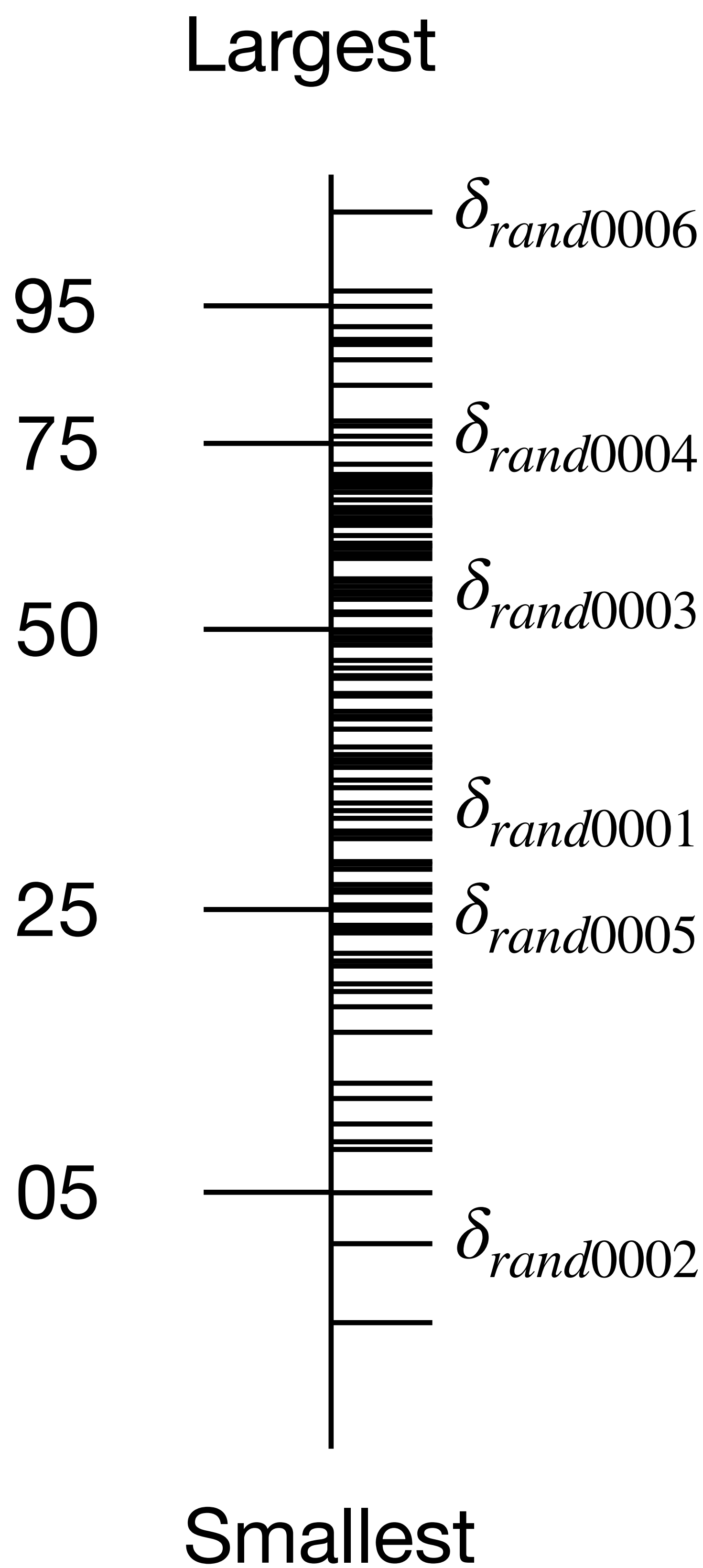

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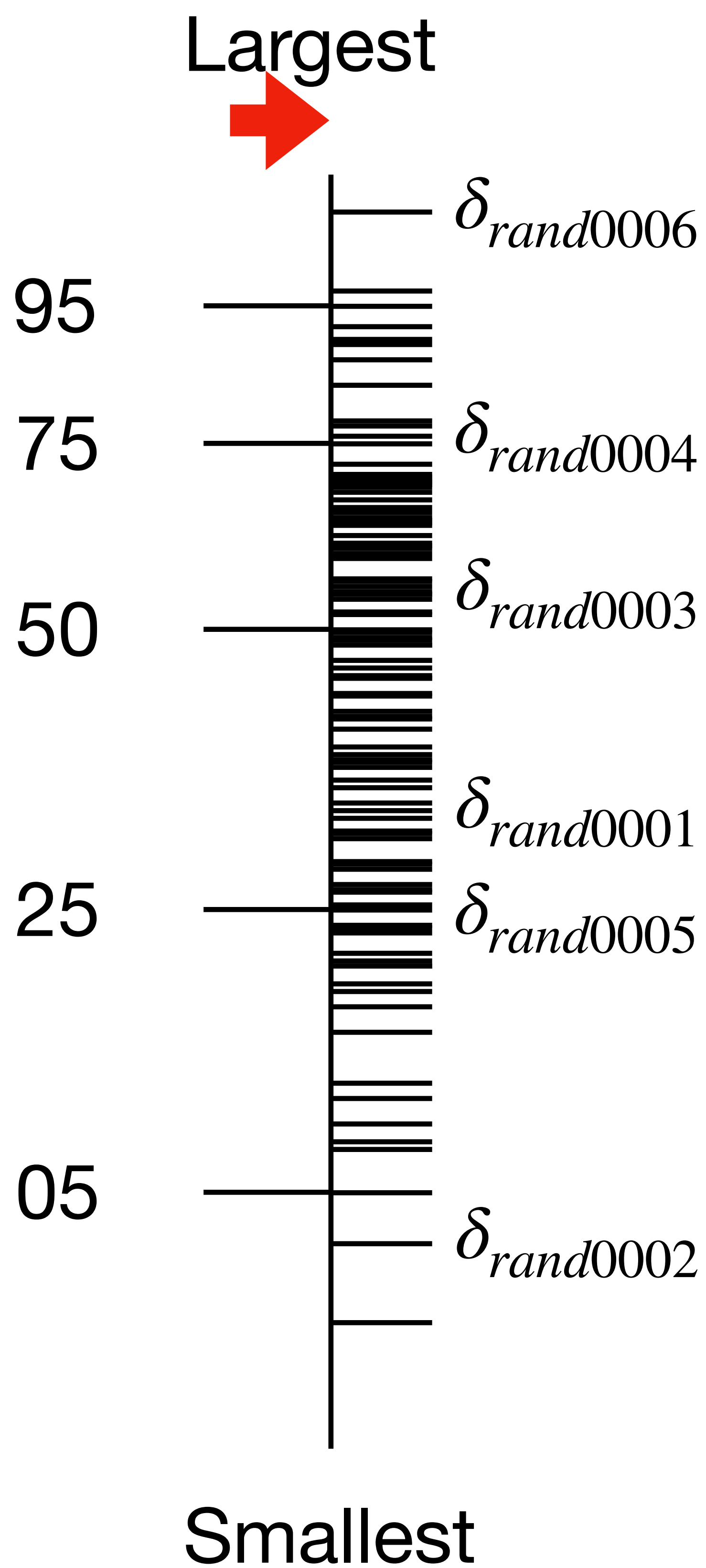

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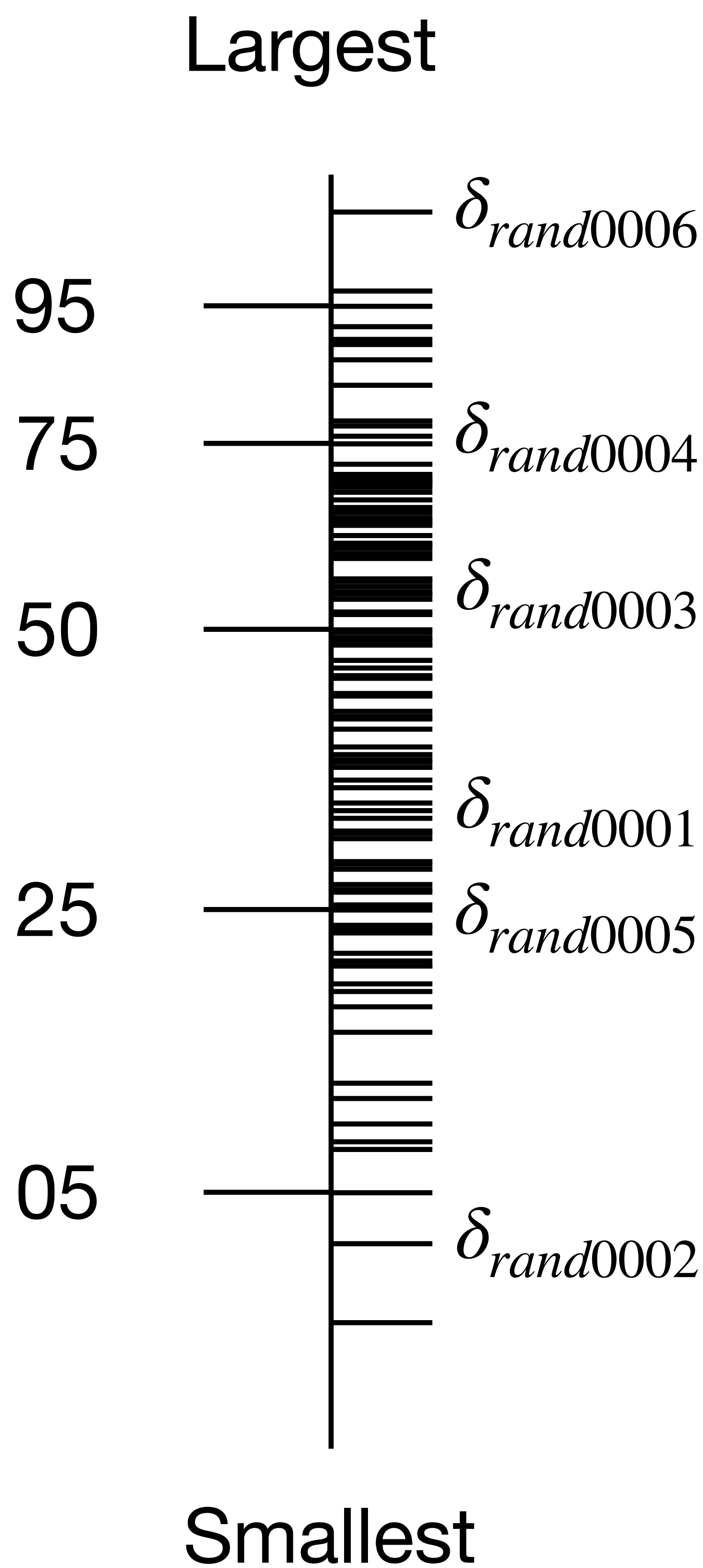

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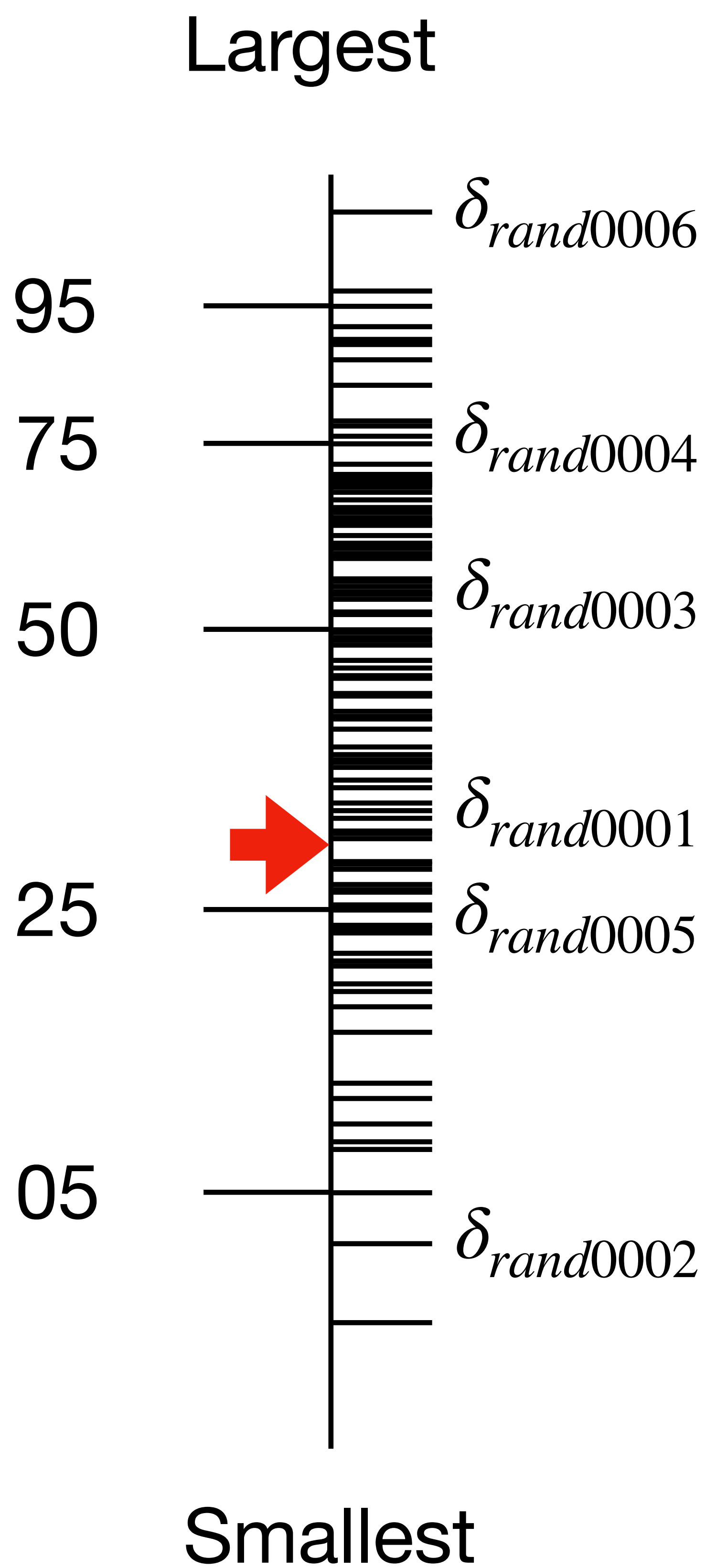

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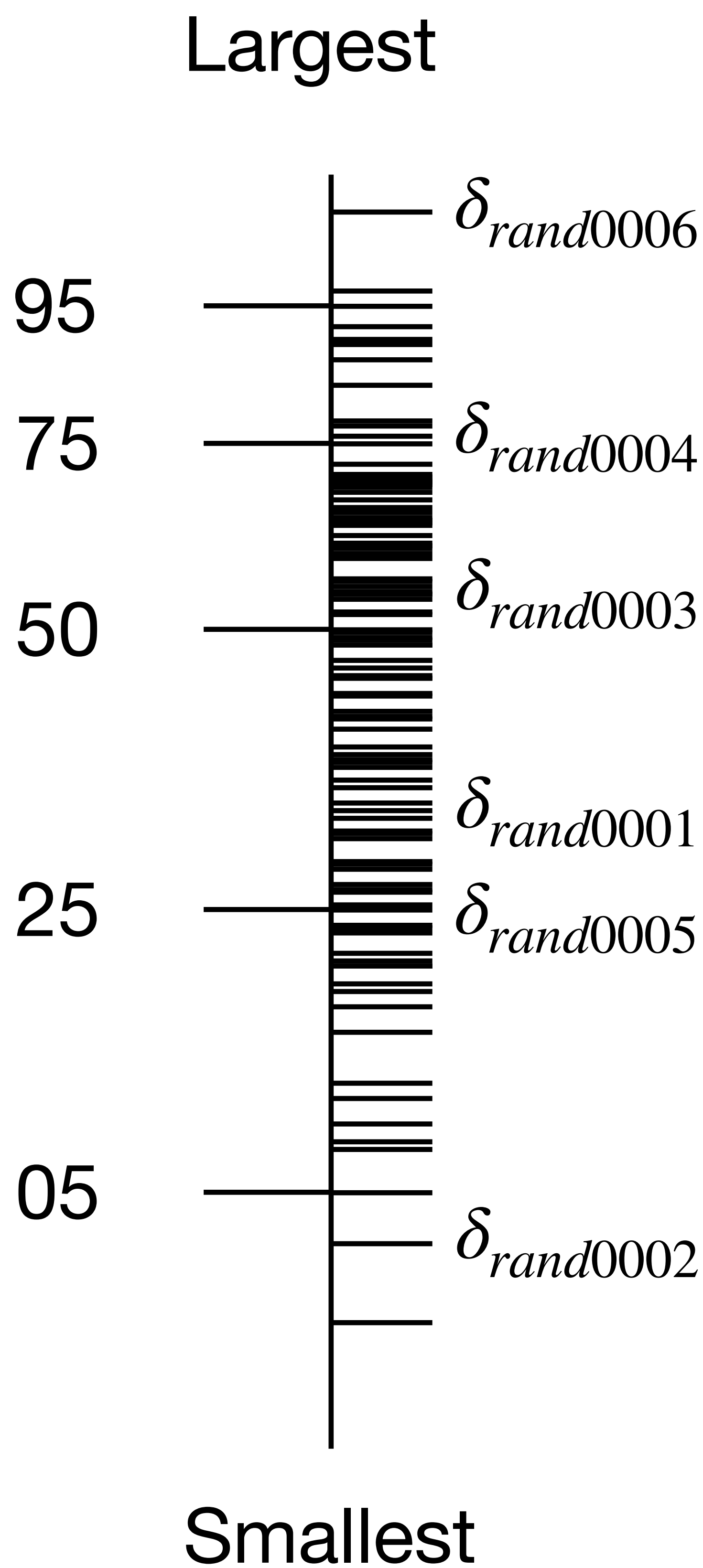

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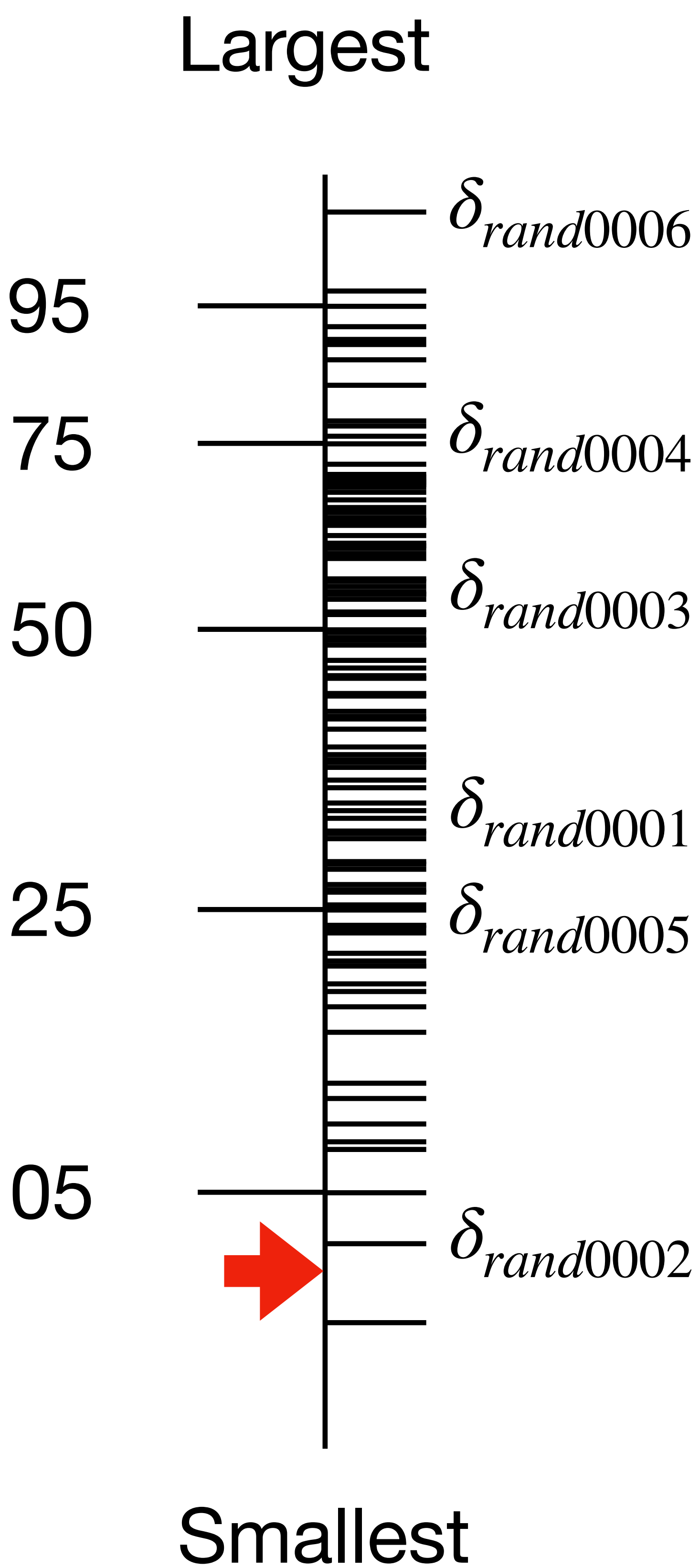

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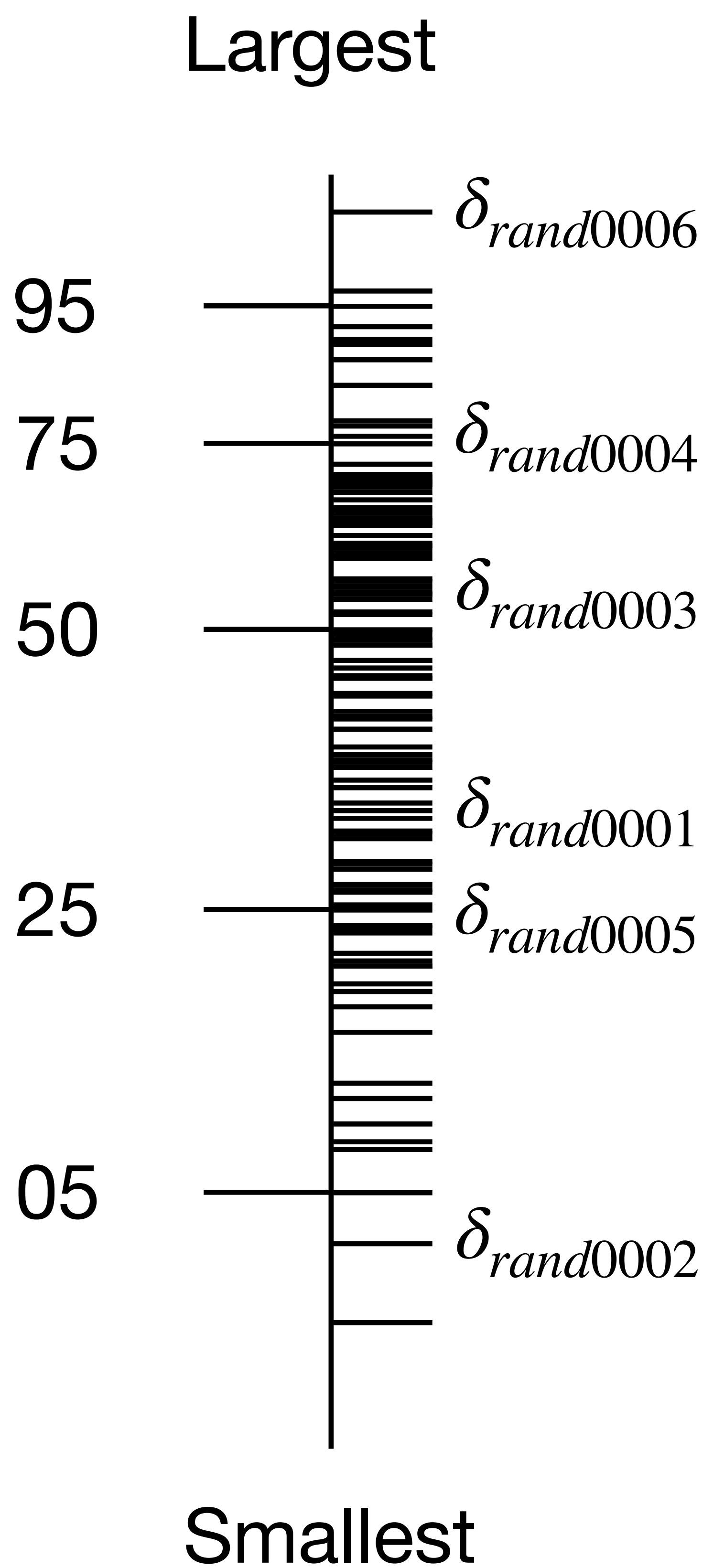

What ranks mean $p < 0.05$?

$$\delta_{real} = \mu_1 - \mu_0$$

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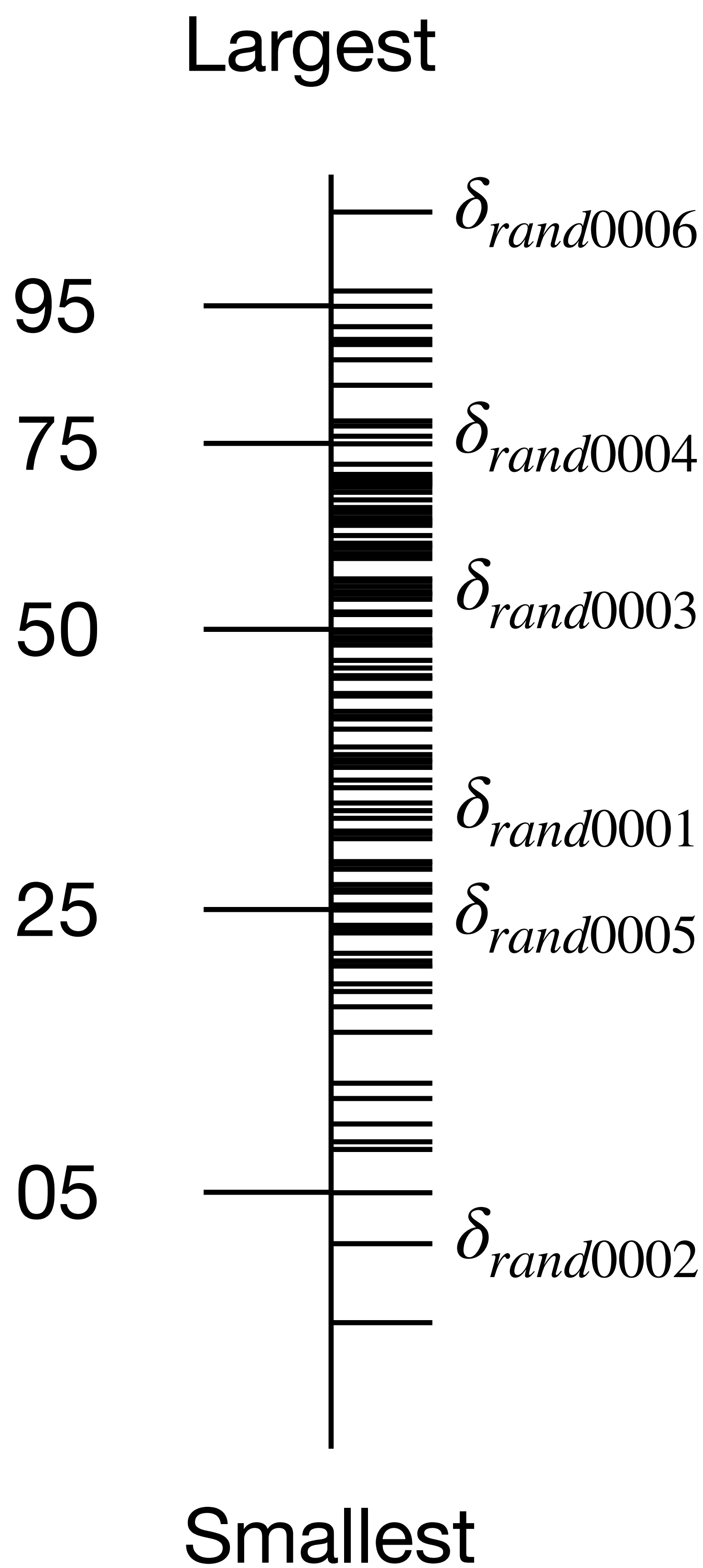

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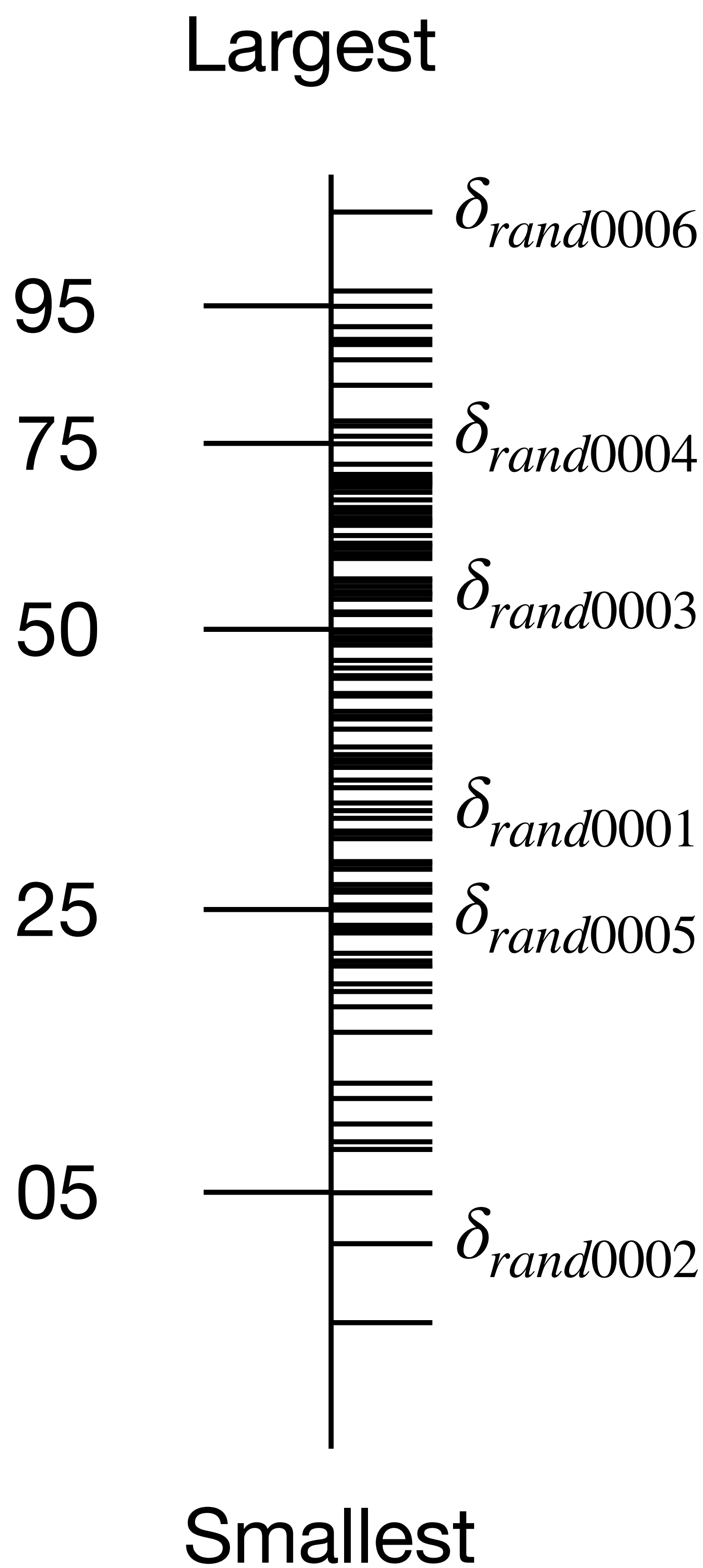

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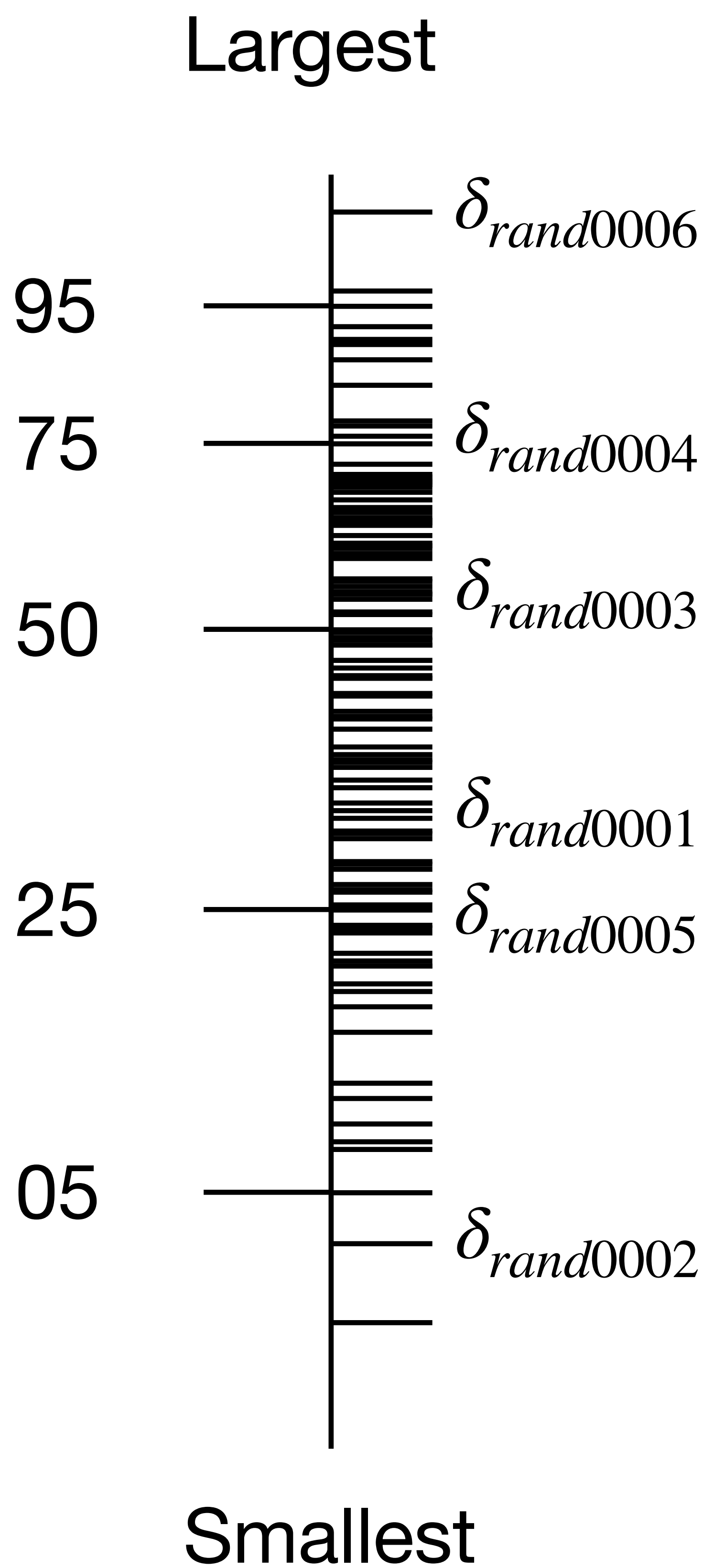

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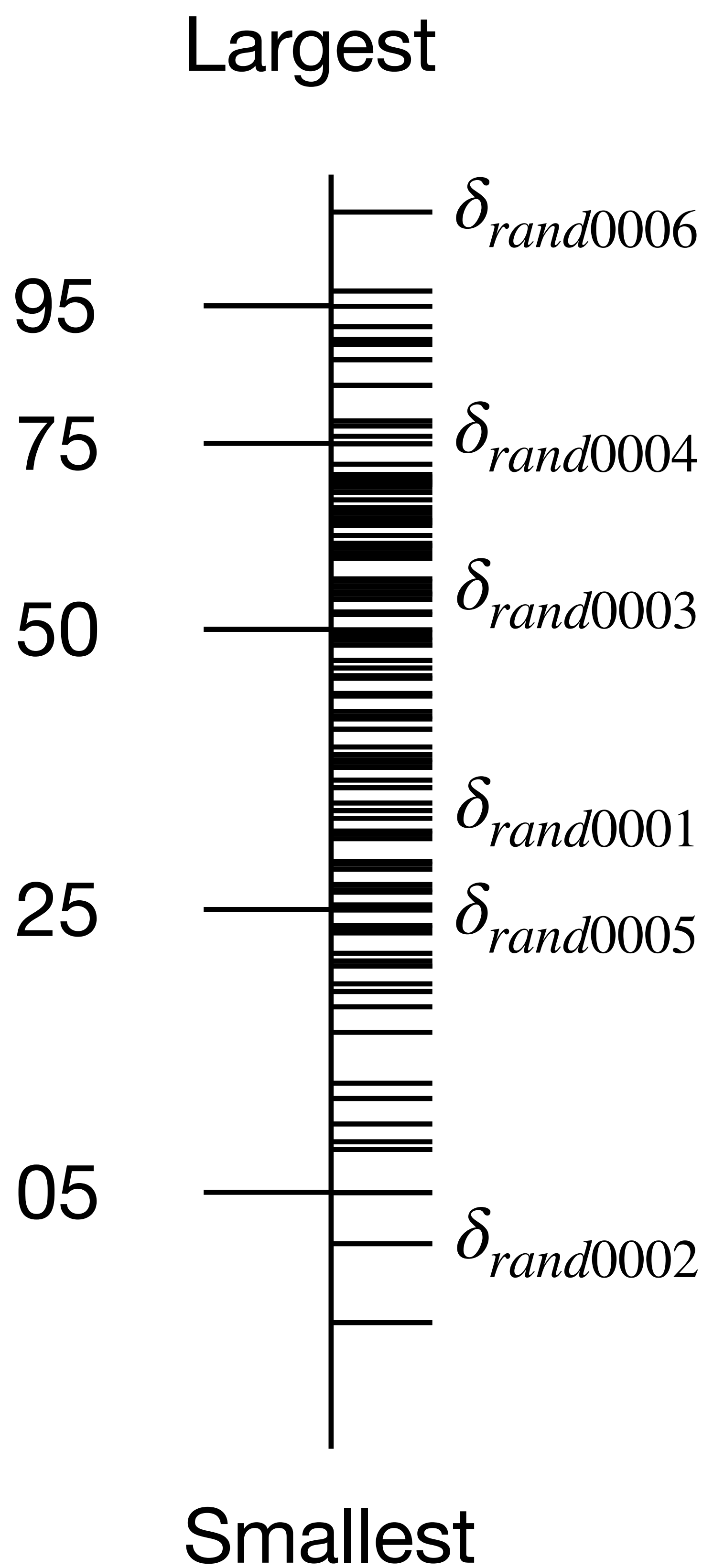

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If we believe delta is negative

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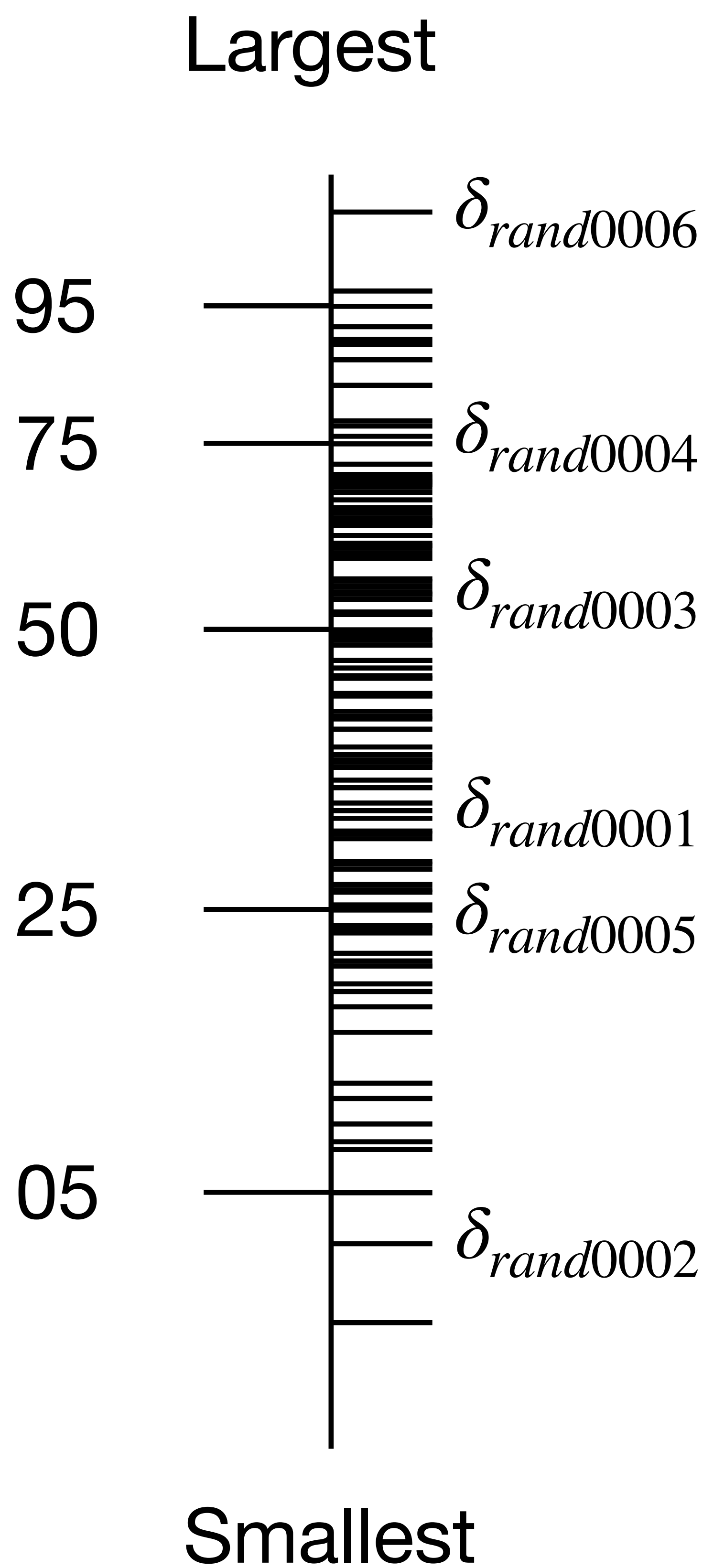

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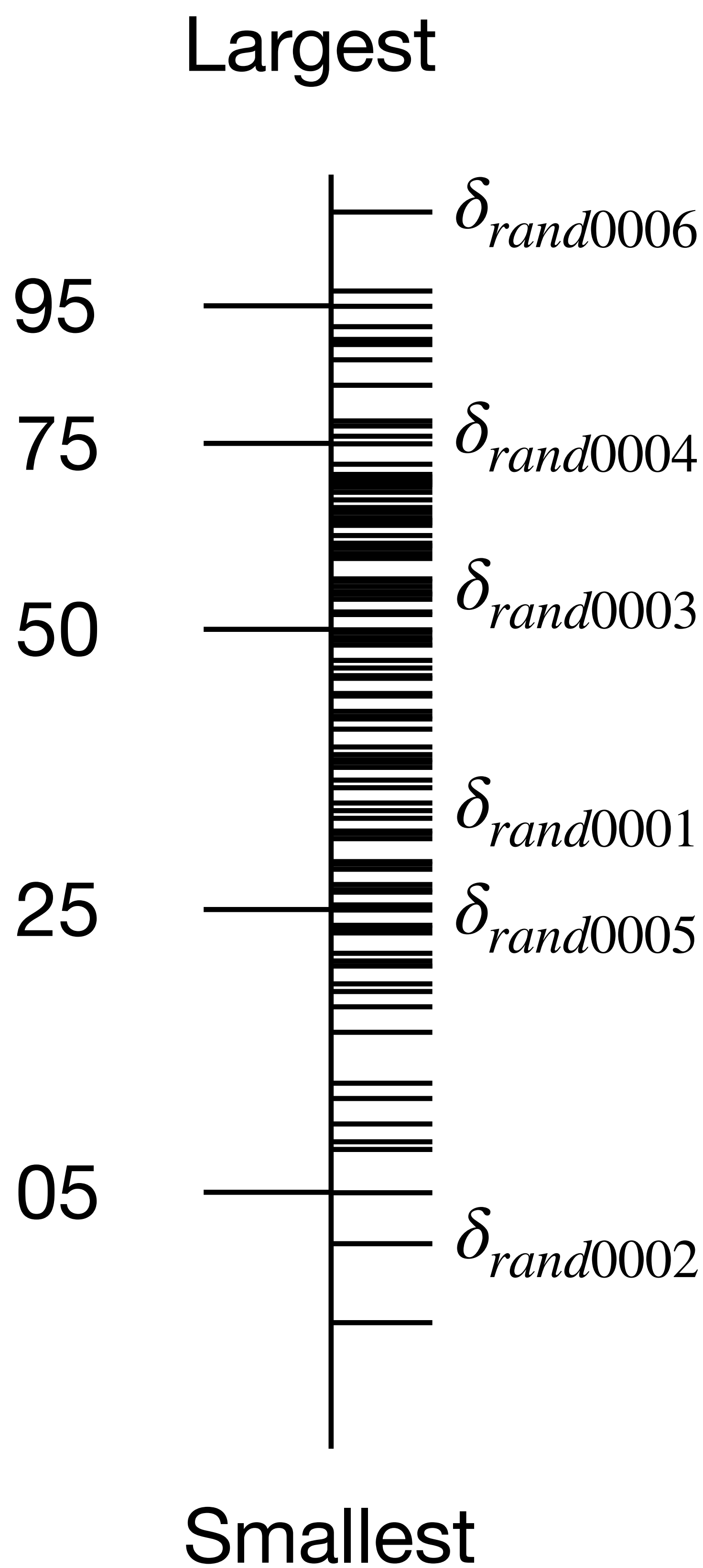

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If delta might be either

>97.5% OR <2.5%

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2^8=256

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You need a credible number of permutations, 10K is customary

For binary labels, 2^(n-1) are possible permutation groups

- 2^n total groups, but each with 2 versions (e.g., 0001, 1110)

- 2^8=256 2^10=1024

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Recipe:

1) Identify the relationship that matters.

2) Destroy ONLY that relationship, by permutation.

Example 2

You have subjects with measures in 2 conditions. Do they differ?

	Cond A	**Cond B**
	34	35
	57	57
	36	45
	97	87
	46	4
	33	23
	75	43
	45	34

$$\delta_{real} = \mu_{A-B}$$

Example 2

You have subjects with measures in 2 conditions. Do they differ?

Where A?	Cond A	Cond B
L	34	35
L	57	57
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Example 3

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    - E.g., 125 samples become 60 in a 5 min scan. Ouch!

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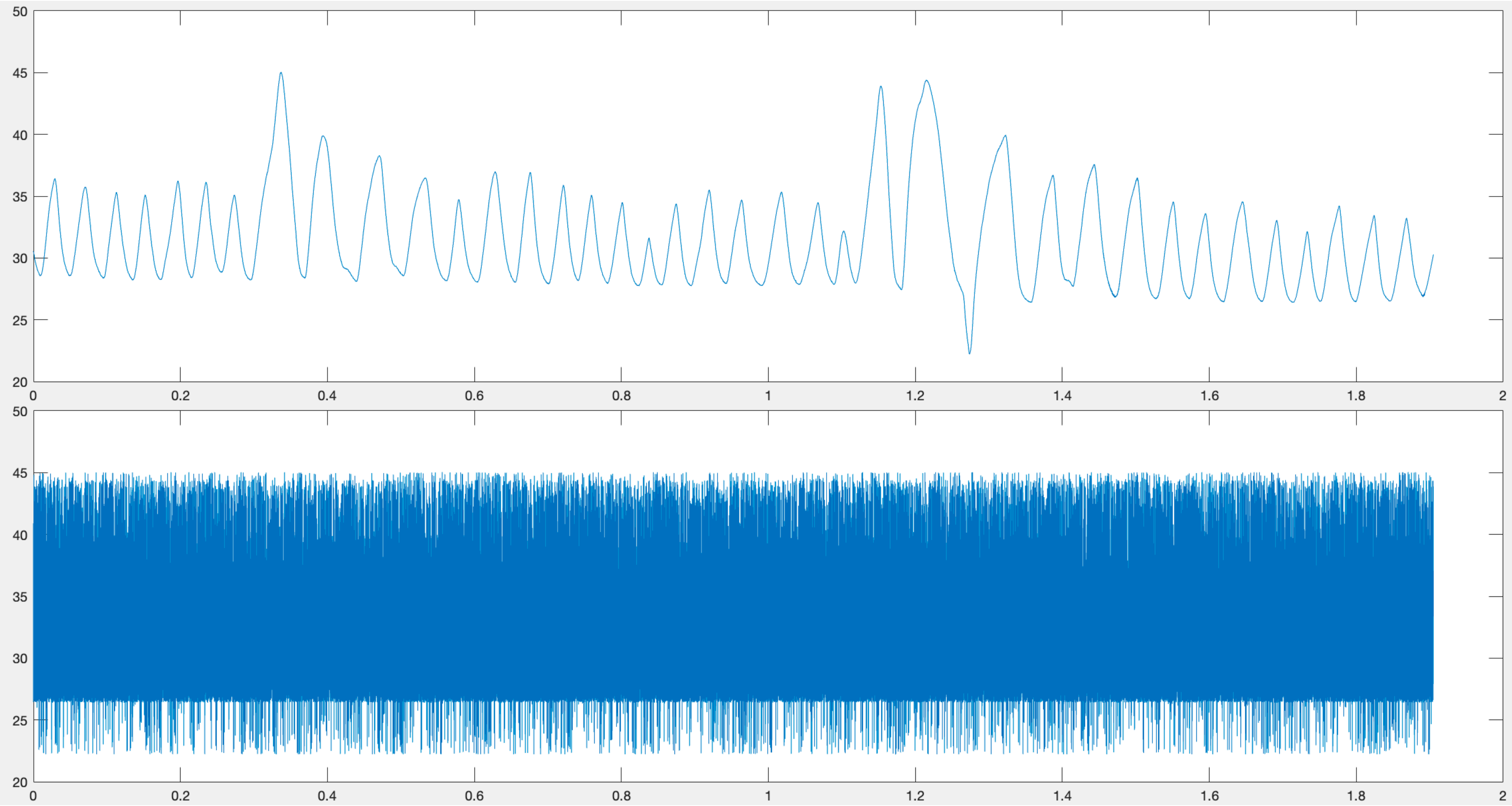
Autocorrelated timeseries



Permute timepoints

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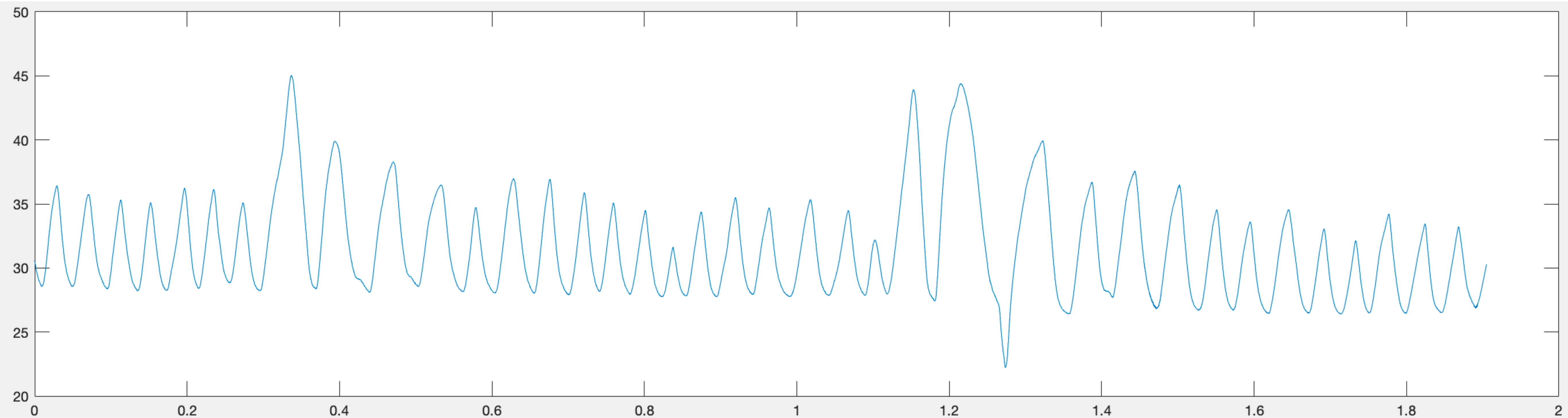

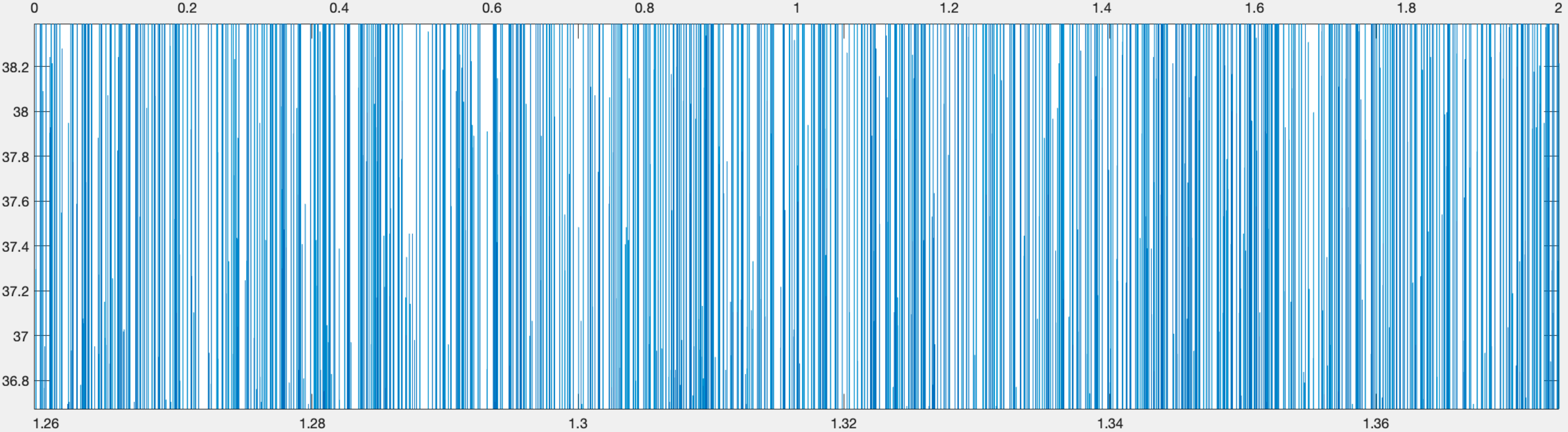

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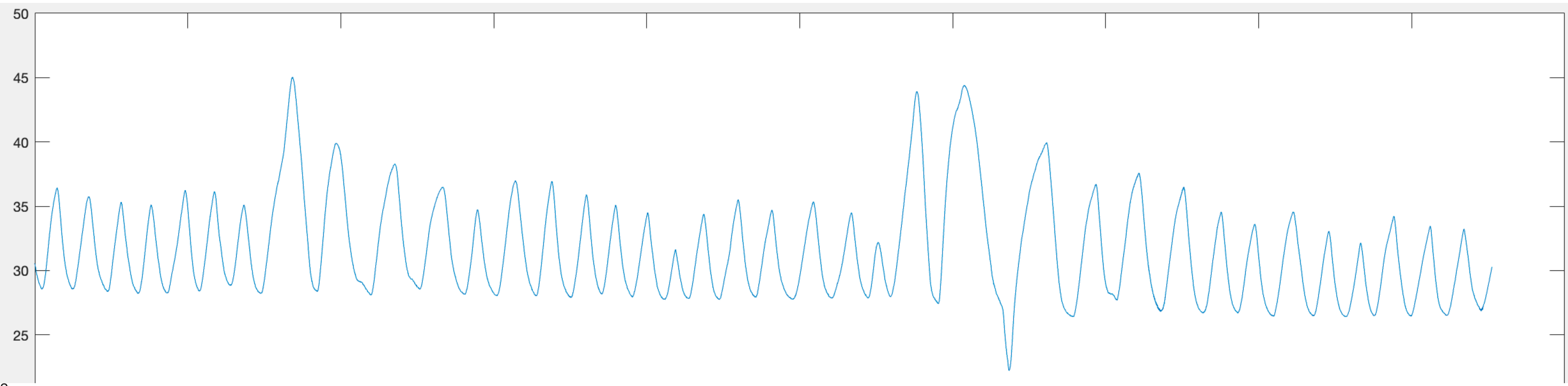

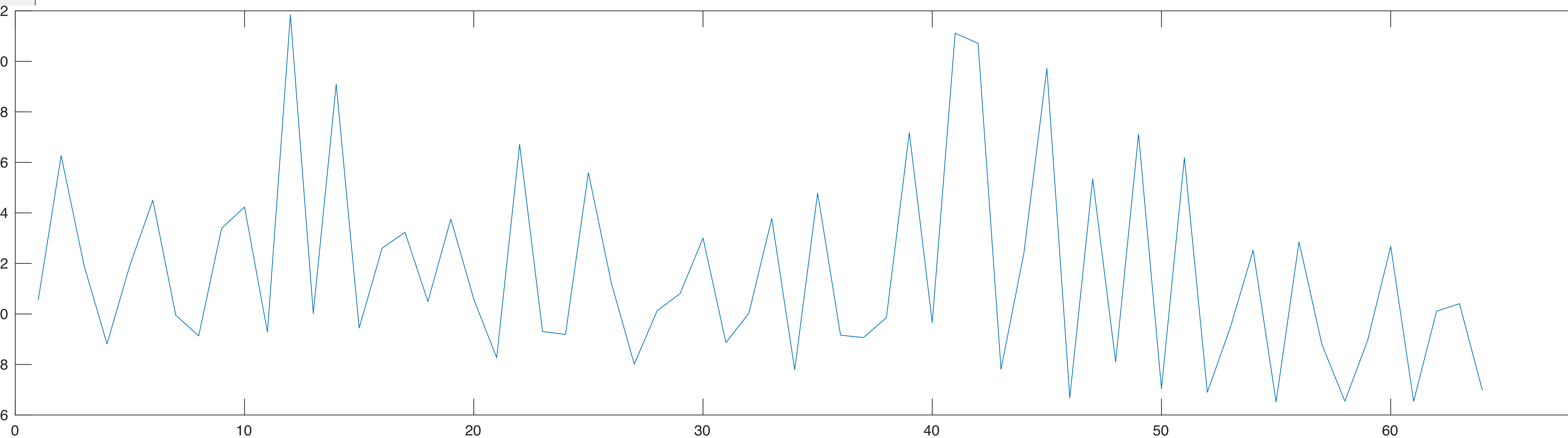

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  - Spatial registration, brain folding, global signals, etc etc etc

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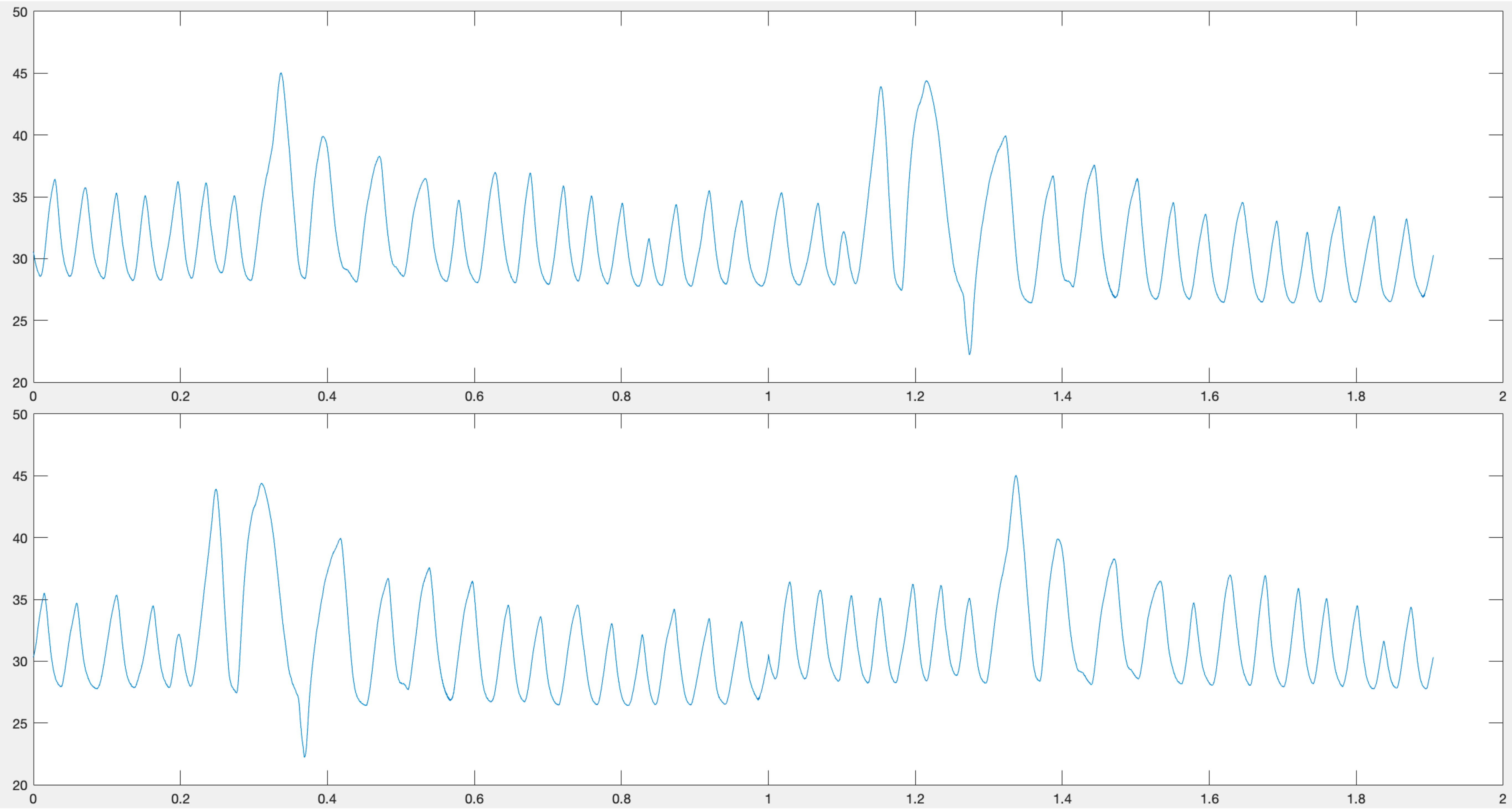

Cyclic shift randomly

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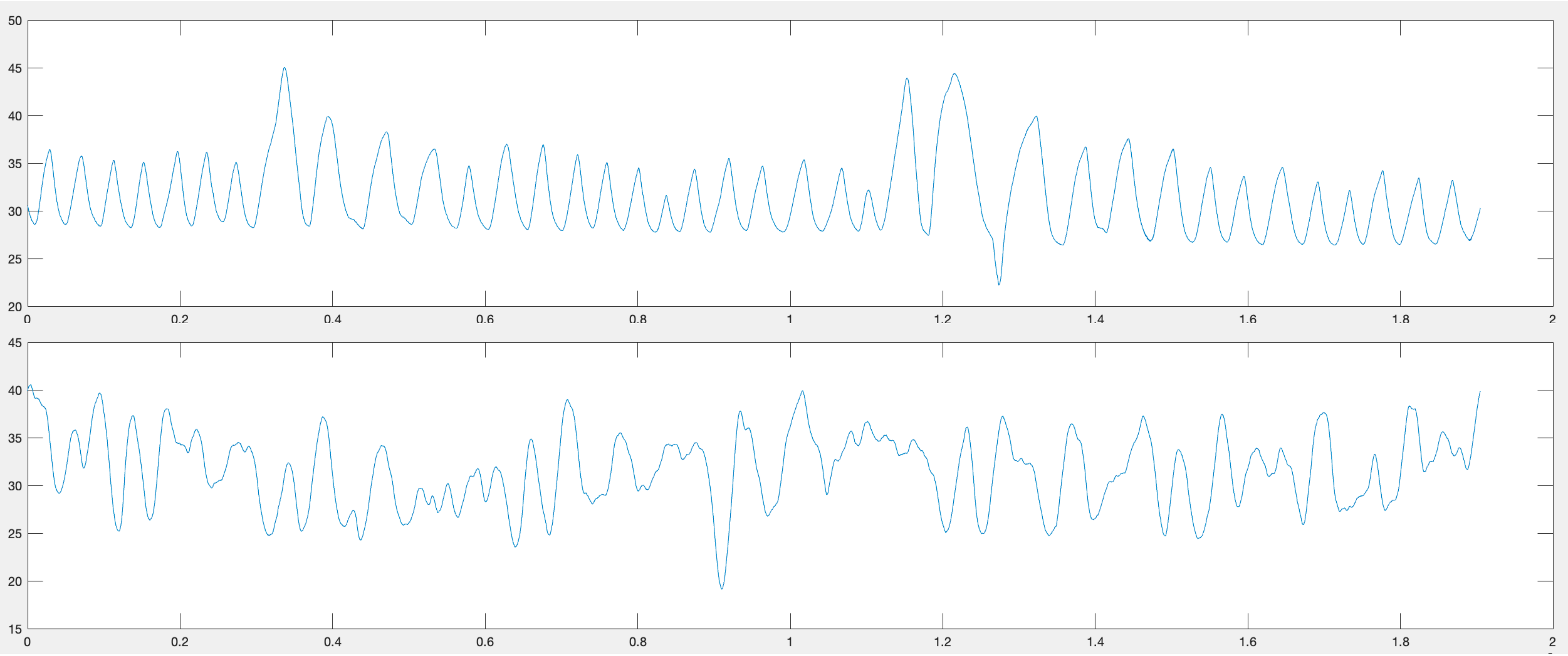

FFT -> permuted phase -> IFFT

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The hard part is choosing the framework for permutation